A Comparative Study on Discrete Orthonormal Chebyshev Moments and Legendre Moments for Representation of Printed Characters

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Abstract

Moment functions are widely used in image analysis as feature descriptors. Compared to geometric moments, orthogonal moments have become more popular in image analysis for their better representation capabilities. In comparison to continuous orthogonal moments discrete orthogonal moments provide a more accurate description of the image features. This paper compares the performance of discrete orthonormal Chebyshev moments with continuous orthogonal Legendre moments for representation of noisy printed characters of the Assamese alphabet. Experimental results show that at all noise levels Chebyshev moments outperforms Legendre moments. The performance of both the moments increases when higher order moments are considered but after some point the performance of Chebyshev moments decreases.

1. Introduction

Research in computer recognition of printed characters is popular for its application in banks, post offices and defence organizations. Other applications are reading aids for blind, library automation, language processing and multimedia system design. A perfect Optical Character Recognition (OCR) system for printed and hand-written characters is an ideal goal of the researchers.

A volume of research results are available on character recognition on European scripts based on Roman alphabet and Kanji and other Chinese/Japanese scripts. A good number of research reports are available on Indian scripts recognition systems [1], [2], [10], [11], [14], [15]. Some pioneering works towards the development of a complete OCR for an Indian language script were done in Bengali(Bangla)[1] and Devanagari [10], [14]. The recognition accuracy of the OCR presented in [1] is reported to be 95.50% at word level (which is equivalent to 99.10% at character level) for a single font clear document.

Most of the character recognition methods developed for Indian languages are based on features extracted from the image of the character. Some of the features are: stroke and run-number based features [1],[2], topological features [2], features obtained from the concept of waterflow [2] and wavelet based feature [11]. The features selected for a particular OCR are based on the specific properties of the script for which it is developed. The features selected for one script may not be suitable for another script. Moments, on the other hand, can be used as features for character representation as well as recognition without considering specific properties of the script. Legendre moments and other orthogonal moments have been used for recognition of Chinese characters with improved success rates of recognition [4],[5]. The orthogonal moments are obtained by weighted averaging of the image pixels and thus expected to be robust against noise. The basis sets or the moment weighting kernels in conventional moments like Legendre and Zernike are polynomials defined over a continuous domain which is completely different from the image coordinate space. The implementation of such moments involves two sources of errors: [7], [8],[9]. (i) the image coordinate space must be normalized to the range where the orthogonal polynomial definitions are valid (ii) the continuous integral must be approximated by discrete summation. A new set of discrete orthogonal moments, known as Chebyshev moments, introduced in [8] eliminate the two problems completely but preserves all theoretical properties of orthogonal moments. An orthonormal version of Chebyshev moments is proposed in [6] which improves the quality of reconstructed images. We present here a comparison of orthonormal Chebyshev moments with Legendre moments for representation of Indian scripts, particularly Assamese and Bengali scripts.

We consider the representation of noisy printed characters. A character is represented by a set of d features, or attributes, viewed as a d-dimensional feature vector. Moments are considered as features in our study. The classification is based on the concept of similarity and the Euclidian distance is taken as a measure of similarity. Experimental results using 52 characters from Assamese alphabet are presented.

2. Moments

Moment functions are used in image analysis as feature descriptors in a wide range of applications like object classification, invariant pattern recognition, object identification, robot vision, pose estimation and stereopsis. For an image $f(x, y)$, the general definition of moment function Φ_{pq} is given by

$$\Phi_{pq} = \int_x \int_y \Psi_{pq}(x, y) f(x, y)\, dx\, dy, \quad p, q = 0, 1, 2, \ldots \tag{1}$$

where $\Psi_{pq}(x, y)$ is continuous function of (x,y) known as the *moment weighting kernel* or the *basis set*. The simplest of the moment functions, with

$$\Psi_{pq}(x, y) = x^p y^q \tag{2}$$

were introduced by Hu [3] to derive shape descriptors that are invariant with respect to image plane transformations.

2.1 Legendre Moments

Legendre moments were introduced by Teague [16] with orthogonal Legendre polynomials as kernel. The (p,q) order Legendre moment is defined as

$$\lambda_{pq} = \int_{-1}^{+1} P_p(x) P_q(y) f(x, y) dxdy, \tag{3}$$

where the pth order Legendre polynomial is given by

$$P_p(x) = \frac{1}{2^p p!} \frac{d^p}{dx^p} (x^2 - 1)^p, \quad x \in [-1, 1], \tag{4}$$

When an analog image is discretized with an $M \times N$ array of pixels, we can approximate λ_{pq} by

$$\widehat{\lambda_{pq}} = \sum_{i=1}^{M} \sum_{j=1}^{N} h_{pq}(x_i, y_j) f(x_i, y_j), \tag{5}$$

where

$$h_{pq}(x_i, y_j) = \int_{x_i - \frac{\Delta x}{2}}^{x_i + \frac{\Delta x}{2}} \int_{y_j - \frac{\Delta y}{2}}^{y_j + \frac{\Delta y}{2}} P_p(x) P_q(y) dxdy. \tag{6}$$

2.2 Chebyshev Moments

The $(p + q)$ order Chebyshev moment is defined as [8]

$$T_{pq} = \frac{1}{\rho(p,N)\rho(q,N)} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} t_p(x) t_q(y) f(x, y),$$

$$p, q = 0, 1, 2, \ldots N - 1. \tag{7}$$

where

$$\rho(p, N) = \frac{N(1 - \frac{1^2}{N^2})(1 - \frac{2^2}{N^2}) \ldots (1 - \frac{p^2}{N^2})}{2p + 1} \tag{8}$$

is the squared norm and $t_p(x)$ is the scaled Chebyshev polynomial of order p.

The problem associated with Equation 7 is that, the value of the squared norm tends to zero as p increases. As a result, Equation 7 gives very large values for the moments when either p or q is large. The problem can be resolved by the orthonormal version of the moments as introduced in [6]. The orthonormal Chebyshev polynomials are defined by the following recurrence relation:

$$\hat{t}_p(x) = \alpha(2x + 1 - N)\hat{t}_{p-1}(x) + \beta \hat{t}_{p-2}(x), \tag{9}$$

$p = 0, 1, \ldots, N - 2; \quad x = 0, 1 \ldots, N - 1.$
where

$$\alpha = \frac{\sqrt{4p^2 - 1}}{p\sqrt{N^2 - p^2}},$$

$$\beta = -\frac{(p-1)\sqrt{2p+1}\sqrt{N^2 - (p-1)^2}}{p\sqrt{2p-3\sqrt{N^2 - p^2}}}$$

The initial conditions of the above recurrence relation are

$$\hat{t}_0(x) = N^{-\frac{1}{2}}$$

$$\hat{t}_1(x) = \frac{\sqrt{3}(2x + 1 - N)}{\sqrt{N(N^2 - 1)}} \tag{10}$$

The discrete orthonormal polynomials defined as above satisfy the following condition for all p:

$$\rho(p, N) = \sum_{i=0}^{N-1} \{\hat{t}_p(i)\}^2 = 1.0 \tag{11}$$

The polynomials given in Equation 9 can be renormalized to minimize propagation of any numerical errors through the recurrence relation, as follows:

$$\hat{t_p}(x) = \frac{\hat{t_p}(x)}{\sqrt{\sum_{x=0}^{N-1} (\hat{t_p}(x))^2}} \tag{12}$$

It has been shown that the reconstruction accuracy improves significantly by renormalization of the orthonormal Chebyshev moments [6]. We use this renormalized version of the orthonormal Chebyshev polynomial in our study.

3. Character Representation with Moments

According to the uniqueness theorem if $f(x, y)$ is piecewise continuous and bounded in a finite region of the (x, y) plane, the moment sequence $\{\Phi_{pq}\}$ is uniquely determined by $f(x, y)$, and vice versa [3]. However, the moment set defined by the Equation 1 is infinite. The moment set defined by Equation 7 is finite, but usually very large. In practice, we can compute only a finite set of moments. Therefore any moment based pattern representation method is feasible only if it works with a small set of moments. Other disadvantages of taking a large set of moments are that they are computationally very expensive and higher order moments are more sensitive to noise. Also the numerical errors due to the approximation of the polynomials in computing the moments increases with the increase of order of moments. Past studies [4] reveal that geometric moments can be used for some pattern recognition problems where the number of patterns is not large and they are not very much similar. But it is not suitable in a situation where the different patterns are very similar to each other. The information redundancy in a orthogonal moment set is minimum and therefore possess better feature representation capabilities. S. X. Liao in [4] has shown that the methods based on Legendre moments are better for image pattern recognition problems than the methods based on central moments. A method using Principal Component Analysis(PCA) on Legendre moments is suggested in[13] which shows better result than the method suggested by Liao. A study on representing printed noisy Assamese characters using Legendre moments and Principal Component Analysis is presented in [12]. Discrete orthogonal moments based on Chebyshev polynomial overcome the drawbacks of continuous orthogonal moments and reported to have better representation capabilities [6],[7],[8]. In this paper we present a comparative study of methods based on the orthonormal Chebyshev moments and the orthogonal Legendre moments for representation of printed Assamese characters in presence of noise.

Let there be n different characters in the alphabet. Let the gray-level image of each character be corrupted by zero-mean Gaussian noise. From each character k different samples are considered. Thus, we have $N = n \times k$ characters corrupted with Gaussian noise of zero mean. These characters belong to n different classes where each class represents one character from the alphabet. Moments (either Legendre or Chebyshev) of order 0 to r are computed for each character using Equation 5 and Equation 7. The characters are then represented in a $(r + 1)(r + 2)/2$ dimensional feature space where the features are the moments. These characters form different clusters and if each cluster represent one class i.e., one character in the alphabet, then the method can effectively recognize noisy characters of the alphabet.

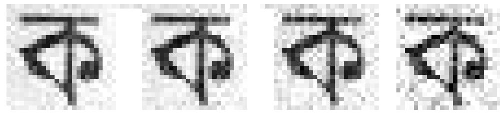

Figure 1. Sample noisy characters

Let, X_{ij} is the jth sample of the ith character ($i = 1, 2, \ldots, n, j = 1, 2, \ldots, k$. Each sample X_{ij} is represented as a column vector of $(r+1)(r+2)/2$ moments. The mean vector X_i of the ith class is given by

$$X_i = \frac{1}{k}\sum_{j=1}^{k} X_{ij}, i = 1, 2, \ldots, n; j = 1, 2, \ldots, k \quad (13)$$

Now, let us consider the hyperspheres in $(r+1)(r+2)/2$ dimensional Euclidian space with centers at X_is and radii R_i, where R_i is the maximum Euclidian distance of samples of the ith character from the center X_i and is given by

$$R_i = \max_j\{\sqrt{(X_{ij} - X_i)'(X_{ij} - X_i)}\}, i = 1, 2, \ldots, n \quad (14)$$

These hyperspheres represent n clusters corresponding to the n characters. If these hyperspheres are non-overlapping, i.e. the clusters are disjoint then the noisy characters belonging to one class are clearly distinguishable from the other classes. Let, $d(X_i, X_j)$ be the distance between X_i and X_j. For the two clusters centered at X_i and X_j, we have the following three cases:

- $d(X_i, X_j) > R_i + R_j$ - the clusters are disjoint
- $d(X_i, X_j) = R_i + R_j$ - the clusters are touching each other
- $d(X_i, X_j) < R_i + R_j$ - the clusters are overlapping

To examine the disjointness of the clusters we find out the closest pair of clusters. The two clusters centered at X_i and X_j are the closest if $d(X_i, X_j) - (R_i + R_j)$ is minimum among all pairs of clusters. If the closest pair of clusters are disjoint, i.e., $d(X_i, X_j) - (R_i + R_j) > 0$, then all clusters are disjoint.

In order to compare of the two moment methods or the same moment method at different noise levels we consider the ratio $DC : SR$, where DC and SR are the **D**istance between **C**enters and the **S**um of **R**adii respectively, of the closest pair of clusters. Larger value of $DC : SR$ implies better clustering. Obviously $(DC : SR) < 1$ and $(DC : SR) = 1$ mean overlapping and touching clusters respectively.

4. Experimental Results

In this section we present the experimental results to compare the performance of the Chebyshev moments with

Table 1. Distance between Centers (DC) and Sum of Radii (SR) of the closest pair of clusters with Legendre moments (σ = standard deviation of noise)

σ	DC & SR	Order of moments considered						
		1	2	3	4	6	7	8
4	DC	4.83	8.23	11.46	12.93	19.17	20.5	21.25
	SR	2.74	2.87	3.06	3.21	3.32	3.43	3.48
6	DC	4.17	8.01	11.21	12.65	22.91	20.07	25.76
	SR	3.99	4.44	4.13	4.25	5.42	4.46	5.51
8	DC	3.77	7.61	15.32	12.48	23.02	25.87	25.93
	SR	4.30	4.77	7.17	5.13	7.38	7.45	7.52
10	DC	3.90	8.67	12.56	15.13	22.12	22.58	23.12
	SR	4.78	5.73	7.28	7.37	8.81	8.89	8.95
12	DC	4.32	8.04	10.36	11.94	18.44	20.16	21.28
	SR	8.08	8.82	7.93	8.23	8.24	9.44	9.53
14	DC	4.62	7.10	11.30	12.85	23.27	25.85	25.97
	SR	7.52	8.09	8.32	8.63	12.24	12.49	12.65
16	DC	3.54	6.23	9.26	11.53	18.60	19.21	21.47
	SR	8.20	8.97	9.56	9.74	10.58	10.36	11.01
18	DC	2.91	7.12	13.23	15.11	21.46	22.59	20.18
	SR	9.31	11.27	12.54	12.72	13.07	13.16	11.36
20	DC	5.10	7.00	13.86	15.89	21.79	22.69	22.21
	SR	12.77	9.96	13.60	13.87	14.50	14.80	13.45
22	DC	3.72	7.72	10.72	12.37	18.41	18.84	19.73
	SR	11.08	12.48	12.74	13.27	13.60	13.78	14.03
24	DC	4.12	8.85	13.85	15.89	21.41	22.50	21.21
	SR	14.19	12.57	15.07	15.47	15.85	16.09	14.98
26	DC	2.60	6.50	9.67	10.78	17.06	18.24	19.12
	SR	9.33	11.64	12.59	13.27	14.94	15.26	15.38
28	DC	2.25	11.13	11.82	14.97	17.93	18.51	19.07
	SR	11.77	19.88	18.48	18.81	19.97	19.55	20.21
30	DC	3.54	6.71	9.80	11.34	17.63	18.95	22.03
	SR	11.55	16.20	16.70	17.59	18.34	18.72	21.08

Legendre moments in representation of the 52 characters of the Assamese alphabet in presence of noise. For each character, an image of size 24 $\times$24 and 256 gray levels is obtained by scanning the character.

Each image is corrupted with Gaussian noise of mean 0 and different standard deviations. At each noise level, five noisy images are generated from each character. Therefore we have altogether 260 patterns belonging to 52 different classes. Each class represents one character from the Assamese alphabet. Figure 1 shows an Assamese character and its three noisy samples with mean 0 and standard deviations$(\sigma) 5, 10, 20$ and 30 respectively.

After generating 260 characters belonging to 52 classes, moments of order 0 to 9 (55 moments) are computed using Equation 5 and Equation 12 for each character pattern. Then the mean vectors X_i and the radii R_i are computed for each class using Equation 13 and Equation 14 respectively. Then the values $d(X_i, X_j)$ and $(R_i + R_j)$ are computed for all pairs of classes. Using these values we find out the closest pair clusters and the ratio $DC : SR$ as explained in the previous section.

The experiment is repeated at different levels of noise, i.e., with $\sigma = 4, 6, 8, \ldots, 30$ and considering distances computed from 3 moments (order 0 and 1), 6 moments (up to order 2), …, 55 moments (Upton order 9).

The result for Legendre and Chebyshev moments are given in Table 1 and Table 2 respectively for order of moments $1, 2, 3, 4, 6, 7$, and 8.

It can be observed from table 1 that with Legendre moments of order 0 and 1 clusters are disjoint if the standard deviation (σ) of noise is 6 or less. For higher noise level (i.e., $\sigma > 6$) the clusters are overlapping. At the same time it can seen from table 2 that with Chebyshev moments of order 0 and 1 the clusters remain disjoint up to a noise level with standard deviation of noise 8. Similarly when the standard deviation of noise is 16, the clusters becomes disjoint only when the Legendre moments up to order 4 (15 moments) are considered. But, Chebyshev moments up to order 3 (10 moments) are sufficient to make the clusters disjoint at the same level of noise. From these two tables it can be observed that at all noise level performance of Chebyshev moments is better than Legendre moments so far as the disjointness of the clusters are considered. But at the same time it can be observed that when we go for higher order moments performance of Chebyshev moments degrades.

The results for Legendre moments and Chebyshev moments are graphically presented in Figure 2 and Figure 3 respectively for few levels of noise. In these two figures the minimum of the ratio $DC : SR$ are plotted against the order of moments computed from Legendre (Figure 2) and Chebyshev (Figure 3) moments. A value of $DC : SR$ above one means that all the cluster are disjoint. If its value is less than one then at least one pairs of clusters are overlapping. From both the graphs it can be seen that the ratio increases with the order of moments considered. This means when higher order of moments are considered the clusters are more apart from each other. Also the higher the noise level the lower is the ratio of DC to SR, i.e., when noise is more then the clusters are closer to each other. Figure 4 and Figure 5 give comparison of the two moments with standard deviation of noise 16 and 22 respectively. From these two figures it can be clearly observed that performance of Chebyshev moments is far better than Legendre moments when order of moments is 8 or less. The performance of Chebyshev moments starts decreasing beyond order of moments 6 and it becomes lower than Legendre moments at order 9. Similar results are observed at other levels of noise also.

5. Conclusion

The paper performs a comparative study on the performance of orthonormal Chebyshev moments with Legendre moments for representation of printed noisy characters of the Assamese alphabet. Experimental results show that the performance of both the moments increases initially along with increasing order of moments. Though the Chebyshev moments show a decline in performance when order increases beyond 7, its performance is better than Legendre

Table 2. Distance between Centers (DC) and Sum of Radii (SR) of the closest pair of clusters with Chebyshev moments

σ	DC & DC	Order of moments considered						
		1	2	3	4	6	7	8
4	DC	32.9	88.4	197.3	208.4	360.1	398.1	397.3
	SR	16.9	25.6	33.7	35.8	45.6	49.7	54.9
6	DC	30.6	89.2	159.0	196.8	360.7	391.1	396.2
	SR	30.5	33.2	45.7	55.0	66.2	71.4	77.3
8	DC	47.5	89.5	164.1	206.6	378.2	400.2	406.7
	SR	42.5	51.3	72.6	66.9	95.5	99.2	99.5
10	DC	42.1	97.4	175.7	272.4	380.6	395.7	402.3
	SR	42.4	59.4	72.0	97.2	120.2	140.6	153.2
12	DC	22.0	85.7	160.7	192.5	363.7	390.5	398.2
	SR	54.8	81.0	84.7	93.0	129.0	140.3	158.4
14	DC	35.1	84.9	175.1	201.4	420.8	381.1	390.2
	SR	61.0	78.6	98.4	108.3	173.4	156.3	189.2
16	DC	33.2	74.5	146.4	180.4	363.2	395.1	408.3
	SR	60.6	87.7	121.9	118.1	163.6	184.4	203.2
18	DC	47.2	83.8	159.8	190.7	377.9	393.8	401.4
	SR	79.2	89.9	142.6	134.3	213.8	233.7	248.7
20	DC	47.5	91.1	168.4	233.9	337.3	376.0	394.6
	SR	103.3	117.3	132.6	158.1	198.6	220.8	246.0
22	DC	38.8	96.4	147.1	185.2	342.1	387.1	385.9
	SR	101.7	116.2	145.6	180.6	215.4	239.7	255.7
24	DC	38.7	86.4	149.1	257.9	362.3	417.8	412.6
	SR	93.4	135.0	201.3	224.0	259.1	283.8	271.2
26	DC	24.3	96.0	187.2	279.1	386.6	397.6	404.3
	SR	85.9	115.3	193.5	226.9	275.4	315.7	345.4
28	DC	19.60	89.4	142.9	182.7	382.0	377.1	387.9
	SR	118.0	161.6	184.1	195.9	289.2	325.7	359.7
30	DC	24.9	88.0	173.3	205.1	345.4	436.8	376.1
	SR	119.9	149.1	179.2	213.4	284.2	353.4	358.4

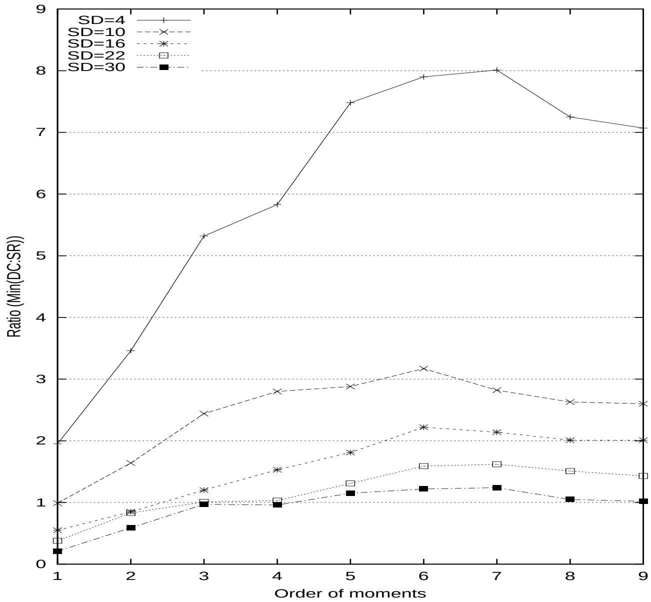

Figure 3. The ratio $DC : SR$ with Chebyshev moments at different noise levels(SD =standard deviation of noise (σ))

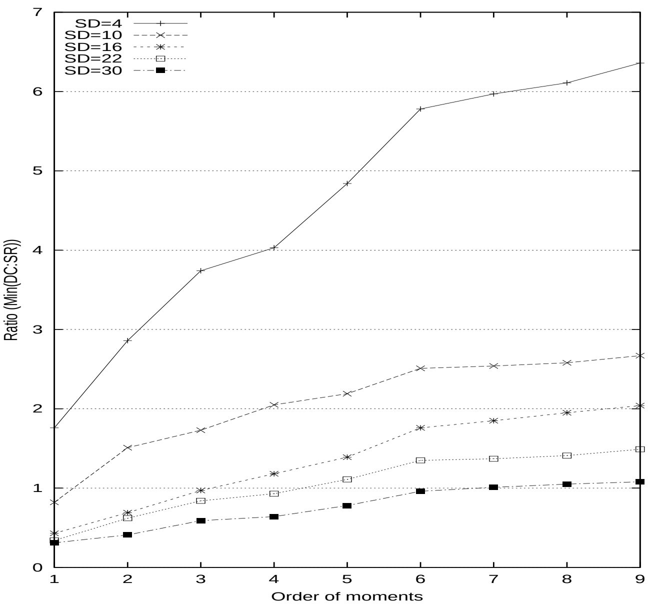

Figure 2. The ratio $DC : SR$ with Legendre moments at different noise levels (SD =standard deviation of noise(σ))

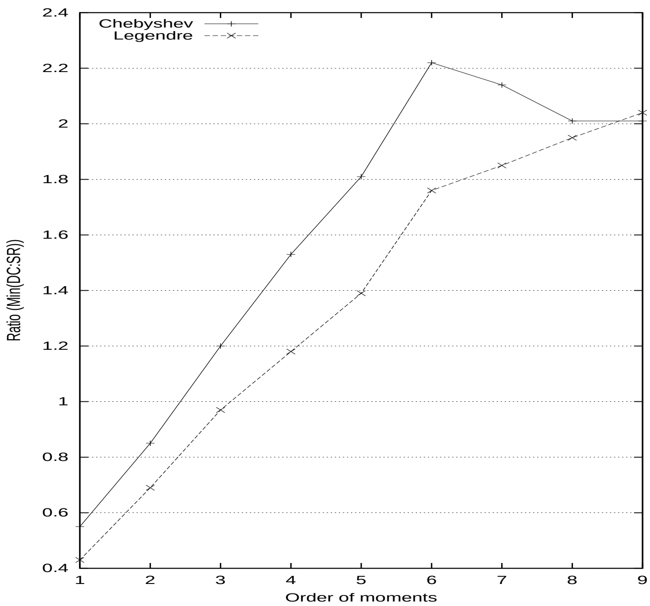

Figure 4. Comparison of Chebyshev moments and Legendre moments at noise level $\sigma = 16$

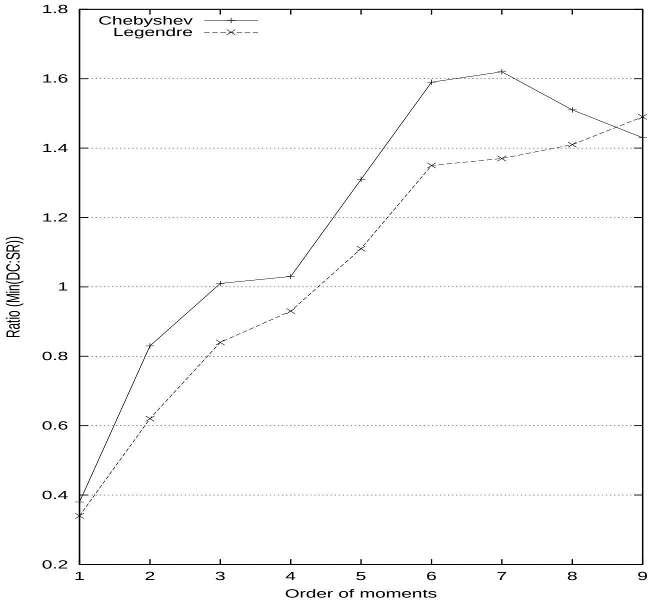

Figure 5. Comparison of Chebyshev moments and Legendre moments at noise level $\sigma = 22$

moments up to order 8. Chebyshev moments up to order 5 can distinguish noisy characters in the alphabet at all noise levels considered. But, Legendre moments up to order 9 is required to distinguish the noisy characters of the alphabet when the standard deviation of noise is 30. Though the experiments were performed for characters of the Assamese alphabet the results of this comparative study may be useful to any character recognition system.

References

[1] B. B. Choudhury and U. Pal. A Complete Printed Bangla ocr System. *Pattern Recognition*, 31(5):531 – 549, 1998.

[2] B. B. Choudhury, U. Pal, and M. Mitra. Automatic Recognition of Printed Oriya Script. *Sadhana*, 27(Part–1):23 – 34, February 2002.

[3] M. K. Hu. Visual Pattern Recognition by Moment Invariants. *IRE Trans. on Information Theory*, 8:179 – 187, 1962.

[4] S. X. Liao. *Image Analysis by Moments*. PhD thesis, University of Monitoba, Canada, 1993.

[5] S. X. Liao and M. Pawlak. On Image Analysis by Moments. *IEEE Pattern Analysis and Machine Intelligence*, 18(3):254 – 266, 1996.

[6] R. Mukundan. Improving Image Reconstruction Accuracy Using Discrete Orthonormal Moments. In *Proc. Intnl. Conf. on Imaging Systems, Science and Technology CISST, Las Vegas*, pages 287 – 293, June 2003.

[7] R. Mukundan, S. H. Ong, and P. A. Lee. Discrete Orthogonal Moment Features using Chebyshev Polynomials. In *Proc. Image and Vision Computing, NewZealand 2000*, pages 20 – 25, November 2000.

[8] R. Mukundan, S. H. Ong, and P. A. Lee. Image Analysis by Tchebyshev Moments. *IEEE Trans. on Image Processing*, 10(9), September 2001.

[9] R. Mukundan and K. R. Ramakrishnan. *Moment Functions in Image Analysis-Theory and Applications*. World Scientific, Singapore, 1998.

[10] U. Pal and B. B. Choudhury. Printed Devnagari Script ocr System. *Vivek*, 10:12 – 24, 1997.

[11] A. K. Pujari, C. D. Naidu, and B. C. Jinaga. An Adaptive and Intelligent Character Recognizer for Telugu Scripts using Multiresolution Analysis and Associative memory. In *Proc. Canadian Conf. on AI, Calagary*, May 2002.

[12] S. Saharia, P. K. Bora, and D. K. Saikia. Representation of Printed Characters with Legendre Moments. In *Proc. 5th Intnl. Conf. on Advances in Pattern Recognition ICAPR 2003, Kolkata*, pages 280 – 283, December 2003.

[13] S. Saharia, S. N. N. Pandit, and M. Borah. Orthogonal Moments and Principal Component Analysis in Image Reconstruction and Pattern Recognition. In *Proc. of the Intl. Sympo. on Intelligent Robotic Systems, Bnagalore*, pages 181 – 188, January 1998.

[14] R. M. K. Sinha. Rule Based Contextual Post Processing for Devanagari Text Recognition. *Pattern Recognition*, 20:475 – 485, 1987.

[15] G. Siromony, R. Chandrashekharan, and M. Chandrashekharan. Computer Recognition of Printed Tamil Characters. *Pattern Recognition*, 10:243 – 247, 1978.

[16] M. R. Teague. Image Analysis via General Theory on Moments. *J. of Opt. Soc. of America*, 70:920 – 930, 1980.