Edgeless Active Contours for Natural Color Images based on Angular Mapping and Level Sets

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Abstract: *We propose an angular map driven active contouring based on curve evolution using level sets to extract object from natural color images. The constraint parameters to stop the intrinsically evolving curve on the angular map of the color space (RGB used) are derived based upon a modified minimal partition problem.*

Keywords: angular color map, active contouring, object detection, level sets, refinement force.

1. Introduction:

Edge based active contouring for natural images, is a challenging task due to two problems. 1) The gradients at the boundaries are not uniform, which suggests that the constraint on curve evolution cannot be derived from gradient information 2) No unique method exists which has similar distinguishing feature of object from its background in the image like that of human eye. This has motivated us to explore suitable constraints for curve evolution method for vector valued natural images, which is not dependent on edge based stopping constraint but on a minimal partition problem [3] over an angular mapped image.

Active contouring involves curve evolution under constraints derived from image features aimed at object detection. The method of active contouring involves constrained propagation of an initial curve either parametric [1] or intrinsic [2,4,5,8], which is launched interactively to detect object boundaries. The corresponding motion PDE (Partial Differential Equation) is derived based upon energy minimization functional using variational approach.
Conventional level set models [8] for boundary detection use gradient of the image as a constraint to stop evolution. Based on this some of the reported work on image analysis are Geometric Active Contours [4], Shape modeling [5], Geodesic active contours [7], statistically unifying region and boundary approach using Geodesic active contours [9]. Accuracy of these edge-based methods depends upon gradient edge response of the image and also relies heavily on well-separated homogeneous regions for detection. Since, this approach is sensitive to isolated noise, it cannot be applied to natural images. An approach for object detection for vector valued images, is proposed in color snakes [6], which essentially is based on using similar edge constraint that is derived on a Riemannian metric and has a Geodesic active contour [7] formulation. Again this fails when applied to natural color images.

Accurate methods for level set based active contouring are possible, by using features other than edge maps. One can find a histogram feature based approach for region segmentation [11] and shape modeling [13] that segments regions and extracts shapes accurately without the necessity for boundary information.

Based on edgeless features for constraining active contours, we propose a level set method, which is an improvement over the work by Chan & Vese [12]. Advantages of their model being (a) Allows for arbitrary initialization of the contour (b) Depending upon a particular feature derived from the image it allows for boundary detection. (c) It avoids reinitialisation of intermediate contours during propagation. The main drawback being it is applicable only to images with finite gray levels and not natural images. The model is similar to Mumford-Shah segmentation [3] functional possessing level sets formulation.

The model we propose here is an improvised version of [12] using an additional refinement force that acts as a spring force to the evolving front, allowing boundary detection and object extraction in a more accurate way especially in color natural images. The paper is organized as follows. Section 2 describes our model for color natural images. In section 3 the implementation issues of the model are dealt. In section 4 results and observations are presented. Lastly, a brief conclusion and scope is provided in section 5.

2. Description of Proposed model:

The energy functional used for minimization in [12] with propagating closed curve '*C*' is

$$F(c_1,c_2,C) = \text{ì}\ Length\ (C) + \text{õ}\ Area\ (inside\ (C)) + \text{ë}_1 \int_{inside(C)} |I_0(x,y) - c_1|^2 dxdy + \text{ë}_2 \int_{outside(C)} |I_0(x,y) - c_2|^2 dxdy \quad \ldots(1)$$

Here $m > 0$ and is a scaling parameter, u, l_1, l_2 are positive constants, and c_1 and c_2 are the average intensity of image I_0 inside and outside the propagating active contour '*C*'.

We propose to modify (1) with addition of a refinement force and extend it to detect objects accurately in color natural images (RGB space). This force helps in further propagation of contour to reach perceptual boundaries of object. Before describing the model we present a brief overview on the angular mapping of the color space [10]. There the authors have used the angular mapping method to extract objects from natural color images with parametric model (Snakes) using edge-based constraints.

2.1 Angular map of color space

Let a color image be represented by a set of two dimensional vector valued image in a selected color space (e.g. RGB space). Let each of these image planes be represented as a matrix. The values of an angular color map are computed by [10],

$$q_1(x,y) = 1 - \frac{2}{p}\arccos\left(\frac{v_{(x,y)}v_{ref}}{\|v_{(x,y)}\|\|v_{ref}\|}\right) \quad \ldots(2.1) \text{ or}$$

$$q_2(x,y) = q_1(x,y)\left[1 - \frac{\|v_{(x,y)} - v_{ref}\|}{\sqrt{3\times 255^2}}\right] \quad \ldots(2.2)$$

Notation $v_{(x,y)}$ stands for a vector value of all the pixels located at the position *(x, y)* within the color image space. For instance, the term $v_{(x,y)} = [R(x,y)G(x,y)B(x,y)]$ is the vector in the RGB color space where *R (x, y), G (x, y)* and *B (x, y)* denote the values of the pixels located at the position *(x, y)* in the *R*, *G* and *B* color planes, respectively. Notations $n_{ref}, q_1, \& q_2$ stand for a reference vector, the value of the angle given by (2.1) between the vector $n_{x,y}$ and the reference vector n_{ref} and the value of the angle given by (2.2) between the vector $n_{x,y}$ and the reference vector n_{ref} respectively. The significant color changes within the original image are thus stored in a matrix $I_{ang}(x,y)$ termed as the angular map, resulting in angular map image I_{ang}. This is presented to the Level set model.

The main difference between the angular maps $\{q_1\}$ and $\{q_2\}$ is that the former takes into account only the angle of the color vectors, whereas the latter of both the angle and the magnitude of the color vectors. Both angular maps $\{q_1\}$ and $\{q_2\}$, however indicate identical color changes within the image. The angular map image for Whale image is shown in Fig.1. The angular map image I_{ang} is normalized and scaled to [0, 255].

Fig.1.The angular maps $\{q_1\}$ and $\{q_2\}$ of the Whale color image computed with reference vector $\mathbf{v}_{ref}$ =[1,1,1].

2.2 Energy minimizing model

The energy minimization functional as a minimal partition problem is illustrated in fig.2 Amongst all the possibilities only fig.2(d) will be a unique solution for the corresponding minimization functional (with usual notations)

$$F(c_1,c_2,C) = F_1(C) + F_2(C) = \int_{inside(C)} |I_0(x,y) - c_1|^2\, dxdy + \int_{outside(C)} |I_0(x,y) - c_2|^2\, dxdy \quad \ldots(3)$$

i.e., (3) reduces to zero only for the case of fig. 2(d). For more details the readers are referred to [12]. Equation (1) is a generalized minimization functional of (3) with several regularizing terms dependent on length and area of the propagating contour. This technique looses accuracy when considered for object detection of natural images. Observe the following proposition.

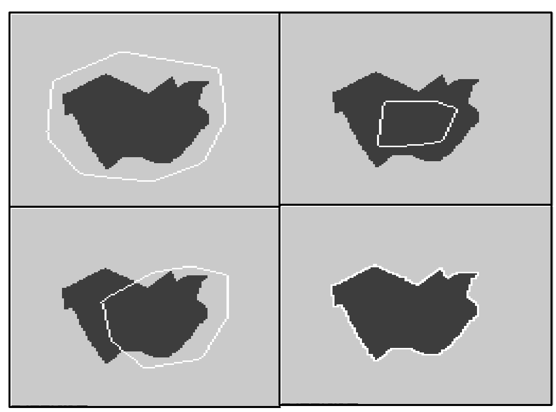

Fig. 2. (Left to Right Top to Bottom) Initial Contour 'C' (a) outside the object (b) inside the object (c) partially overlapped on the object (d) perfectly segments the object. Observe that (d) is the only solution of minimal partition problem

Proposition 1: Accuracy cannot be achieved in natural image segmentation without an additional refinement force in (1).

Proof: To recover the exact boundary in color natural images additional refinement force is required to be added due to the absence of global minima for the perceptual boundaries of the object, to the energy minimization functional, without which the propagating curve will lock on to several local minima. The best force and the only one that remarkably avoids these inherent local minima is a contrast dependent additional difference force $(c_1 - c_2)^2$, which will pull (spring force) the contour near to approximated global minima and near the boundary.

The modified energy minimization functional in (1) for detecting objects in natural image with refinement term will thus be

$$F(c_1,c_2,C) = \mu Length(C) + \lambda_1 \int_{inside(C)} |I_{ang}(x,y) - c_1|^2 \, dxdy + \lambda_2 \int_{outside(C)} |I_{ang}(x,y) - c_2|^2 \, dxdy + \lambda_3 (c_1 - c_2)^2 \quad \ldots (4)$$

Here I_{ang} is the angular map of vector image. In the above functional the area and length metric can always be expressed in terms of one another, hence the area term is made zero.

2.3 Level Set Method of the proposed model:

Level set function is an intrinsic representation of the propagating curve $C(s)$, which propagates in the direction normal (N direction fig. 3(a)) to the curve with speed dependent on curvature. For more details on its characteristics we refer the reader to pioneering works in [2, 5, 8].

The functional in equation (4) is solved for infimum using calculus of variations approach with a higher dimensional level set function. Let Ω be an open domain over which all parameters are defined. For the parametric extrinsic initial contour $C(s)$ the corresponding contour in the higher dimensional level set function Φ is $\{(x,y) \mid \Phi = 0\}$. The level set function is a distance-mapped function (we use Euclidean distance) with respect to this zero valued contour on it, corresponding to $C(s)$, such that (see fig. 3(a))

$$\Phi = \begin{cases} 0 & for\ all\ poinsts\ on\ C(s) \\ < 0 & for\ region\ outside\ C(s) \\ > 0 & for\ region\ inside\ C(s) \end{cases} \quad \ldots (6)$$

Defining $H(\Phi)$ as the Heaviside function corresponding to region inside the curve $\{\Phi \geq 0\}$ and $\delta(\Phi) = H'(\Phi)$ is the Dirac Delta function having the support regi on near the propagating front.

Level Set Function

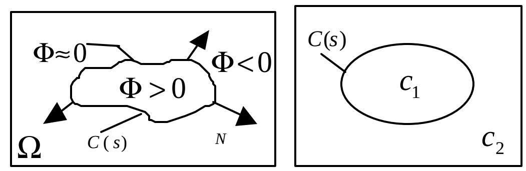

Fig 3. (a) Level set function and the embedded initial curve $\Phi \approx 0$ that propagates in the normal direction N. (b) Piecewise approximation of angular map I_{ang} of color image.

The corresponding minimization functional using the refinement force in Φ on Ω will be given by (5). The intensity average terms c_1 and c_2 are estimated over the angular map of color image, i.e., I_{ang} using

$$c_1 = \frac{\int_\Omega I_{ang}(x,y) H(\Phi(x,y)) dxdy}{\int_\Omega H(\Phi(x,y)) dxdy} \quad \ldots (7a)$$

$$c_2 = \frac{\int_\Omega I_{ang}(x,y)(1 - H(\Phi(x,y))) dxdy}{\int_\Omega (1 - H(\Phi(x,y))) dxdy} \quad \ldots (7b)$$

Without the regularization terms the solution of the minimization becomes a piecewise approximation of the image I_{ang}, as the particular case of Mumford & Shah [3] segmentation model. See fig. 3(b).

$$I_{ang}(x,y) = c_1 H(\Phi(x,y)) + c_2 (1 - H(\Phi(x,y))) \ldots (8)$$

Since the interest is on the region near the initialized contour, the Heaviside and Dirac Delta functions are approximated in such a way that, the level set during evolution remains smooth avoiding formation of shocks without stressing only on the region of interest (near the propagating curve). Regularizing Heaviside function thus ensures smooth evolution.

$$H_\varepsilon(z) = \tfrac{1}{2}\left(1 + \tfrac{2}{\pi} \arctan\left(\tfrac{z}{\varepsilon}\right)\right) \quad \ldots (9)$$

The support region of (9) extends over complete length extending up to ∞. The Dirac Delta function is derived as $\delta_\varepsilon = H'_\varepsilon$ that doesn't decay down to zero. The main advantage of this approximation is that, since the region of support extends to complete Ω (strong near the propagating contour) the whole function Φ simultaneously evolves & hence there is no need of reinitialization of

$$F(c_1,c_2,\Phi) = \int_\Omega \left(\mu \delta(\Phi) |\nabla \Phi| + \lambda_1 |I_{ang} - c_1|^2 \, H(\Phi) + \lambda_2 |I_{ang} - c_2|^2 \, (1 - H(\Phi)) - \lambda_3 (c_1 - c_2)^2 \delta(\Phi) \right) dxdy \ldots (5)$$

propagating contour [8, 5]. Normally, when the level set function evolves it deviates from its characteristic distance mapped function, thus requiring an intermediate reinitialization.

Solution of the PDE in (5) based on principles of Calculus of Variations and gradient descent flows using Euler technique over the level set formulation, yields motion PDE that can be solved iteratively. With $t \geq 0$ as the time marching parameter,

$$\frac{\partial \Phi}{\partial t} = d_e(\Phi)\left\{\begin{matrix} m\, div\left(\frac{\nabla \Phi}{|\nabla \Phi|}\right) - l_1 (I_{ang} - c_1)^2 + \\ l_2 (I_{ang} - c_2)^2 - l_3 (c_1 - c_2)^2 \end{matrix}\right\} \quad (10)$$

With $\Phi(x, y, 0) = \Phi_0(x, y) \quad in \quad \Omega$

Note: The grid spacing '*h*' and 'e' are constants and equal to 1. We choose $l_1 = l_2 = 1$

3. Numerical Methodology of the model:

The Level Set function $\Phi(x, y, t)$ is implemented using signed Euclidean distance mapping with respect to the embedded propagating curve (zero valued contour) on Level Set.

$$\Phi(x, y, t = 0) = \pm d \qquad \text{... (11)}$$

The curvature function $k = div\left(\nabla \Phi / |\nabla \Phi|\right)$ in (10) is implemented by adopting proper entropy satisfying upwind scheme [12], which uses combination of forward and backward differences. The final level set model is given in (12) with forward difference in time and is solved for each point on the grid $(i, j)\ on\ \Omega$ using iterative relaxation technique.

The approximate zero level set or the propagating curve is constructed, after each iteration in (12) which is obtained by determining the boundary pixels (zero-crossing) in the level set.

3.1 Convergence Test:

The iterative algorithm in (12) is repeated till the condition for convergence or perceptual global minima is reached. A simple method that tests the convergence is by keeping track of whether the propagating front remains stationary over a specified number of consecutive iterations. The condition checks for the stability of contour length over consecutive P iterations, where P>1.

$$\frac{1}{P}\sum_{\substack{ZLS \\ i=0 to P-1}} (\Phi(x,y)=0)^{m+1-i} - \sum_{ZLS} (\Phi(x,y)=0)^{m+1} \overset{t}{\approx} 0 \qquad \text{…(13)}$$

Where 'm' as the iterative parameter

3.2 Algorithm:

The steps involved in the active contouring for color natural images can be depicted as (RGB Space)

1. Compute the angular map image I_{ang} using equation (2.1) or (2.2).
2. For fast solution use pyramidal representation of I_{ang} (optional).
3. Compute $c_1(\Phi^n)$ & $c_2(\Phi^n)$ using (7) and evolved curve by finding zero-crossing pixels [5].
4. Using iterative relaxation technique, evolve level set function Φ^{n+1} from Φ^n by (12).
5. Check for convergence using (13) with $t = \pm M$, where M is a finite tolerance specified in pixel length, if this is not satisfied then repeat steps 3 to 5.

4. Experimental results and observations:

In our experimentation we have used RGB color images, since it is the most common representation used for digital images. The angular map $\{q_1\}$ has been computed between normalized RGB image & the reference vector $n_{ref} = [111]$. Other reference vectors may also be used, depending on *a priori* knowledge of the color information.

We have tested the proposed algorithm on several color natural images with single object having variations in color both within the object and in the background. The results are compared with Chan & Vese [12] model. Results indicate that our model performs in a much superior way with an added refinement force that is able to reach perceptual boundaries of the object.

The following constants were chosen for weighting coefficients in (12). As indicated, the grid spacing '*h*' and 'e' are equal to 1 and $l_1 = l_2 = 1$. The coefficient of length term is chosen as $m = 0.01 \times 255 \times 255$, which acts as a scaling term, and time marching increment parameter $\Delta t = 0.01$. l_3 the coefficient of refinement force $(c_1 - c_2)^2$ is chosen as shown in Table I. This coefficient l_3 needs to be tuned and depends on the contrast value between the approximated regions (inside and outside the propagating front) and is low for high contrast and high for low contrast. The initial contour is launched interactively on the angular map

$$\Phi_{i,j}^{n+1} = \Phi_{i,j}^{n} + \Delta t \cdot d_e(\Phi_{i,j}^{n})\left(m \cdot k - l_1 (I_{cp,i,j} - c_1(\Phi^n))^2 + l_2 (I_{cp,i,j} - c_2(\Phi^n))^2 - l_3 (c_1(\Phi^n) - c_2(\Phi^n))^2\right) \text{...(12)}$$

I_{ang} of the color image. Equation (12) is iteratively evaluated to avoid formation of shocks using iterative relaxation technique. For convergence test in (13), P is chosen to be 5 and tolerance limit M is taken as 5-pixel length. For the results see figures 4 and 5.

Following observations were made in our experimentation with color natural images. Due to the existence of non-uniform variations in the chroma levels within the image, exact boundary detection or perceptual boundary detection is still a big challenge. Another observation is that, though the proposed level set method is capable of propagating in both directions, due to inherent variations in color components of natural image (local minima) one cannot have a global minimum. Hence there is a need for the initial contour to be near and outside the object that increases the accuracy and converges fast.

Without the refinement force the contour cannot reach exact boundary locations. With the initial contour near the boundary, the convergence is fast enough and requires a maximum of 25 to 100 iterations. All the implementations were done on IBM PC P-III, 500MHz using Matlab v5.2.

5. Conclusion:

We have presented a level set based edgeless active contouring model for natural color images in the RGB space based on angular mapping. The model is an improved version of that in [12]. Our model has been tested on wide range of color images having variations in color within the object as well as the background. We are now exploring into a unified approach to object detection in natural images using level sets.

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TABLE - I

Image (RGB)	Size	l_3
Whale Image	232x164x3	2.5
Bird image	169x128x3	5.0
Butterfly Image	200x145x3	5.0
Gar Fish	161x128x3	2.0
Bass Fish Image	128x128x3	5.0
Duck	1024x692x3	2.5

TABLE – I: Various images depicted along with their size and values of l_3

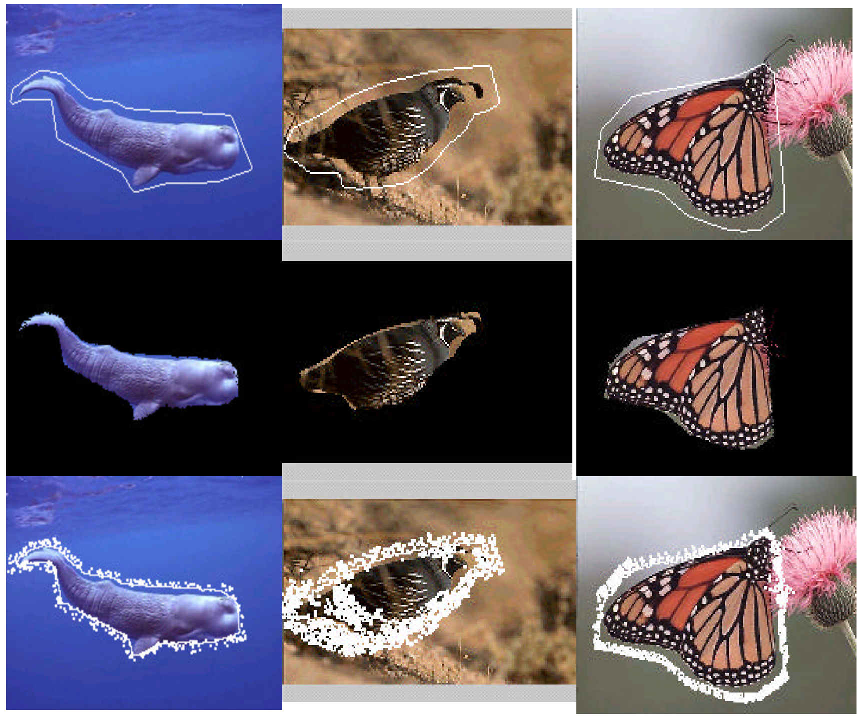

Figure 4: Results for Whale Image, Bird Image & Butterfly Image Top Row: Original with interactively initialized contour, Middle Row: Extracted objects by our model with Refinement force, where the region inside the propagated contour is depicted. Bottom Row: Strayed final contour for Chan and Vese Model without refinement force. (These are color images)

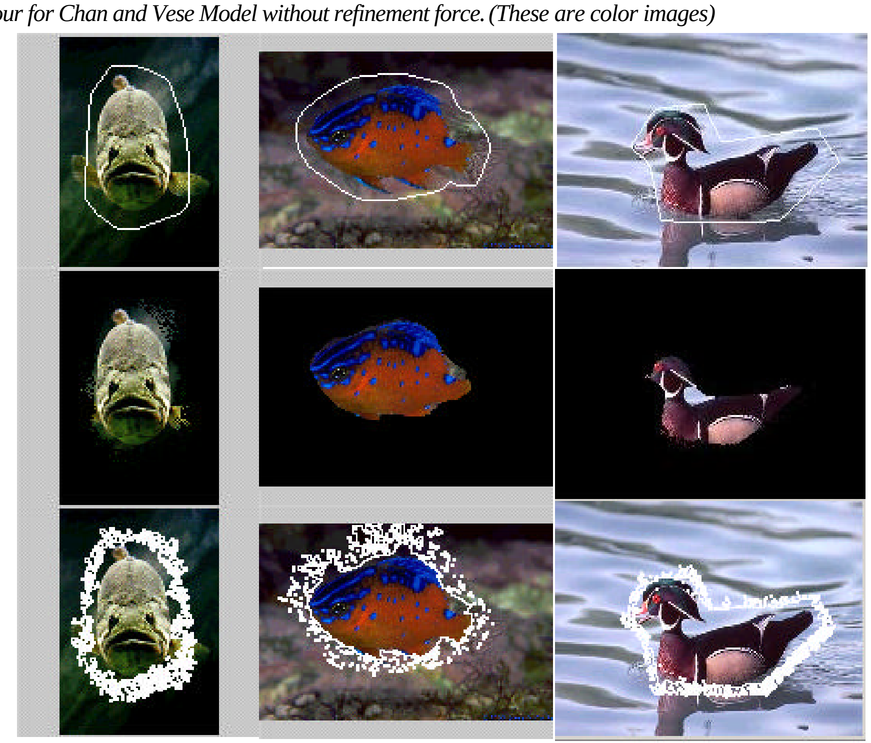

Figure 5: Results for Bass and Gar fish Images and duck image. First Row: Original with interactively initialized contour, Middle Row: Results of our model with Refinement force, where the region inside the propagated contour is depicted. Third Row: Strayed final contour for Chan and Vese Model without refinement force.(These are color images)