Simulation studies for the performance analysis of the reconstruction of a line using stereoscopic projections

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Abstract

The process of reconstruction of a line in 3-D space using stereoscopic projections obtains the set of parameters representing the line. This method is widely used in many applications of 3-D object recognition and machine inspection. However in certain applications which require a large degree of accuracy, a performance analysis of the process of reconstruction in the presence of noise in the image planes is necessary. In this paper a set of inverse perspective equations for reconstruction of a line in 3-D space are derived based on coplanarity equations. Simulation studies were conducted to observe the effect of noise on errors in the process of reconstruction. Performance analysis illustrating the effect of noise and parameters of imaging setup on errors in reconstruction are presented. Smaller resolution of the image and certain geometric conditions of the line and imaging setup produce poor performance in reconstruction. Results of this study are useful for the design of an optimal stereo-based imaging system for best reconstruction with minimum error.

Key words: *Perspective views, Reconstruction, Stereo vision, Least square regression, Error Analysis, Inverse perspective equations, Parameter, Noise.*

1 Introduction

Stereo vision is an effective method to estimate depth and structural parameters of 3-D objects from a pair of 2-D images [15] and [18]. Points, lines, conics, general curves and surfaces are the set of primitives which are used to model and represent 3-D objects. Hence reconstruction of these features has received great attention by many researchers in the field of vision [1], [3], [4], [8], [10], [12], [13], [14], [16], [17], [18], [19] and [20]. In the case of a line, the most widely used method is to reconstruct a pair of planes in 3-D from the pair of projections, which pass through the respective centre of projections of the cameras [2]. Intersection of these two planes gives the parameters of the line. Xie and Thonnat in [19] reconstruct a line by obtaining the two end points of the line from their corresponding pair of projections. Both these concepts are based on triangulation.

Image digitization and presence of noise in the imaging set up affect the accuracy of the process of reconstruction. This is crucial when accurate estimates are necessary for a particular application. Errors in reconstruction depend on the parameters of the line as well as on the geometry of the imaging setup. To provide an optimal design of the imaging setup (sensor and geometry) for best reconstruction as well as to evaluate the robustness of the system, it is necessary to analyze the performance of the system in the presence of noise. This is possible with the help of error analysis using simulation studies only, as it is impossible to generate a large set of real world data for all possible scenaries of the imaging setup and properties (parameters) of the line to be reconstructed.

Blostein and Haung [3] present an error analysis method in the case of stereo for obtaining 3D point positions. They derive closed form expressions for the probability distribution of error in position along each coordinate direction namely, horizontal, vertical and range. Based on this methodology the probability that range error dominates over errors in the point's horizontal or vertical position has been determined. Rodriguez and Aggarwal [14] have derived a probability density function of the range estimation error and the expected value of the range error magnitude in terms of the various design parameters of the stereo imaging setup. They have given a stochastic analysis of the quantization error in the stereo imaging setup. Simulation experiments were conducted for a stereo imaging system to validate the model. Hartley [8] has given a fast algorithm for projective reconstruction of a scene consisting of a set of lines from three or more images with uncalibrated cameras. The algorithm is rapid and quite reliable, provided the degree of error in the image-to-image correspondence is not excessive. He has also demonstrated that for images with higher resolution where the relative errors may be expected to be smaller, the algorithm shows enhanced performance.

The work presented in this paper deals with the performance analysis of the process of reconstruction of a line in 3-D space from two arbitrary perspective views. Coplanarity equations are derived for the projection of a line in 3-D space on the 2-D image plane. A set of inverse perspective equations representing the analytical relationship between the parameters of a line in 3-D space and the parameters of the corresponding pair of projections on the 2-D image planes have been derived based on coplanarity equations. Parameters of a line in a 2-D image plane are obtained using linear least square regression of the pixel coordinates of the line. This method of reconstruction is different from the typical methods of obtaining the 3-D line using the intersection of two planes [2] and obtaining the pair of end points of a line [19]. Typical applications of this methodology are accurate reconstruction of the trajectory of a ball in the field of sports (Cricket, Tennis etc.), estimation of the flight path of missiles or meteorides in space, non-contact method of measurement of the length of thin rods, cables and objects with linear edges or segments.

Performance analysis of the method of reconstruction is based on simulation studies. Noise with Gaussian distribution is added to the pixel coordinates of the projections of the line on the pair of image planes. This simulates the effect of noise in the sensor and signal acquisition system as well as errors in the preprocessing tools used to detect line segments from gray level images. Error between the original and the reconstructed line in 3-D space is estimated and used as a criteria to analyze the performance of the system. These results provide an optimal range of values of the parameters to be used for the design of a stereo-based imaging system for best reconstruction. Certain conditions of the viewing geometry where the reconstruction process has a poor performance are also obtained from these studies.

In the following section, the basic set of equations to formulate the problem are provided and the problem of reconstruction is introduced. In section 3, the methodology for the process of reconstruction of a line is discussed in detail. Section 4 gives a performance analysis of the process of reconstruction in the presence of noise in the image planes. Section 5 concludes with discussions and contributions.

2 Basic Model of Imaging Setup

The collinearity equations represent the mathematical process of image formation, linking the co-ordinates of a point on an object in 3-D space to the corresponding co-ordinates of its projection in the 2-D image plane. The collinearity equations are derived using the criteria that all the three points, namely, the center of perspective projection, the image point and the object point lie on the same straight line. The detailed imaging set-up using two cameras is shown in Figure 1, where f_1 and f_2 are the focal lengths of the first and the second cameras respectively. The relation between the coordinates of the point $W(x_w, y_w, z_w)$ and that of the image point $P_1(X_1, Y_1, f_1)$ is given by the perspective equation [9]:

$$X_1 = f_1 \frac{x_w}{z_w} \quad , \qquad Y_1 = f_1 \frac{y_w}{z_w} \tag{1}$$

The 3-D co-ordinates of point $W(x_w, y_w, z_w)$ with respect to second camera C_2, is given by

$$\begin{bmatrix} x'_w \\ y'_w \\ z'_w \end{bmatrix} = \lambda \begin{bmatrix} \cos\alpha_1 & \cos\beta_1 & \cos\gamma_1 \\ \cos\alpha_2 & \cos\beta_2 & \cos\gamma_2 \\ \cos\alpha_3 & \cos\beta_3 & \cos\gamma_3 \end{bmatrix} \begin{bmatrix} x_w - x_d \\ y_w - y_d \\ z_w - z_d \end{bmatrix} \tag{2}$$

where the rotation matrix in equation (2) is given in [6], λ is a scale factor between the two reference frames and without loss of generality this is considered to be 1, in this work. Using equation (2), the relation between the object space point $W(x_w, y_w, z_w)$ and the image point $P_2(X_2, Y_2, f_2)$ is given by the perspective equations [9]:

$$\begin{aligned} X_2 &= f_2 \,[(x_w - x_d)\cos\alpha_1 + (y_w - y_d)\cos\beta_1 \\ &+ (z_w - z_d)\cos\gamma_1][(x_w - x_d)\cos\alpha_3 \qquad (3) \\ &+ (y_w - y_d)\cos\beta_3 + (z_w - z_d)\cos\gamma_3]^{-1} \\ Y_2 &= f_2 \,[(x_w - x_d)\cos\alpha_2 + (y_w - y_d)\cos\beta_2 \\ &+ (z_w - z_d)\cos\gamma_2][(x_w - x_d)\cos\alpha_3 \qquad (4) \\ &+ (y_w - y_d)\cos\beta_3 + (z_w - z_d)\cos\gamma_3]^{-1} \end{aligned}$$

Equations (1), (3) and (4) are the collinearity equations for a pair of arbitrary perspective views [9]. The process of reconstruction of a line in 3-D space involves the formulation of a set of inverse perspective equations using these collinearity equations. This will give the direction cosines, namely l, m, n, of the line in 3-D space as well as the coordinates (x_w, y_w, z_w) of a point W on the line. This method is discussed in detail in section 3.

3 Process of reconstruction

Two separate 2-D digital images of a line in 3-D space are obtained using a pair of cameras as illustrated in figure 2. The process of matching a pair of corresponding projected line segments in these two images is not dealt with in this paper, as we are primarily concerned with the mathematical formulation of the inverse perspective equations for reconstruction of a line in 3-D space as well as its performance analysis. The correspondence between the pair of projections of a line is thus assumed to be established in this work. In the following section, a new methodology of reconstruction of a line in 3-D space, from two arbitrary perspective images is given.

3.1 Estimation of line parameters

Let, l, m, n be the direction cosines of a line segment S in 3-D space passing through the point $W(x_w, y_w, z_w)$. Figure 2 shows the projections of this line on the pair of image planes I_1 and I_2. Let s_1 and s_2 be the corresponding line segments on the pair of image planes. ρ and Θ are the parameters which represent a line segment in 2-D space, in normal form, as

$$x \cos\Theta + y \sin\Theta = \rho \tag{5}$$

The parameters of the line (l, m, n, x_w, y_w, z_w) in 3-D space and the respective inclination parameters (Θ_1 and Θ_2) of the corresponding line segments s_1 and s_2 are related by the following *coplanarity equations*

$$\frac{1}{\tan\Theta_1} = -\frac{z_w m - y_w n}{z_w l - x_w n}, \tag{6}$$

$$\frac{1}{\tan\Theta_2} = -\frac{N_{yz} l + N_{zx} m + N_{xy} n}{D_{yz} l + D_{zx} m + D_{xy} n}, \tag{7}$$

where $N_{xy}, N_{yz}, N_{zx}, D_{xy}, D_{yz}, D_{zx}$ are functions of elements of rotational and translation matrix in equation (2). The process of reconstruction of a line presented in this paper is based on inverse perspective equations which are derived from the collinearity equations (1) - (4) and coplanarity equations (6) and (7). This process is described below.

The pixel co-ordinates of a line segment s_i, on image plane i $(i = 1, 2)$, are used to obtain the parameters (ρ_i, Θ_i) of the line using linear least square regression. Linear least square regression is used to minimize the effect of discretisation and noise on the 2-D digital image plane. Once Θ_1 and Θ_2 are obtained from the corresponding pair of projections of a line using least square regression, the next process is to obtain the parameters of the line (x_w, y_w, z_w, l, m, n) in 3-D space. The steps to obtain the 3-D coordinates (x_w, y_w, z_w) only of a point W, on the line S, is given below:

(i) Obtain the equation of the plane $O'PQ$ (see figure 3) passing through the point O' and line s_2 on the image plane I_2 as, $A_2 x + B_2 y + C_2 z + 1 = 0$.

(ii) Select a point $P_1(X_1, Y_1, f_1)$ as the centroid of the line s_1, on image plane I_1.

(iii) Obtain the equation of the 3D line OP_1.

(iv) Intersection of line OP_1 and the plane $O'PQ$ gives the coordinates of a point $W(x_w, y_w, z_w)$ on line S, as:

$$(x_w, y_w, z_w) = \frac{-1}{A_2 X_1 + B_2 Y_1 + C_2 f_1}(X_1, Y_1, f_1) \tag{8}$$

Using the constraint $l^2 + m^2 + n^2 = 1$ and equations (6) - (8), we get the solution of the inverse perspective projection of a line as:

$$(l, m, n) = \left(K_l n, \quad K_m n, \quad \frac{1}{\sqrt{1 + K_l^2 + K_m^2}} \right) \tag{9}$$

where

$$\begin{bmatrix} K_l \\ K_m \end{bmatrix} = \frac{1}{Dr} \begin{bmatrix} D_{zx} C_2 + N_{zx} S_2 & z_w S_1 \\ -(D_{yz} C_2 + N_{yz} S_2) & -z_w C_1 \end{bmatrix} \begin{bmatrix} x_w C_1 + y_w S_1 \\ D_{xy} C_2 + N_{xy} S_2 \end{bmatrix} \tag{10}$$

where

$$Dr = z_w \{ (D_{zx} \mathcal{C}_2 + N_{zx} \mathcal{S}_2) \mathcal{C}_1 - (D_{yz} \mathcal{C}_2 + N_{yz} \mathcal{S}_2) \mathcal{S}_1 \}$$

$$\mathcal{S}_1 = \sin\Theta_1, \ \mathcal{S}_2 = \sin\Theta_2, \ \mathcal{C}_1 = \cos\Theta_1, \ \mathcal{C}_2 = \cos\Theta_2$$

The parameters (l, m, n, x_w, y_w, z_w) of the reconstructed line in 3-D space are obtained using the inverse perspective equations (8) - (10). The parameters of the imaging setup required as input are $f_1, \ f_2, \ x_d, \ y_d, \ z_d$ and $(\alpha_i, \ \beta_i, \ \gamma_i), \ i = 1, 2, 3$.

3.2 Special cases of inverse perspective equations:

The inverse perspective equations for the reconstruction of a line in 3-D space use a pair of equations (10). This pair of equations use parameters Θ_1 and Θ_2 of the 2-D line segments s_1 and s_2 on the pair of image planes respectively. Equations (9) - (10) can not be used for reconstruction, when either or both the terms K_l and K_m, used in equation (9) become indeterminant. Similarly, when the 3-D line S projects to only a single point in one or both the image planes, the equations for reconstruction suggested in the previous section can not be used. This section provides solutions for such special cases of inverse perspective equations.

Case I:
In equation (10), if

$$z_w\{(D_{zx}\mathcal{C}_2 + N_{zx}\mathcal{S}_2)\mathcal{C}_1 - (D_{yz}\mathcal{C}_2 + N_{yz}\mathcal{S}_2)\mathcal{S}_1\} = 0,$$

then the solution for reconstruction is:

$$(l, m, n) = (-\sin\Theta_1, \cos\Theta_1, 0)$$

Case II:
If line S projects to a single point $P_1(X_1, Y_1)$ in the first image plane only, then the solution for reconstruction can be given as:

$$(l, m, n) = \frac{1}{(X_1^2 + Y_1^2 + f_1^2)}(X_1, Y_1, f_1)$$

Case III:
If line S projects to a single point $P_2(X_2, Y_2)$ in the second image plane only, then the solution for reconstruction is given by:

$$\begin{bmatrix} l \\ m \\ n \end{bmatrix} = \begin{bmatrix} \cos\alpha_1 & \cos\alpha_2 & \cos\alpha_3 \\ \cos\beta_1 & \cos\beta_2 & \cos\beta_3 \\ \cos\gamma_1 & \cos\gamma_2 & \cos\gamma_3 \end{bmatrix} \begin{bmatrix} l' \\ m' \\ n' \end{bmatrix}$$

where

$$(l', m', n') = \frac{1}{(X_2^2 + Y_2^2 + f_2^2)}(X_2, Y_2, f_2)$$

4 Error Analysis of the reconstruction process

Error analysis for the methodology of reconstruction is essential to estimate the performance of the system. Noise with Gaussian distribution is added to the pixel coordinate values of the projection of a line on the image plane. This perturbs the location of pixels forming the projection of the line on the image plane, simulating the effect of noise. The level of noise is characterized by the variance, σ, of the Gaussian distribution, which is considered to be in the range [0-10]. Bresenham's algorithm [5] was used to obtain the discrete set of points as projections of the line on the pair of image planes. We use the following pair of criteria for estimating the error in reconstruction:

(i) Error in orientation (angle between the original and the reconstructed lines):

$$\theta_{\mathbf{e}} = \cos^{-1}(l_1 l_2 + m_1 m_2 + n_1 n_2)$$

(ii) Error in position (Shortest Distance between the two lines):

$$\mathbf{D_e} = (x_{w1} - x_{w2})\, l'' + (y_{w1} - y_{w2})\, m'' + (z_{w1} - z_{w2})\, n''$$

where
$(l'', m'', n'') =$

$$\frac{(m_1 n_2 - m_2 n_1, n_1 l_2 - n_2 l_1, l_1 m_2 - l_2 m_1)}{\sqrt{(m_1 n_2 - m_2 n_1)^2 + (n_1 l_2 - n_2 l_1)^2 + (l_1 m_2 - m_1 l_2)^2}}$$

where (l_1, m_1, n_1) and (l_2, m_2, n_2) are direction cosines of the original and the reconstructed lines passing through points (x_{w1}, y_{w1}, z_{w1}) and (x_{w2}, y_{w2}, z_{w2}) respectively.

Using simulation studies, these errors are estimated for different combinations of the geometry of the imaging setup, parameters of the line and levels of noise added to the image feature (line). Results of performance studies, shown in figures 4-9, are obtained by taking the mean of 100 different observations of simulated experiments conducted using the parameters as specified in each corresponding figure. Each of the 3-D plots in figures 4-9, illustrate that errors vary non-linearly with respect to the level of noise in the image planes, and parameters of the imaging setup and the line being reconstructed. In order to provide a proper visualization of such non-linear multivariate error functions, only one of the parameters of the imaging setup or line is altered, keeping all others constant.

Figures 4, 5 and 6 illustrate the effect of the direction cosines of the line on the error in reconstruction. For the graphs in figures 4, 5 and 6, the values for the various parameters of the imaging setup are chosen to be : $x_w = y_w = 10.0$, $z_w = 200.0$, $x_d = 10.0$, $y_d = 20.0$, $z_d = 30.0$, $f_1 = f_2 = 1.0$, $N = 320$ (N is the resolution of the digital image), $\alpha_1 = \beta_2 = \frac{2\pi}{3}$ and $\gamma_3 = \frac{\pi}{6}$ (given α_1, β_2, and γ_3, the other six Eulerian angles [6] are found using the constraint of the orthogonal matrix used in equation (2)). Only one component of the direction cosines of the line is altered, keeping the other two identical. It is observed from figures 4, 5 and 6 that the errors in reconstruction are large when the values of the direction cosines are near the extreme limits of the range [0-1]. Errors in orientation of the line, $\theta_{\mathbf{e}}$, was found to be large compared to that of the position, $\mathbf{D_e}$. In figure 7, the value of N (image resolution) is varied from 30 to 350 and the direction cosines are $l = m = n = \frac{1}{\sqrt{3}}$. Figure 7 shows that errors are appreciably high when the resolution of the image is low (i.e., $N \simeq 32$). Errors are negligible when the image resolution is more than 200.

For the plots in figures 8 and 9, the value of z_w is varied from 100 to 500 and $l = m = n = \frac{1}{\sqrt{3}}$. In figures 8 and 9, the parameters of the viewing geometry, α_i, β_i and γ_i, $i = 1, 2, 3$, are changed simultaneously in such a manner that the 3-D line lies within the common field of view of both the cameras. Figure 8 shows that error in orientation is very high when the value of depth z_w is greater than 250 and negligible when less than 200. Figure 9 shows that the error in position, $\mathbf{D_e}$ is negligible when the depth z_w is less than 350.

All 3-D graphs in figures 4-9, show that for noise levels in the range $0 \le \sigma \le 10$, the errors $\theta_{\mathbf{e}}$ and $\mathbf{D_e}$ are mostly within acceptable limits ($0^o \le \theta_e \le 10^o$ and $0 \le D_e \le 2$ respectively), except for certain specific conditions of the viewing geometry and orientation and position of the line. The errors are negligible for small levels of noise in the range $0 \le \sigma \le 2$, which is realistic. Negligible error in the process of reconstruction of a line in the noise free case ($\sigma = 0$), is the result of digitization (sampling) of spatial coordinate values in the digital image plane. This is more if the image has low resolution as illustrated in figure 7. As the line tends to be parallel to one of the principle coordinate axis ($l, m, n \simeq 0$ or 1), the errors in reconstruction are large. With increase in the level of noise in the image planes the error in orientation, $\theta_{\mathbf{e}}$, increases rapidly than error in position, $\mathbf{D_e}$, of the line. Hence based on our studies, for best results we recommend the following range of values of the parameters of the imaging setup and line to be reconstructed, $0.2 \le l \le 0.9$, $0.2 \le m \le 0.7$, $0.2 \le n \le 0.8$, $z_w < 250$ and $N > 200$. Other parameters of the line and viewing geometry do not affect the accuracy of the reconstruction process to a great extent. The dimensions of all distances used in the simulation studies are normalized with respect to the focal length of the cameras which is considered to be unity.

5 Conclusion

In this paper, a new method of reconstruction of a line in 3-D space from two arbitrary perspective views has been discussed. Inverse perspective equations to obtain the parameters of a line in 3-D space from two arbitrary perspective projections have been derived in this paper, based on coplanarity equations. The process of reconstruction may be used for any automatic inspection or quantitative measurements of 3-D objects with regular geometrical features, for real world vision systems.

A rigourous performance analysis has been provided, using simulation studies, to illustrate the effect of noise and parameters of the line and the imaging setup on errors in reconstruction. Results of simulation studies presented in this paper are useful for the design of an imaging system for accurate reconstruction of lines or edges of 3-D objects, as well as to evaluate the performance of such a system. Smaller resolution of the image, larger depth and certain orientations of the line in 3D have been found to produce a poor performance in the process of reconstruction. It is observed that for larger levels of noise present in the image planes, the errors in the orientational parameters of the reconstructed line are much larger than that in the positional parameters.

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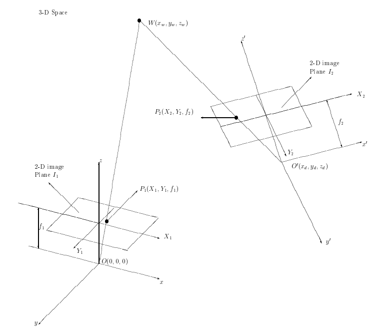

Figure 1: Imaging system with two arbitrary perspective views of a point W in 3-D space. f_1 and f_2 are the focal lengths of first and second cameras respectively.

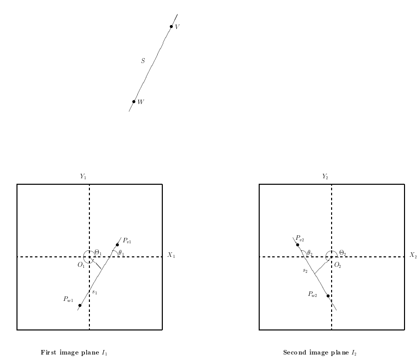

Figure 2: Two arbitrary perspective projections s_1 and s_2 of a line segment S in 3-D space.

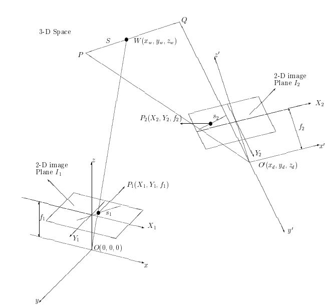

Figure 3: Given a pair of projections s_1 and s_2 for a line S in 3-D, the co-ordinates of an arbitrary point W on the line S can be obtained by selecting a point P_1 on line s_1. The intersection of line OP_1 and plane $O'PQ$ in 3-D, gives the coordinates of the point W corresponding to the point P_1 on image plane I_1

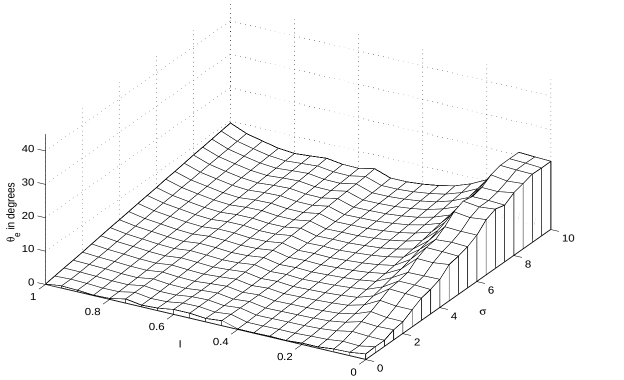

Figure 4: 3-D Plot showing the errors in reconstruction, l varies from $[0-1]$, $m = n = \sqrt{(1-l^2)/2}$, σ varies from $[0-10]$, $\alpha_1 = \beta_2 = \frac{2\pi}{3}$, $\gamma_3 = \frac{\pi}{6}$, $f_1 = f_2 = 1.0$, $x_d = 10.0, y_d = 20.0, z_d = 30.0$, $x_w = y_w = 10.0, z_w = 200.0$ and $N = 320$.

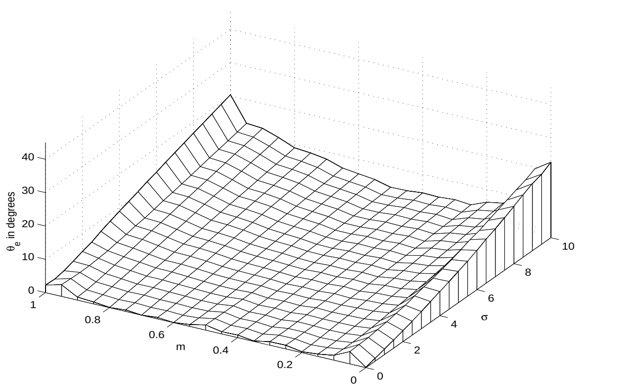

Figure 5: 3-D Plot showing the errors in reconstruction, m varies from $[0-1]$, $l = n = \sqrt{(1-m^2)/2}$, σ varies from $[0-10]$, $\alpha_1 = \beta_2 = \frac{2\pi}{3}$, $\gamma_3 = \frac{\pi}{6}$, $f_1 = f_2 = 1.0$, $x_d = 10.0, y_d = 20.0, z_d = 30.0$, $x_w = y_w = 10.0, z_w = 200.0$ and $N = 320$.

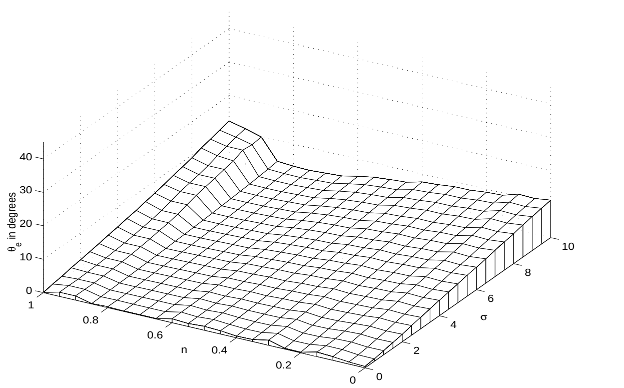

Figure 6: 3-D Plot showing the errors in reconstruction, n varies from $[0-1]$, $l = m = \sqrt{(1-n^2)/2}$, σ varies from $[0-10]$, $\alpha_1 = \beta_2 = \frac{2\pi}{3}$, $\gamma_3 = \frac{\pi}{6}$, $f_1 = f_2 = 1.0$, $x_d = 10.0, y_d = 20.0, z_d = 30.0$, $x_w = y_w = 10.0, z_w = 200.0$ and $N = 320$.

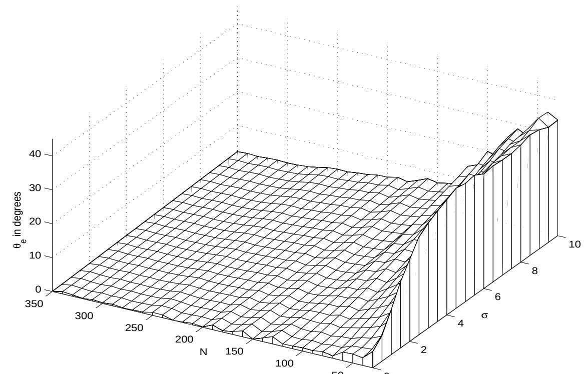

Figure 7: 3-D Plot showing the errors in reconstruction, N varies from $[30-350]$, $l = m = n = 1/\sqrt{3}$, σ varies from $[0-10]$, $\alpha_1 = \beta_2 = \frac{2\pi}{3}$, $\gamma_3 = \frac{\pi}{6}$, $f_1 = f_2 = 1.0$, $x_d = 10.0, y_d = 20.0, z_d = 30.0$, $x_w = y_w = 10.0$ and $z_w = 200.0$.

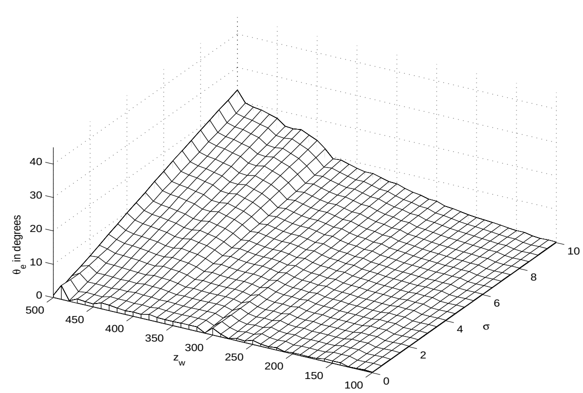

Figure 8: 3-D Plot showing the errors in reconstruction, z_w varies from $[100-500]$, $l = m = n = 1/\sqrt{3}$, σ varies from $[0-10]$, $\alpha_1 = \beta_2 = \frac{2\pi}{3}$, $\gamma_3 = \frac{\pi}{6}$, $f_1 = f_2 = 1.0$, $x_d = 10.0, y_d = 20.0, z_d = 30.0$, $x_w = y_w = 10.0$ and $N = 640$.

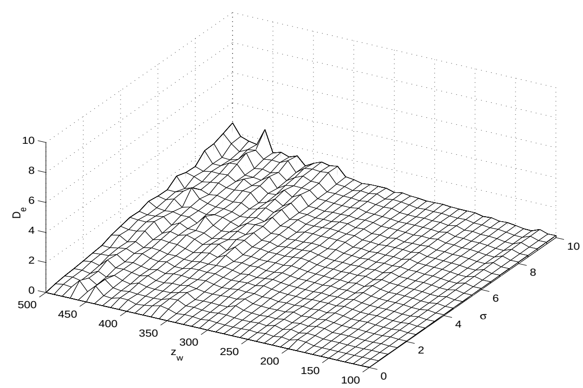

Figure 9: 3-D Plot showing the errors in reconstruction, z_w varies from $[100-500]$, $l = m = n = 1/\sqrt{3}$, σ varies from $[0-10]$, $\alpha_1 = \beta_2 = \frac{2\pi}{3}$, $\gamma_3 = \frac{\pi}{6}$, $f_1 = f_2 = 1.0$, $x_d = 10.0, y_d = 20.0, z_d = 30.0$, $x_w = y_w = 10.0$ and $N = 320$.