Fractal Based Image Segmentation

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Abstract

In this paper, we have proposed an algorithm for image segmentation, using the fractal codes. The basic idea behind this algorithm is to use fractal codes for the image segmentation. This method uses compressed codes instead of the gray levels of the image. Therefore it is cost effective since it operates directly on the fractal codes of the image. Hence, the proposed scheme can be directly used on the images accessed from the image database where images are kept in fractal–compressed format.

1. Introduction

Fractal image coding was proposed originally by Barnsley [1], and applied to the grayscale images by Barnsley [2] and Jacquin [3]. A lot of improvements have been done in image compression methods [7], e.g. image quality, compression ratio, coding speed etc. In this paper, we are proposing algorithm for the Fractal Based Image Segmentation. The encoding method is same as conventional fractal coding method, but the compressed code, called the fractal codes, is that usually used for the image reconstruction is used by us for the image segmentation. The advantage of this approach is that it operates directly in the fractal domain.

The essence of the fractal compression method is to complete the missing pixel data of an image piece using one of the similar "known" image pieces, which are called a reference image region. The phrase "the most similar" refers to the one image piece that has the best matching image piece in the partial information commonly known for both the completing image piece (i.e. destination region) and candidate reference image pieces. In fractal compression and representation a destination region is matched with a reference region by comparing the pixel values in the sample grid by down-sampling the reference region. The fractal image segmentation is purely based on same principle [4].

2. Proposed Algorithm

The development of the algorithm was carried out in two parts. First, was to extract the pixel chains using the convergence of the Partition Iterated Function System (PIFS) [5]. The pixel chains so extracted were verified by decoding the images using Sub-sampling Fractal Decoder and the results are compared with those obtained using the Iterative Fractal Decoder. The comparison of the results of the two decoders is presented in Section 2.3. The second task was to extract the dynamics of the convergence of PIFS and use this for the segmentation of the image (see Section 3). The major steps in the algorithm are shown in Figure 1.

2.1. Fractal Encoder

This stage is required for images that are not fractal encoded. Fractal encoder takes the image as an input and produce fractal codes for it. We have implemented the Partitioned Iterated Function System (PIFS) [3] for the generation of fractal codes. The PIFS scheme is based on self-similarity present in an image i.e. under suitable affine transformation, larger blocks of the image look like smaller blocks in the same image. The smaller image blocks are called "range block "and the larger image blocks are called "domain block".

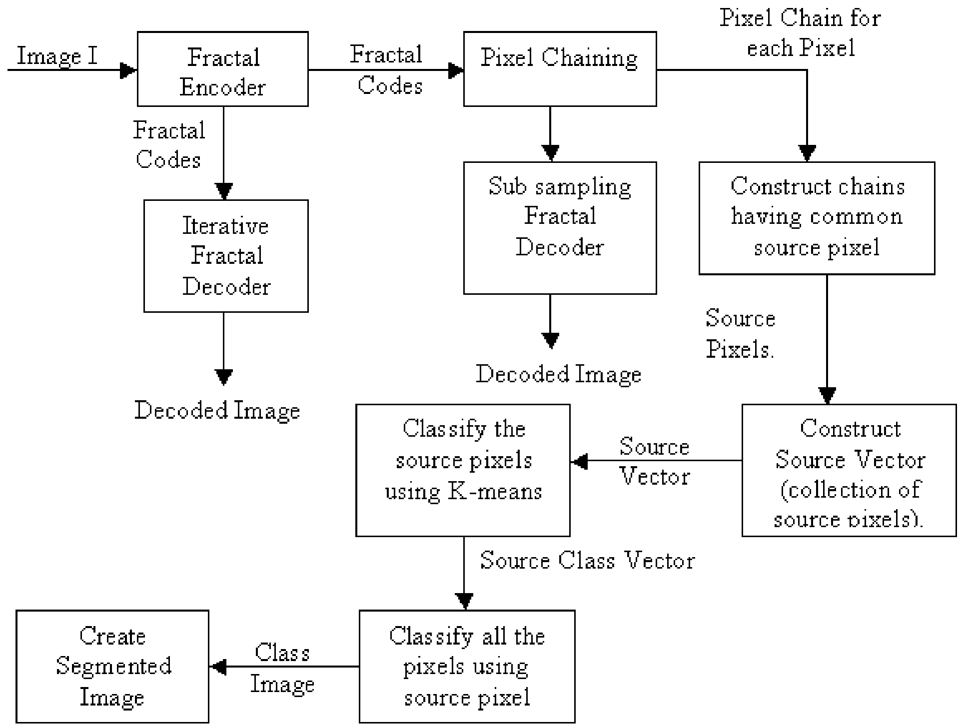

Figure 1. Block Diagram Description of the algorithm

Let us consider a w x w image I, with each range block of size r x r. Let $R = \{R_1, R_2, \ldots, R_m\}$ be the set of m non-overlapping range blocks. Let each domain block be of size $2r$ x $2r$ resulting in $D = \{D_1, D_2, \ldots, D_n\}$ as the set of n overlapping domain blocks of the image I.

Determining the fractal code of each range block is effectively finding the pair best matched domain block and the corresponding map for each range block. An exhaustive search is carried out on the set of domain blocks whereas for each selected domain block a contractive map (from domain block to range block) is constructed in two steps.

The first step is transformation of the rows and columns from domain block to range block. This can be achieved by using any one of the eight transformations on the domain blocks, called *isometry*, as described by Jacquin [3]. Once the transformed domain blocks are computed, an estimate of pixel value of a range block from the pixel values of the transformed domain block is obtained by fitting a straight line using the method of least squares as described below.

We like to estimate r_i's from d_i's using an equation

$$r_i = a\, d_i + b \qquad \forall i = 1,2,\ldots,n$$

If there exists no values of a and b that satisfies the above condition, then one can find the 'best' values for a and b by minimizing

$$MSE = \sum_{i=1}^{n} \left(r_i - ad_i + b\right)^2$$

Finally the domain block, with the corresponding map applied to it, resulting in the closest approximation of the concerned range block is selected. The selected domain block number, isometry and the parameters of the contractive map (a and b) are stored as the fractal code of the range block.

2.2 Pixel Chaining

This is a mechanism that captures the dynamics of the convergence of PIFS [5] and applies only to the Subsampling encoding techniques. In average compression scheme, while transforming from domain block to range block, we take the average of the consecutive pixel whereas in case of Subsampling scheme, we take the first pixel and drop the next.

According to the principle of the convergence, for each pixel in the image there is a unique pixel, called its *associated reference pixel*, given by the fractal transformation. If the reference pixel is known, the corresponding destination pixel value can be derived in one step. By viewing this reference pixel as a destination pixel, another, new reference pixel value is associated to it. Continuing this procedure, a chain of pixel is built. This chain stops when it hits a pixel already in the chain. Once it stops, tracing the chain back will generate all the pixel values of the chain.

Here, the goal is to find the pixel chain for a particular pixel using the fractal codes. At the time of encoding, we have stored parameters a, b, domain number and isometry (fractal code). Once we get the pixel chain, we can calculate the attractor (intensity value) for each pixel in chain using the principle of convergence of the PIFS [5]. The steps required for construction of pixel chain is explained below

1. Pick the target pixel (x_1, y_1) from the image, where x_1 and y_1 specifies the row and column of the pixel in the image.
2. Then, determine the target range block for the pixel in which it lies.
3. Determine the relative position of the pixel in the target range block determined in Step 2.
4. Extract the domain number d_n from the fractal code of the target range block.
5. Create the matrix M_d of domain size, which stores the co-ordinates of each pixel in the domain d_n.
6. Transform matrix M_d into another matrix M_r of range size, using Subsampling scheme.
7. Corresponding to the relative position of the target pixel co-ordinate in the target range block, extract the value of this co-ordinate from the M_r Range block. This itself will become a next target pixel.
8. Repeat the procedure until chain converges. The convergence criterion is when the target pixel maps onto a pixel in the same pixel chains.
9. Calculate the attractor (intensity value) for each pixel in the extracted pixel chain by moving backwards from the converged pixel (also called the source pixel).
10. Repeat the same procedure, until the attractor for all the pixels in the image are obtained.

2.3 Dynamics of Convergence of PIFS

The process starts with a fractal image compression scheme that uses an image partition of destination region and uses the Subsampling method when matching different domain and range blocks. Given a coded image ***P***, for any pixel *x* in the image, there is a pixel chain, which can be calculated using the algorithm described in Section 2.2.

The most important task is to capture the dynamics of the pixel chain. By doing some experiments, which are described further, we have observed the following three dynamics of a pixel chain. These are stated below.

1. For pixels lying inside an object, all pixels in the chain to which this pixel belongs also lie inside the object.
2. For pixels lying outside an object, all pixels in the chain to which this pixel belongs also lie outside the object.
3. For pixels lying on the boundary of the object, all the pixels in chain to which this pixel belongs also lie on the boundary of the object.

These dynamics reveal that all the pixels in particular chain can form a basis for segmenting an image into its various components. Thus, the segmentation criterion becomes that the pixels in the same pixel chain will be in the same-segmented portion of the image. And, all possible number of pixel chains defines the maximum number of segments possible in a given image. However, it was found that the number of pixel chains detected was very large for any image. A first stage of reducing the number was to combine all pixel chains that have at least one common pixel in their respective chains.

Each of these combined pixel chain has convergence point (we shall call this the *source pixel*), and tracing the chain back from the *source pixel* we can compute the attractor values of all the intermediate pixels in the chain as described in convergence of PIFS [5]. The next level of reduction of the number of segments that has been done is by clustering the combined pixel chains on the basis of the attractor value of their *source pixel* i.e. we have used the attractor value of the *source pixel* as a parameter for the clustering of all pixel chains by using the K-means algorithm. The K-means algorithm requires the maximum number of segments that can be expected in that image to be specified as a parameter. Determination of the optimum number of segments to be expected in an image was carried based on the method outlined in [6].

Figure 2. Dynamics of Pixel Chain

We have implemented a system to capture the dynamics of the pixel chaining process. It plots the pixels that belong to a particular pixel chain and links them. Couple of snap shots is shown in Figure 2 for the Lena image.

3. Implementation and Results

The entire sequence starting from pixel chaining to determining the source pixels for the pixel chains is carried out for the fractal-encoded images. After clustering the source pixel using K-means [6], we obtain the class of each source pixel. We generate the class image by assigning class to each pixel in the pixel chain, according to the class value of the source pixel associated with it. The proposed algorithm has been tested on several images. An example of the segmented images so obtained is presented here in Figure 3. The optimum number of regions for the Lena image was found to be four by using the aforementioned methodology. The four non-overlapping segmented regions of the image have been shown as four separate images in this figure.

The approach has also been tested on images from the ORL database [8] that have been fractal encoded and then segmented using the proposed approach.

4. Conclusions and Discussion

We have proposed a scheme for segmenting an image in the fractal domain. The most important advantage of the proposed technique of image segmentation using PIFS code is that it utilizes the

Figure 3. The four segmented images of the Lena image obtained by the proposed segmentation algorithm

coded (fractal) version of the input image instead of the intensity values of the original image to give fair quality segmentation, i.e. the segmentation is carried out in the fractal domain itself. Therefore it is cost effective as compared to conventional segmentation methods using gray levels for fractal encoded images since the proposed scheme can be directly used on the images accessed from the image database where images are kept in fractal-compressed format.

The quality of segmentation achieved is encouraging. Since the performance of proposed segmentation algorithm depends on the PIFS or fractal code, it is natural that the quality of the segmented image is dependent on the correctness of these fractal codes.

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