One-Dimensional Processing for Edge Detection using Hilbert Transform

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Abstract

This paper presents a new method for edge detection using one-dimensional processing. The Discrete Hilbert Transform of a Gaussian function is used as an edge detection filter. The image is smoothed using 1-D Gaussian along the horizontal (or vertical) scan lines to reduce noise. Detection filter is then used in the orthogonal direction, i.e., along vertical (or horizontal) scan lines to detect the edges. The proposed method differs from the traditional approaches based on 2-D operators in the sense that smoothing is done along one direction and the detection filter is applied along the orthogonal direction. The traditional 2-D operators smooth the image in all directions, thus resulting in some loss of edge information. Performance of the proposed method is compared with Canny's method for a set of real-world images. We also compare the performance of the proposed filter with the first order derivative of Gaussian (1-D Canny operator) for different 1-D edge profiles.

1 Introduction

Edges represent the discontinuities in the intensity in an image. Edges created by occlusions, shadows, roofs, textures etc., may have different local intensity profiles. Edge detection is a process that measures, detects, and localizes the changes in intensity. Edge detection is an important step in the process of segmentation because edges have the desirable property of drastically reducing the amount of data to be processed subsequently, while preserving information about the shapes of objects in the scene. Most vision systems use an edge description of the scene as input to high level image understanding processes. Edges are used to infer motion in a sequence of images [1]. Edge points or connected edge chains also play a crucial role in most passive stereopsis paradigms.

The most natural way of detecting changes in the image intensity is to take the first or second order derivatives and look for maxima or zero crossings in the output. Traditional edge detection operators like Robert, Sobel and Laplacian [2] were based on this observation. Piggio *et al* [3] reported that numerical differentiation is an ill-posed problem because its solution depends continuously on the data. Using regularization techniques they reported that it is necessary to preprocess the image data by a filter similar to the Gaussian. Marr and Hildreth [4] prefiltered the image by a Gaussian function before taking the second derivative in the form of a Laplacian. Canny [5] approximated an optimal finite length filter by the first derivative of the Gaussian. Diriche [6] proposed a recursive filtering structure that drastically reduces the computational effort required for smoothing, performing the first and second directional derivatives and obtaining the Laplacian of the image. Shen and Casten [7] showed that Infinite Symmetric Exponential Filter (ISEF) is an optimal filter for both mono and multi step edge detection.

Stochastic approaches like Markov random field models and autoregressive models consider the image as a random field and try to detect the changes of various statistical properties characterizing an edge, assuming mainly a step edge model. The main drawback of these techniques is that as the models gets more complicated, the computations increase. Surface fitting approaches [8], [9] for edge detection involve fitting polynomial or non-polynomial functions to estimate the first and second derivatives. But all these methods are also computationally expensive, although results are sometimes encouraging.

The problem of using Gaussian filter for smoothing the image is that it will disturb the edge localization, while suppressing noise. But smoothing is important, as the differential operators are sensitive to noise [2], [10]. Thus edge detection methods using 2-D operators smear the edge information, if a smoothing operation precedes it. To overcome this difficulty, a method of one-dimensional processing is proposed in [11], [12]. The advantage of 1-D processing is that smoothing is done in one direction, i.e., along rows (or columns) of an image, and the detecting filter is applied along the orthogonal direction i.e., along columns (or rows). This method of 1-D smoothing will not disturb the gray-level transitions of an edge in the orthogonal direction. Thus the desired location of the edge can be obtained efficiently by 1-D processing.

Hilbert Transform [13], [14] provides a means of separating signals based on *phase selectivity* and uses phase shifts between the pertinent signals to achieve the desired separation. In this paper, we extend the same logic to detect the change in image intensities by using Discrete Hilbert Transform of the Gaussian as an edge detection filter. This method of detection differs from the conventional operators in the sense that they detect the changes in intensity by performing the first or second order derivative operation on the image intensities.

This paper is organized as follows: Section 2 gives an introduction to the detection filter and illustrates the results on different types of 1-D edge profiles. In Section 3, we describe the method for edge detection. The performance of the proposed method on a set of real-world images is illustrated in Section 4. Finally, the paper is concluded in Section 5.

2 Edge Detection filter

We use the Discrete Hilbert Transform [14] of the Gaussian function as an edge detection filter. Let $g(n)$ be a finite Gaussian sequence of length L represented as:

$$g(n) = \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{n^2}{2\sigma^2}}, \qquad n = -L/2, ..., L/2 \quad (1)$$

where σ is the spatial spread of the Gaussian. Detection filter $h(n)$ can be obtained as:

$$h(n) = DHT\{g(n)\} \quad (2)$$

where, $DHT\{.\}$ represents the Discrete Hilbert Transform operator.

Fig.1(a) shows the shape of the Gaussian function for $\sigma = 6$ and the detection filter is shown in Fig.1(b). The first derivative of the Gaussian (1-D Canny operator) for the same value of σ is shown in Fig.1(c). Comparing both these filters, the proposed filter has slower rate of decay than 1-D Canny operator for the same σ. This characteristic of the proposed filter makes the performance in noisy cases better compared to 1-D Canny operator. This will be illustrated with results on 1-D noisy edge profiles at the end of this section.

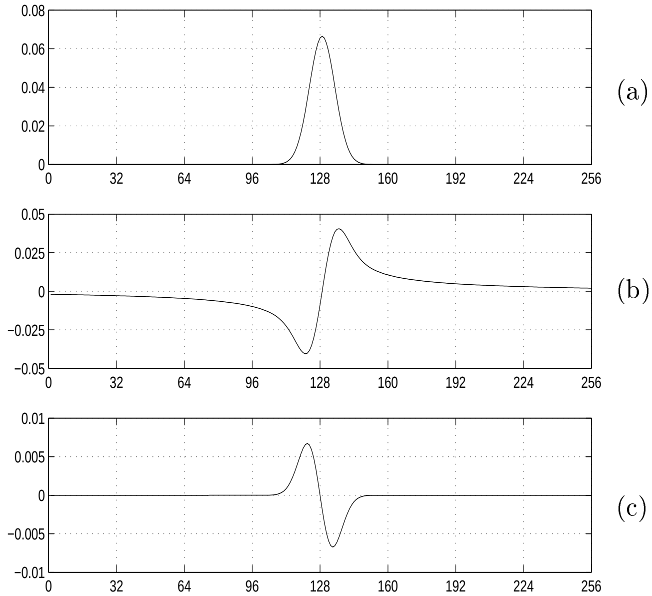

Figure 1: (a) Gaussian function for $\sigma = 6$. (b) The proposed detection filter. (c) First order derivative of Gaussian (1-D Canny operator)

In Figs.2 to 4, we present the comparative study of proposed filter and 1-D Canny operator for different types of edge profiles. Fig.2(a) shows a step edge, for which the convolved output of the proposed detection filter is shown in Fig.2(b). We will mark the center of an edge at a local minimum in the convolved output. For comparison, the result obtained by 1-D Canny operator is shown in Fig.2(c).

A "ridge" profile is shown in Fig.3(a) and the corresponding convolved outputs of the proposed filter and 1-D Canny operator are shown in Fig.3(b) and 3(c) respectively.

In Fig.4 is the signal has more than one sharp variation point. The signal shown in Fig.4(a) contains three different edge profiles and the convolved outputs of the proposed filter and 1-D Canny operator are shown in Fig.4(b) and 4(c). All the results of the comparative study presented in Figs.2 to 4, show that the qualitative nature of the output for both these operators are same for noise-free signals. The magnitude of the peak in the response of the proposed filter, at the location of the edge, is much larger compared to that of 1-D

Canny operator.

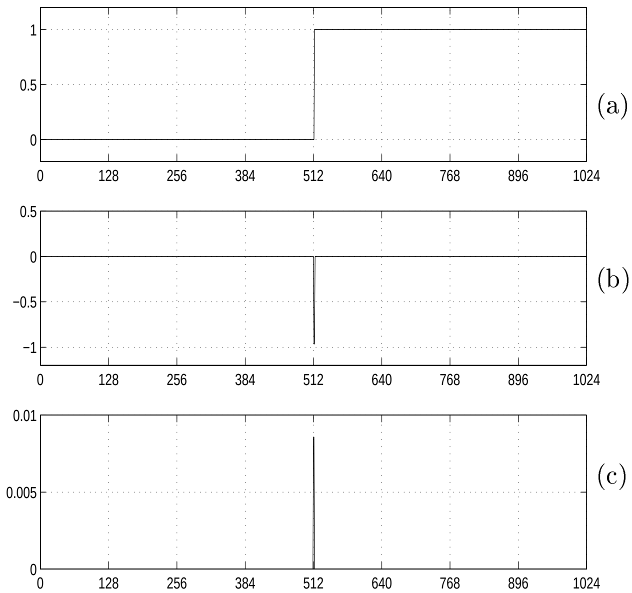

Figure 2: (a) An ideal step edge. (b) Convolved output of the proposed filter for $\sigma = 0.25$ (c) Convolved output of 1-D Canny operator for $\sigma = 0.25$

In Figs.5 and 6 the compares the performance of the proposed filter and 1-D Canny operator for two different levels of noise added to the step edge. Figs.5(b), 5(d), 6(b) and 6(d) shows the outputs of the proposed filter, whereas Figs.5(c), 5(e), 6(c) and 6(e) shows the outputs the 1-D Canny operator. In the location of the edge the peak is always more prominent than all the other local peaks. This is not the case with 1-D Canny operator, specifically for lower values of σ. This peak of the output which appears at the location of the edge will help in locating the edge better using the hysteresis thresholding strategy suggested by Canny [5]. This will be evident in the results with real-world images shown in Section 4.

3 Edge detection in images using 1-D Processing

In this section, we extend the method of 1-D edge detection using Hilbert filter to detect edges in 2-D images. This technique is similar to those proposed in [11], [12]. Image is first smoothed along all horizontal scan lines using 1-D Gaussian filter. The response of the 1-D Gaussian filter can be expressed as:

$$a(m, n_r) = i(m, n_r) * g(m) \tag{3}$$

where $*$ denotes the 1-D convolution operator, $g(m)$ represents the 1-D Gaussian filter, $i(m, n_r)$ represents

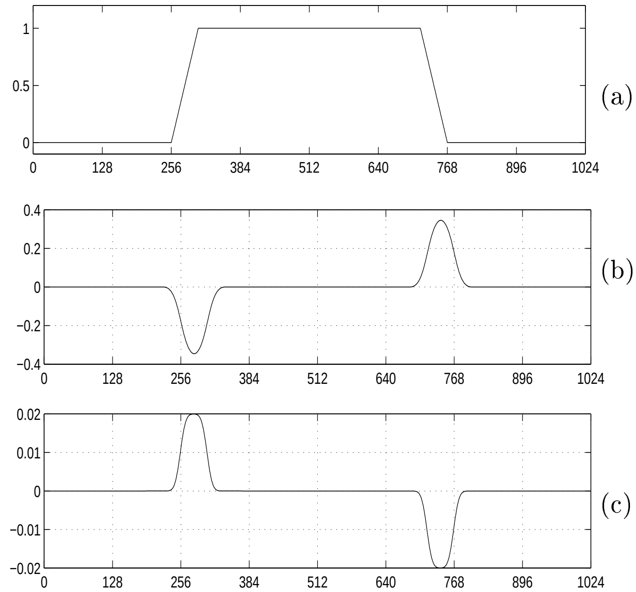

Figure 3: (a) A ridge profile. (b) Convolved output of the proposed filter for $\sigma = 8$. (c) Convolved output of 1-D Canny operator for $\sigma = 8$.

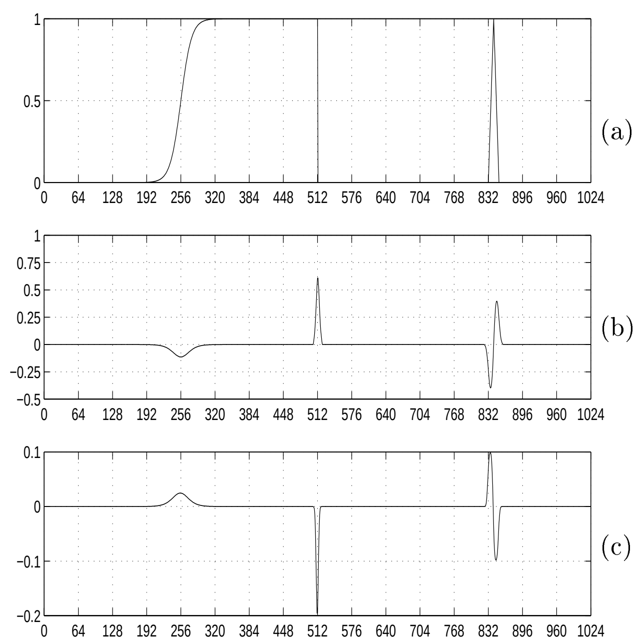

Figure 4: (a) A signal containing three sharp variation points. (b) Convolved output of the proposed filter for $\sigma = 2$. (c) Convolved output of 1-D Canny operator for $\sigma = 2$.

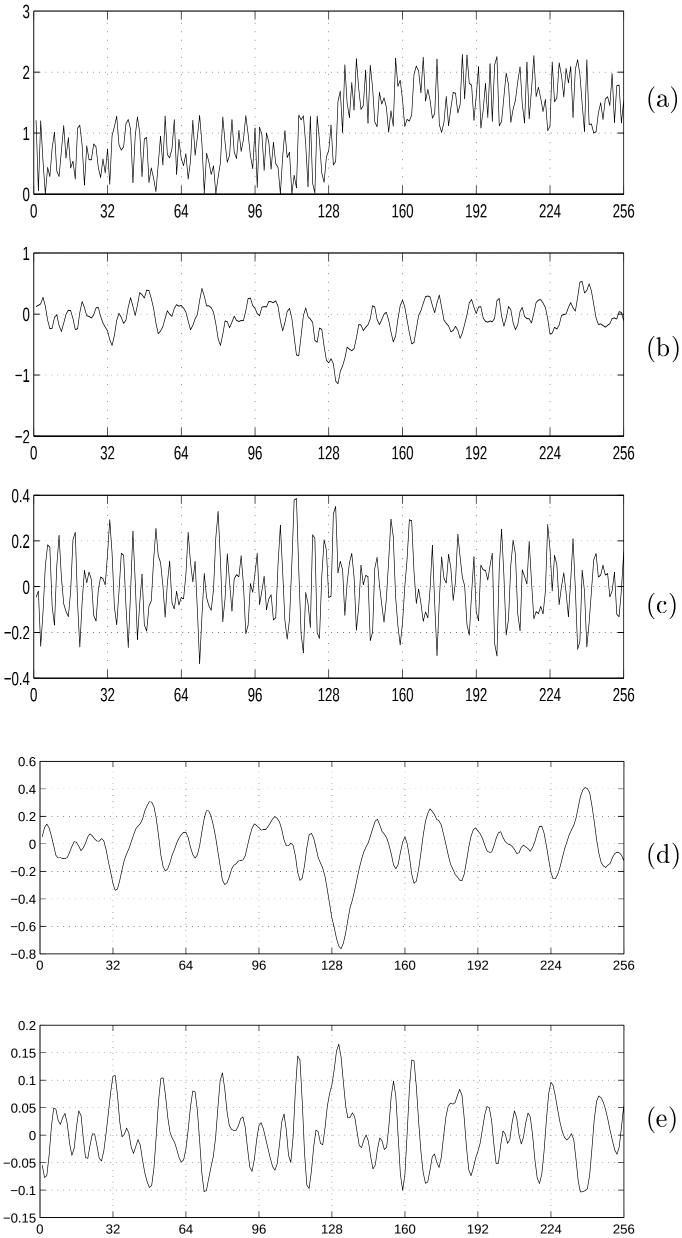

Figure 5: (a) Noisy step edge. (b), (d) Convolved outputs of the Proposed filter for $\sigma = 1, \sigma = 2$. (e), (f) Convolved outputs of 1-D Canny operator for $\sigma = 1$, $\sigma = 2$.

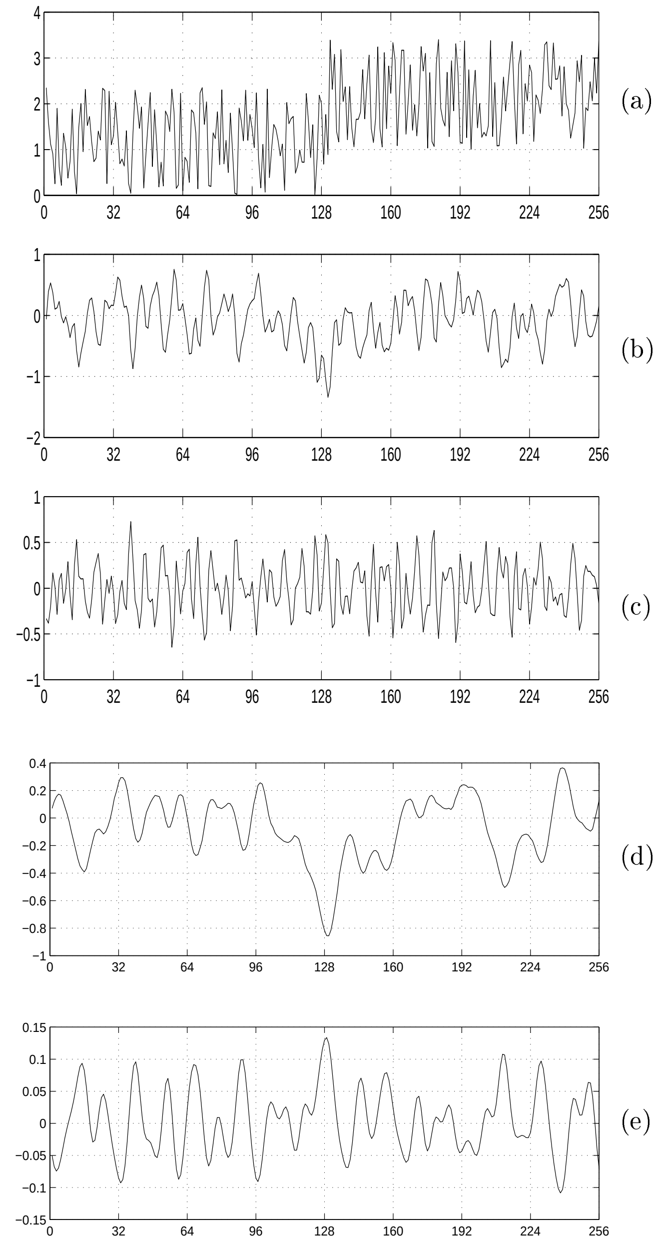

Figure 6: (a) Noisy step edge. (b), (d) Convolved outputs of the proposed filter for $\sigma = 1, \sigma = 3$. (c), (e) Convolved outputs of 1-D Canny operator for $\sigma = 1$, $\sigma = 3$.

the r^{th} row of the image i, and $a(m, n_r)$ is the corresponding filter response. The response is computed for all rows in the image to obtain $a(m, n)$.

For the 1-D Gaussian filter output $a(m, n)$, obtained from Eqn.(3) for all rows, the detection filter is applied along each column m_c to detect the edges oriented along the horizontal lines of the pixels. The result is given by

$$e(m_c, n) = a(m_c, n) * h(n) \quad (4)$$

where $h(n)$ denotes the detection filter and $a(m_c, n)$ represents the c^{th} column in the 1-D Gaussian filtered image $a(m, n)$. The resultant image $e(m, n)$ obtained by applying Eqn.(4) for all the columns produces the horizontal components of edge strengths in the image.

Similarly, the vertical components of edge strengths are detected by applying the 1-D smoothing operator along all vertical scan lines of the image and further processing with the proposed filter along the orthogonal direction (i.e., along horizontal scan lines of pixels).

Finally, the partial edge information obtained in horizontal and vertical directions are combined and thresholded to obtain the edge map of the original image. We have used the hysteresis thresholding strategy proposed by Canny [5] in our studies. The smoothing filter's σ is selected 1.3 times that of the detection filter. This choice provides the best results, which was observed from the preliminary studies. The choice of the spatial width of the Gaussian depends on the nature of the image and edges. In fact, for real-world images one should combine the results obtained by a set of operators with different σ's.

4 Results and Discussion

The performance of the proposed method is illustrated with a set of real-world images in this section. Fig.7 shows the edge detection results on the picture "Lena". Fig.7(b) shows the edge map obtained using the proposed method. For comparison, the edge information extracted by the 2-D Canny method with $\sigma = 0.85$ is shown in Fig.7(c). The σs are chosen to provide similar spatial spread of the Gaussian functions in both the methods, although the choice of σ was found to be not very critical. Hysteresis thresholding strategy is used in both the methods to obtain the edge map. A careful observation reveals that the edges obtained using the proposed method are more smoother, detailed and continuous than those obtained using the 2-D Canny's method.

The advantage of the proposed method is significant in the case of noisy images. The results of edge extraction using the proposed method and the 2-D Canny methods are shown in Figs.7(e) and 7(f), respectively, for the noisy image shown in Fig.7(d). The results clearly demonstrate the superior performance of the proposed method for noisy images. The main reason for this advantage is that noise smoothing is performed on the same pixels using only 1-D filtering, thus preserving the edge information in the orthogonal direction.

Fig.8 shows the edge detection results on an aerial image of an urban area. The edges extracted by the proposed method and 2-D Canny's method are shown in Figs.8(b) and 8(c), for the image shown in Fig.8(a). For the corresponding noisy image (see Fig.8(d)), the edge maps obtained by the proposed method and 2-D Canny's method are shown in Fig.8(e) and 8(f). Fig.8(a) shows a gray level image of an outdoor scene. The edges detected by the proposed method and 2-D Canny's method are shown in Figs.9(b) and 9(c). Figs.8(e) and 8(f) shows the results obtained by the proposed method and 2-D Canny methods for noisy image shown in Fig.9(d). We verified the comparative performace of the proposed method and 2-D Canny method for noisy images with different values of SNR. It was observed that the proposed method produced better performance, in general. Results of different SNRs are not shown due to lack of space.

The noise suppression and continuity of the edges obtained by the proposed method are better than that obtained by the 2-D Canny's method. The proposed method gives also significant computational advantage compared to the 2-D Canny's method. The proposed method takes only about $1/10^{th}$ the time required 2-D Canny's method.

5 Conclusion

In this paper we have proposed a new method for edge detection based on Hilbert transform. It exploits the 1-D property of edges by smoothing along one direction and detecting the edge information along the orthogonal direction by using the Hilbert transform of the Gaussian as an edge detection filter. Comparison of the results with 2-D Canny's method show that proposed method works better for noise-free as well as noisy images, besides providing the significant computational advantage due to 1-D processing. Results of the proposed method will be useful for applications like GIS, object recognition and machine inspection.

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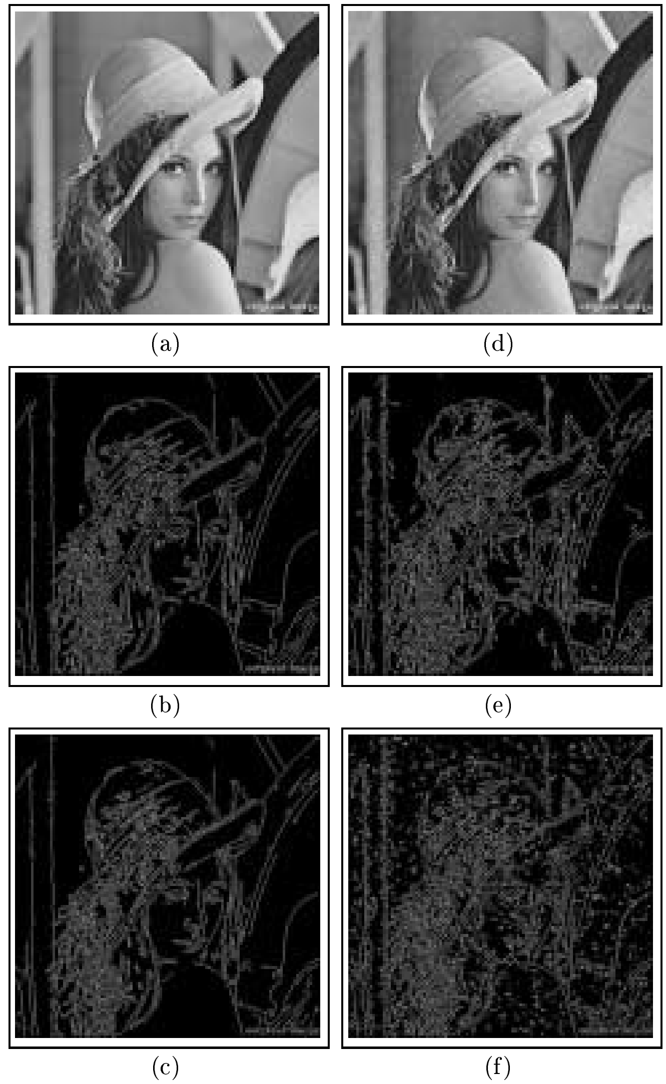

Figure 7: (a) Gray level image ”Lena”. (b) Edge map obtained by the proposed method, for the image shown in (a). (c) Edge map obtained by 2-D Canny method, for the image shown in (a). (d) Noisy image of (a). (e) Edge map obtained by the proposed method, for the image shown in (d). (f) Edge map obtained by 2-D Canny operator, for the image shown in (d).

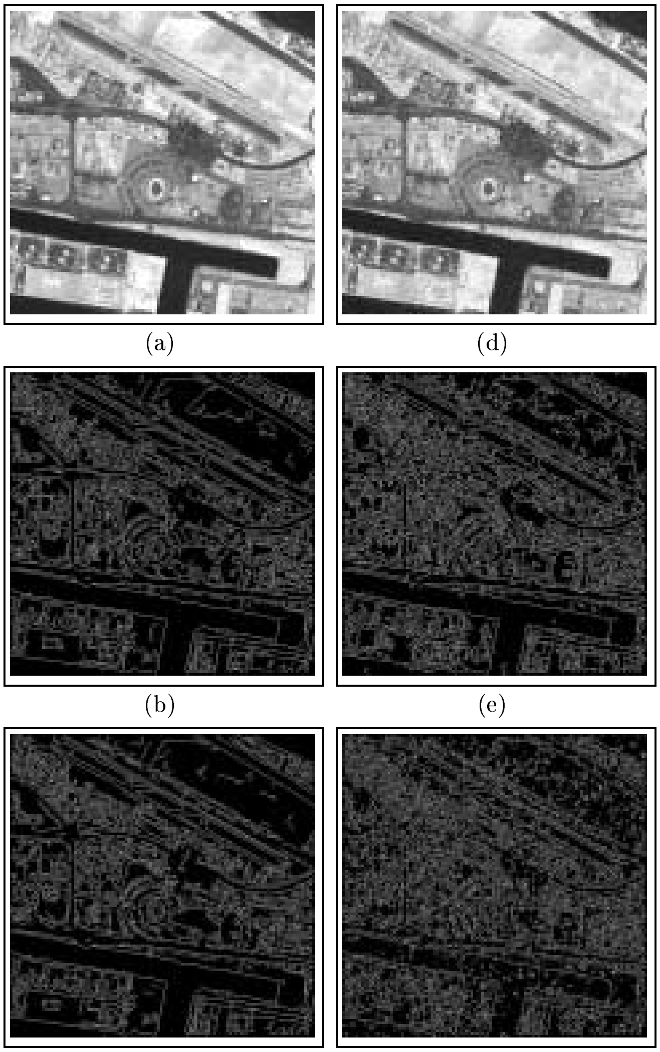

Figure 8: (a) Aerial image of an urban area. (b) Edge map obtained by the proposed method, for the image shown in (a). (c) Edge map obtained by 2-D Canny method, for the image shown in (a). (d) Noisy image of (a). (e) Edge map obtained by the proposed method , for the image shown in (d). (f) Edge map obtained by 2-D Canny operator, for the image shown in (d).

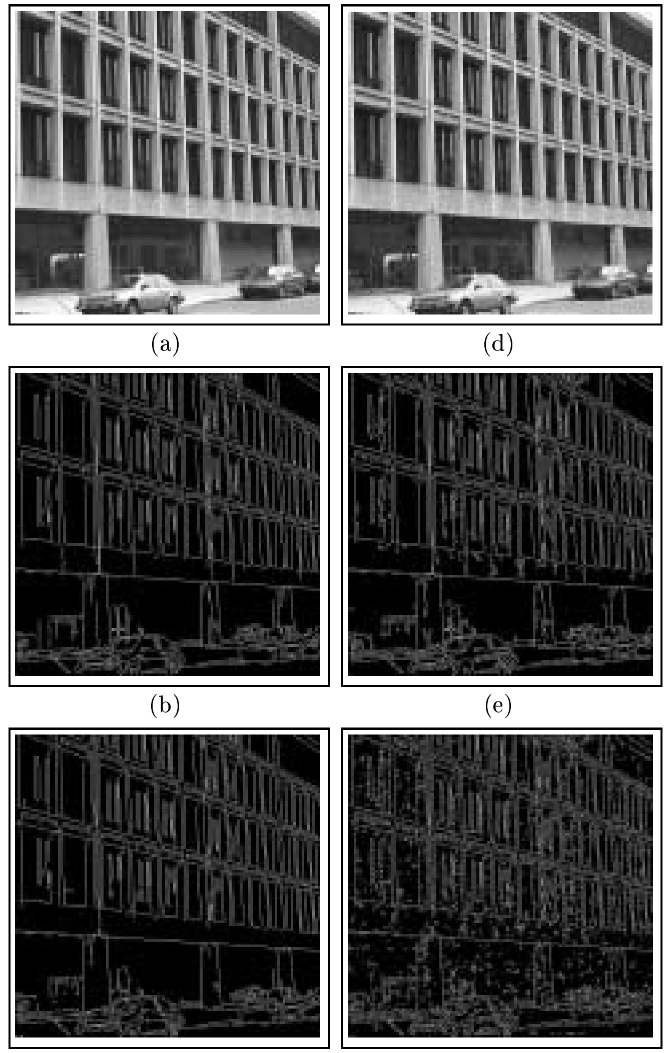

Figure 9: (a) Gray level image of an outdoor scene. (b) Edge map obtained by the proposed method, for the image shown in (a). (c) Edge map obtained by 2-D Canny method, for the image shown (a). (d) Noisy image of (a). (e) Edge map obtained by the proposed method, for the image shown (d). (f) Edge map obtained by 2-D Canny operator, for the image shown (d).