Mathematical Morphology Based Corner Detection Scheme: A Non-Parametric Approach

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Abstract

A simple non-parametric approach for detecting corner points using mathematical morphology is proposed in this paper. The proposed method has two stages, the first stage deals with the extraction of significant corner clusters from an image, while the second stage concentrates on detection and localization of true corner points from the extracted corner clusters. The proposed method unlike other morphology based methods detects all true corner points present in the extracted corner clusters. The experimental results reveal that the proposed method outperforms existing morphology based corner detection schemes.

Keywords: Mathematical morphology**,** Corner points, Region of support, Curvature estimation, Non-parametric, Small eigenvalue.

1. Introduction

Corner points on shape curves are effective primitives for shape representation and analysis [1]. Corner points on a digital boundary are found at locations where nature of curve changes significantly. Corner points provide critical information of a shape which is useful in pattern analysis and recognition problems. Therefore corner detection in images is an important aspect of computer vision applications.

Many approaches [12], [13], [2], [15], [6], [11], [15], [18] have been developed for corner detection. They are all based on the analysis of chain-coded boundaries of objects. Most of them involve complex floating-point computation and selection of region of support at every point on a boundary curve. Determination of region of support and then computation of curvature at every boundary point is time consuming task, especially, for very large objects.

With computational simplicity and effectiveness, mathematical morphology has been applied successfully in image and signal processing [16], [3]. The existing morphological corner detectors [9], [17], [19] are derived from Meyer and Beucher [8]. Zhang and Zhao [19] have proposed a method for corner detection using mathematical morphology. This algorithm introduces morphological residue for detection of corner points. A corner point is obtained for each corner cluster. A corner point corresponding to a corner cluster is defined to be the point of intersection of the boundary of the cluster and the normal line passing through the center of the cluster. Though the approach is fast, the detected corner point is not necessarily a true corner point and therefore the corner points are shifted from the actual location. To overcome this problem Lin et. al., [6] suggested a modification to the method [19] in which the process of shrinking a cluster into a single point is redefined. In the modified approach a corner point in a cluster is defined to be the point with maximal N-hit number.

Though the aforementioned morphology based methods (to the best of our knowledge these are the only two approaches which are based on morphology) deserve appreciation for their simplicity and less computational overhead, they are of little use in real pragmatic applications, because they are parametric approaches, i.e., they require a parameter to decide the size of the structuring element. In fact, the performance of the methods is heavily dependent t on the selection of proper sized structuring element. Selecting a suitable value for the parameter is indeed a tough task. If the size is too small then some unwanted corner points are detected, on the other hand if size is too large then more and more actual corner points are undetected. In addition, these methods tackle the problem of detecting corner points with the assumption that there exists only one corner point in a corner cluster. But in reality a corner cluster may contain more than one corner points. For example, consider a corner cluster shown in Fig. 1(a). Corner point detected due to the application of the method proposed by Zhang and Zhao [19] is shown in Fig. 1(b). It can be observed from Fig. 1(b), that the method proposed by Zhang and Zhao [19] has not only failed to detect all true corner points in the corner cluster but also failed to detect an actual corner point (shown in filled circle in Fig. 1(b)). On the other hand, the method proposed by Lin et. al., [6] has detected exactly one actual corner point (shown in Fig. 1(c) marked with filled circle) by making use of N-hit number computed for the boundary points of corner cluster. But, this method also failed to detect all corner points. Therefore it is clearly understood from the above example that the existing morphology based methods fail to detect all true corner points in a corner cluster and some time they even detect spurious corner points.

In view of this, a method which alleviates the problems associated with the existing morphology based corner detection schemes is proposed in this paper. The proposed method detects all true corner points in every corner clusters. The proposed method has two stages, the first stage deals with the extraction of corner clusters from a given image by the use of morphological operators whereas the second stage concentrates on detection and localization of all true corner points present in the extracted corner clusters. Unlike the other morphology based methods, the proposed method does not require any input parameter. The result of the proposed method is unaltered even if the size of the structuring element is varied.

Rest of the paper is organized as follows, section 2 describes the proposed method for corner detection, section 3 presents the experimental results, discussions are drawn in section 4 and finally paper concludes in section 5.

2.1 Corner cluster extraction

Before presenting the proposed method, we would like to review the following definitions.

Let A and B be two subsets of Z^2.

The *dilation* of A by B, written as $A \oplus B$, is given by $A \oplus B = \{x \mid \hat{B}_x \cap x \ ? \ \varnothing\}$, where $\hat{B}$ is the reflection of B about the origin and B_x is the translation of B by x.

The *erosion* of A by B, written as $A \ \Theta \ B$, is given by $A \ \Theta \ B = \{x \mid B_x \subseteq x\}$, where, B_x is the translation of B by x.

The *closing* of A by B, written as $A \bullet B$, is given by $A \bullet B = ((A \oplus B) \ \Theta \ B)$.

The *opening* of A by B, written as $A \circ B$, is given by $A \circ B = ((A \ \Theta \ B) \oplus B)$.

The subset B used in the above definition is called a *structuring element*. In most of the applications, the structuring element chosen is a simple shape of smaller size. Many useful properties of dilations, erosions, closings and openings can be found in the pioneer works of Matheron [7] and Serra 16]. If B is

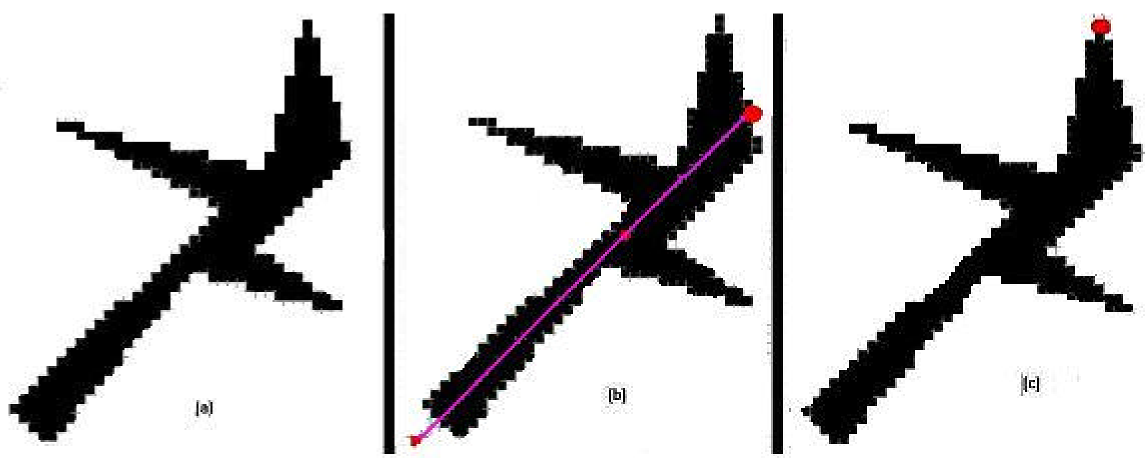

Fig. 1: (a) Corner cluster, (b) corner point detected by Zhang and Zhao [19] method and (c) corner point detected by Lin et. al., [6] method.

2. Proposed method

This section describes a novel, non-parametric method for corner detection. The proposed method has two stages. The first stage of the proposed method deals with the extraction of corner clusters from a given image by the use of morphological operators, while the second stage concentrates on detection and localization of corner points from the extracted corner clusters.

a disk shaped structuring element, the effect of $A \circ B$ is to smooth away convex portions of A, and the effect of $A \bullet B$ is to fill in concave portions of A. Therefore, as in the works of Noble [9] and Shapiro et. al., [17], it is intuitive to use the set difference $A - (A \circ B)$ (this is called the peak extractor when A is a numerical function [16] to locate convex corner clusters and $(A \bullet B) - A$ (this is called a valley extractor when A is a numerical function) to locate concave corner clusters. However, as pointed out earlier, the results of $A - (A \circ B)$ and $(A \bullet B) - A$

consist of areas around corner points in addition to corner points themselves.

In this work the appropriate positions of convex and concave corner clusters are obtained by respectively performing the operations $(A - (A \circ B))$ and $(A \bullet B - A) \oplus E$, where E is the rhombus structuring element. Convex and concave corner clusters are combined by performing union operation. Boundary curve of each corner cluster is obtained by intersecting the union image with the boundary of original image.

Once the boundary of every convex and concave corner clusters are obtained, it is recommended to apply novel non-parametric boundary based localization technique to detect and localize all true corner points present in every corner cluster. The novel boundary based localization technique is explained in the following subsection.

2.2 Corner point localization

The second stage of the proposed corner detection method deals with the corner localization. The proposed methodology for corner localization from boundary of the extracted corner cluster determines the region of support of each point on the basis of its local properties. The connected sequences of points on either side of the point of interest are called the arms of the point. Set of points which come before the point of interest is regarded as *left arm* and set of points which come after the point of interest is regarded as *right arm* irrespective of the direction of traversal of the curve. Number of points comprising an arm is regarded as the size of the arm. The points in the left arm, the right arm and the point itself together constitute the region of support of that point. For each boundary point p three features: (i) the size of the region of support of p, (ii) the curvature at p, which is the reciprocal of the angle made at p due to its left and right arms and (iii) the limit value, the number of boundary points for which the point p is an end point of their region of support, are computed.

Let $C = \{p_i \mid p_i = (x_i, y_i) \in Z^2, i = 1, 2, 3, \ldots, n\}$, where, p_{i+1} is a neighbour of p_i be a boundary of an extracted corner cluster described in a sequence.

Let p_i be the point of interest for which the region of support has got to be determined to decide if p_i is a corner point. The region of support of p_i consists of left arm L_i, right arm R_i and the point p_i itself. The left arm L_i and the right arm R_i are respectively the sequences of points which describe the segments of the curve, extending from the point p_i in backward direction and forward direction. Therefore, $L_i = \{p_j \mid p_j = (x_j, y_j) \in Z^2, j = i-1, i-2, \ldots, i-l\}$ and $R_i = \{p_j \mid p_j = (x_j, y_j) \in Z^2, j = i+1, i+2, \ldots, i+r\}$, here l and r denote respectively the sizes of left and right arms which are decided adaptively based on local properties of the curve and are not necessarily equal. Thus, the region of support is not necessarily symmetric.

The local property that we recommend in this work is the small eigenvalue associated with the covariance matrix of a sequence of points. It has been shown [4] that for a straight-line segment, the small eigenvalue in the continuous domain is zero, regardless of its length and orientation. Hence, on either side of the point of interest p_i, it is proposed to choose the maximum sequences of points that bear approximately zero small eigenvalue as respective arms.

To compute the right arm for the point of interest p_i, we initially begin with the set $S_{i+1} = \{p_i, p_{i+1}\}$ and the small eigenvalue of S_{i+1} denoted by $\lambda_{S_{i+1}}$ is computed, of course this $\lambda_{S_{i+1}}$ is obviously zero as small eigenvalue associated with any two points is zero. The set S_{i+1} is updated to $S_{i+2} = \{p_i, p_{i+1}, p_{i+2}\}$ by adding the point p_{i+2} which is next in the sequence, and the small eigenvalue $\lambda_{S_{i+2}}$ of S_{i+2} is computed. This process of updating the set by adding the next point in sequence and computing the small eigenvalue associated with the updated set is repeated until $\lambda_{S_{i+r}} < \lambda_{S_{i+r+1}} < \lambda_{S_{i+r+2}}$ for some r. Once this condition is satisfied, the value of r is taken as the size of the right arm and $R_i = \{p_j \mid p_j = (x_j, y_j) \in Z^2, j = i+1, i+2, \ldots, i+r\}$ is the right arm of p_i.

Similarly, the left arm $L_i = \{p_j \mid p_j = (x_j, y_j) \in Z^2, j = i-1, i-2, \ldots, i-l\}$ is determined by choosing points in backward direction until $\lambda_{S_{i-l}} < \lambda_{S_{i-l-1}} < \lambda_{S_{i-l-2}}$ for some l which is then taken as the size of the left arm of p_i. Therefore the region of support of p_i is a sequence of $l+r+1$ connected points $\{p_{i-l}, p_{i-l+1}, p_{i-l+2}, \ldots, p_i, p_{i+1}, p_{i+2}, \ldots, p_{i+r}\}$. Here, the points p_{i-l} and p_{i+r} are thus called the end points of the region of support of p_i and their limit values are incremented by one.

It can be noticed that the size of the determined region of support varies from point to point depending on the local property of the curve within the vicinity of the point of interest and thus the proposed method determines adaptively the region of support which is not necessarily symmetric.

Once the region of support of a point is determined, the curvature at that point is then estimated as the reciprocal of the angle made at that point due to its left and right arms. Determination of region of support of all points helps in computing the size of the region of support of each point, curvature at each point and also to compute how many times (limit value) a point has been the end point of the region of support of other points. It is observed experimentally that the limit value, the size of the region of support and the curvature of an actual corner point are relatively larger than the respective values of its neighbours. The same thing is revealed in Fig-2. Fig-2 shows a closed boundary curve along with the graphs of the size of the region of support, curvature value and limit value verses the points on the boundary curve.

The above observations do not necessarily mean that the existence of a local maximum in any one of the graphs alone is sufficient for locating the true corner points. However the existence of local maximum in all three graphs at a point guarantees that the point is a true corner point. Therefore the following rules are worked out to guide the process of locating true corner points.

1. Since points lie on a straight line bear 180^0, their curvature values are negligibly small, therefore this rule suggests to select the points which are not straight line points based on curvature value.
2. Since all the points on smooth curve segment are in general associated with almost same curvature value, and actual corner points bear curvature values larger than that of their neighbors, it is suggested to select the set of connected points, such that the variations in their curvature values are considerably large. This rule helps in selecting only the set of points with in the vicinity of actual corner points and there by eliminating the points on smooth curve segment.
3. Select the points which bear local maximum curvature and local maximum limit values with a relatively longer region of support.

It could be noticed that these rules do not require any a prior knowledge about a curve for selecting true corner points. However they make use of local properties of the curve which vary enormously in size and extent while deciding a point as a true corner point.

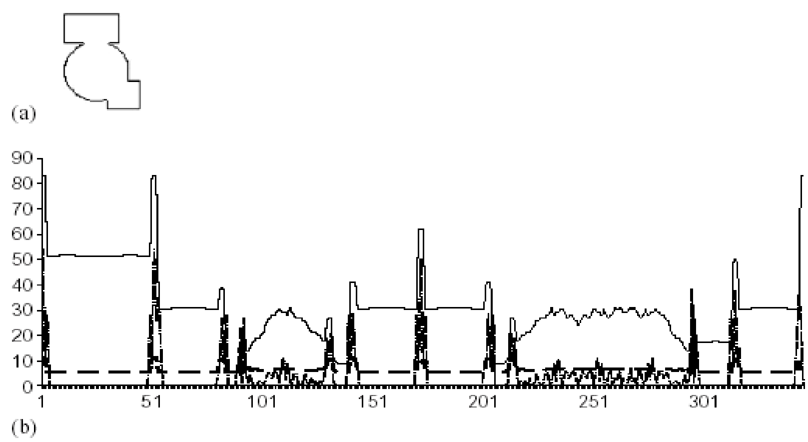

Fig-2: (a) A Boundary curve, (b) Graphs of the curvature (dashed line), size of the region of support (solid line) and limit value (dotted line) versus boundary points of Fig-1 (a).

The process of corner localization is applied to every extracted corner cluster. It can be noticed that the second stage of the corner detection can efficiently be implemented on SIMD parallel computers (Hwang and Briggs, 1985).

3. Experimental results

In order to reveal the superiority, robustness and suitability of the proposed methodology in pragmatic scenario, we have conducted experiments on several shapes (including the shapes considered by many researchers). For the purpose of establishing the superiority of the proposed model over the method proposed by Lin et. al., (1998), similar experiments on the same shapes are also conducted. Intentionally we have not considered the method proposed by Zhang and Zhao (1997) in this comparative study, as it was argued in the work (Lin et. al., 1998) that the results of Lin et.al., (1998) method is superior to the results of Zhang and Zhao (1997) method. In all experiments the radius of the disk structuring element is fixed up to 15 pixels. Fig.3 shows the results of the proposed method for corner detection along with the results of Lin et. al., (1998) method. First column of the Fig. 3, shows the shapes considered for the experimentation, columns 2 and 3 respectively show the results of the proposed method and Lin et. al., (1998) method. It can be clearly observed from the Fig. 3 that the proposed method has detected all true corner points, whereas the method proposed by Lin et. al., (1998) has failed to detect all true corner points.

4. Discussion

The major problem with the existing boundary based corner detection schemes is that they are less efficient since at every boundary point, it is required to compute the region of support and as well the curvature at that point to decide if the point is a corner point. In addition they are parametric. The advantage of these methods is that all true corner points are detected. On the other hand, the morphological operation based approaches appear to be more efficient since they selectively decide the parts of the boundary which have corner points to locate the true corner points. But they are less effective as they failed to detect all true corner points. Unlike these, the proposed method being an integrated approach is both efficient and effective. The suggested corner localization scheme not only makes the approach capable of detecting all true corner points but also make the entire approach non-parametric. This has been revealed by the results of the experiments conducted.

5. Conclusion

A simple method for corner detection using mathematical morphology is presented in this paper. The proposed method has two stages, the first stage deals with the extraction of significant corner clusters from the image, whereas the second stage concentrates on the detection and localization of true corner points from the extracted corner clusters. The proposed method unlike the other morphology based methods detects all corner points present in the extracted corner clusters. In addition the proposed method is non-parametric. The experimental results reveal that the proposed method outperforms the existing morphology based corner detection schemes.

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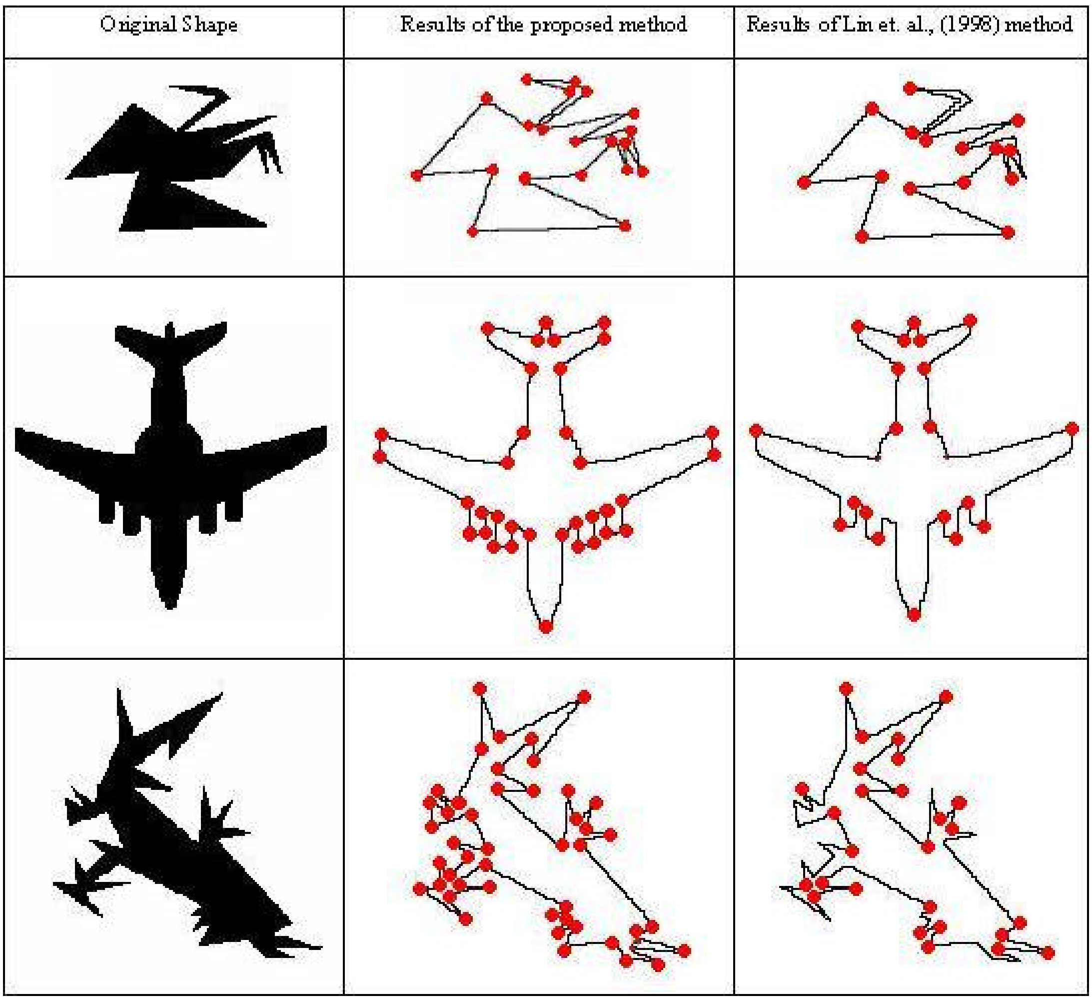

Fig. 3: Results of the experimentation