An Alternative Curvature Measure for Topographic Feature Detection

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Abstract. The notion of topographic features like ridges, trenches, hills, etc. is formed by visualising the 2D image function as a surface in 3D space. Hence, properties of such a surface can be used to detect features from images. One such property, the curvature of the image surface, can be used to detect features characterised by a sharp bend in the surface. Curvature based feature detection requires an efficient technique to estimate/calculate the surface curvature. In this paper, we present an alternative measure for curvature and provide an analysis of the same to determine its scope. Feature detection algorithms using this measure are formulated and two applications are chosen to demonstrate their performance. The results show good potential of the proposed measure in terms of efficiency and scope.

1 Introduction

A feature detection system for 2D digital images is a part of the back-end of computer vision systems. It is commonly preceded by an image enhancement system and operates on the intensity values of the image pixels. The detected features which are used by a higher level system for further processing and understanding of the scene, are of two types: (a) perceptual features such as edges, corners, contours, boundaries etc., and (b) topographic features such as ridges, valleys, watersheds etc.

The notion of topographic features, such as ridges, valleys, watersheds etc., is formed by visualising the 2D image function as a surface in 3D space. Features of this kind are detected by exploiting the properties of the image surface. One such useful property, which can be used to detect image features, is the curvature of the image surface. It has been successfully used to detect ridges, valleys, thin nets and crest lines from digital images [1] [2]. In general, curvature can be used to detect features where the image surface bends sharply. Such features are characterised by points of maximal curvature on the image surface.

Detection of features using curvature information requires efficient techniques to calculate the curvature of image surfaces. A common approach to curvature based feature detection is based on the differential geometry of image surfaces

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[3] [4]. For every point (m, n) on an image, a surface $f(m, n)$ is fit to the 'neighborhood' of the point prior to computing the first and second partial derivatives. There exist two curvature measures, namely, the maximum and minimum principal curvatures along two orthogonal principal directions which measure the bend in the surface and are typically used to detect ridges and valleys from digital images. A pixel is defined as a ridge pixel if the magnitude of the maximum principal curvature (MPC) at that pixel is a local maximum in some direction. Depending on the reference coordinate system, a high negative curvature indicates a strong ridge strength while a high positive curvature indicates a strong valley strength, or vice versa. The direction along which the MPC is a maximum is the direction perpendicular to the orientation of the ridge (or valley) at that pixel. An algorithm for the curvature-based feature detection approach described above can be found in [4]. The algorithm involves four steps: Fit a surface $I(m, n)$ to the neighborhood of the point of interest (a local graph representation); compute the first and second partial derivatives of the image function; determine the principle curvatures of the surface; and finally evaluate curvature measures to find desired features.

Curvature can also be estimated without the knowledge of a local graph representation [5] [6]. Such methods calculate the principle curvatures of the surface numerically from an ensemble of directional curvature estimates. Estimation of curvature using these methods seem to perform about as accurately as the analytic techniques [7]. Furthermore, lack of need for a local graph representation reduces the computational load relative to the analytic techniques.

In this paper, we define a curvature measure, called the Surface Tangent Derivative (STD), as an estimate of the curvature of image surfaces. Though slightly different from the true curvature measure, STD lends itself for a computationally efficient implementation. This ability makes it a superior measure than the standard curvature for use in *real time* feature detection applications.

The paper is organised as follows. The proposed curvature measure (STD) is derived in section 2. An analysis of the same is presented in section 3 and an efficient implementation of the measure is presented in section 4; algorithms for detecting two different kinds of topographical features are presented in section 5; and section 6 presents validation of STD on two different applications followed by concluding remarks.

2 The Surface Tangent Derivative (STD)

The curvature at a point on the image surface is a measure of the bend in the surface along a particular direction. Because of this direction-specific nature of curvature, one can define the curvature of the image surface along a particular direction, in terms the curvature of the 1D profile of the image intensity values along that direction. In this section, we present an alternative measure of surface curvature of 2D digital images using such an approach. Before presenting the new measure, we shall first review the definition for curvature of a 1D function.

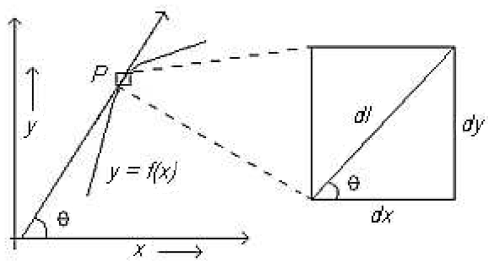

Fig. 1. Illustration of curvature of a 1D function

Let $y = f(x)$ be a 1D function. Let the tangent at a point $P : x$ on this function make an angle θ with the x-axis as shown in Figure 1. If dl is the differential arc length at the point P, then the extrinsic curvature of the function $f(x)$ at this point is defined as:

$$k(x) = \frac{d\theta}{dl} = \frac{d\theta}{\sqrt{dx^2 + dy^2}} = \frac{\frac{d\theta}{dx}}{\sqrt{1 + (\frac{dy}{dx})^2}} \tag{1}$$

Since, θ is the angle made by the tangent with the x-axis, it can be computed as:

$$\theta = tan^{-1}\left(\frac{dy}{dx}\right) \tag{2}$$

Hence, the numerator term $\frac{d\theta}{dx}$ can be computed as:

$$\Upsilon(x) = \frac{d\theta}{dx} = \frac{d}{dx}\left[tan^{-1}\left(\frac{dy}{dx}\right)\right] = \frac{\frac{d^2y}{dx^2}}{1 + \left(\frac{dy}{dx}\right)^2} \tag{3}$$

Substituting the above expression in equation 1, we get:

$$k(x) = \frac{\frac{d^2y}{dx^2}}{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}} \tag{4}$$

which is the true curvature measure. As the point P moves on the curve $y = f(x)$, the tangent angle θ changes. This change over a given arc length dl is the true curvature measure $k(x)$. On a closer examination of equation 1, we can see that the numerator term $\Upsilon(x)$ represents the rate of change the tangent angle with respect to the projection of the arc length over the x axis. Comparing the equations 3 and 4, we see that the two expressions differ only by the power of the denominator. Significantly, $\Upsilon(x)$ will peak sharply at the locations of medial points of ridge profiles (as does $k(x)$), where the first derivative of the profile function vanishes and the second derivative is a negative maximum. We propose using $\Upsilon(x)$ as an alternative to the true curvature measure $k(x)$ since it also provides information about the rate of change the tangent angle as a point moves along a curve. In the case of 2D images, $\Upsilon(x)$ corresponds to a derivative of the angle made by a surface tangent line with the image plane, in some direction. Accordingly, we distinguish it from the true curvature measure, by calling it as the Surface Tangent Derivative (STD). We will now analyse STD as a curvature measure. The analysis is aimed at providing information about the of STD as a feature detector, specifically for ridges/valleys. A theoretical analysis of the scope of STD is also useful to determine its limits as compared to the true curvature measure. A 1D profile-based analysis of the for true curvature measure has been reported in [8]. We follow a similar analysis for STD to assess its scope relative that of true curvature.

3 The Scope of Proposed STD Curvature Measure

A ridge/valley detection technique detects the medial lines of surface structures using their curvature information. Medial lines are loci of 'medial points' of the cross section profiles of ridges/valleys. Therefore, it has been shown [8] that 1D profile functions can be used to perform an analysis of curvature based ridge/valley detection, by reformulating the original 2D framework to detect medial points of 1D profile functions. We begin with some definitions. Let $f : \Re \rightarrow \Re$ be a 1D function. If a point $x = a$ is a point of local maximum of the function $y = |f(x)|$, then it is a point of magnitude maximum (PMMAX) of the function $y = f(x)$. Similarly, if a point $x = a$ is a point of local minimum of the function $y = |f(x)|$, then it is a point of magnitude minimum of the function (PMMIN) $y = f(x)$.

Lemma 1: Let $f : \Re \rightarrow \Re$ be a 1D function for which derivatives upto the second order exist. If

$$(a)\ \left[\frac{dy}{dx}\right]_{x=a} = 0 \ \ and, (b)\ \left[y\frac{d^2y}{dx^2}\right]_{x=a} < 0$$

then, x=a is a PMMAX of the function $y = f(x)$.

Proof: Follows from definition of derivatives. See [8] for details.

We shall now state the criterion for curvature based medial point detection using the STD.

Definition 1: Let $f : \Re \rightarrow \Re$ be a 1D function for which derivatives up to the second order exist. A point $x = a$ is a medial point of the function $y = f(x)$ if it is a PMMAX of $\Upsilon(x)$.

The PMMAX of the curvature is where the first derivative of the curvature vanishes. The derivative of the curvature is found by differentiating the expression in equation 3.

$$\frac{d\Upsilon}{dx} = \frac{\frac{d^3y}{dx^3}\left(1 + \left(\frac{dy}{dx}\right)^2\right) - 2\frac{dy}{dx}\left(\frac{d^2y}{dx^2}\right)^2}{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^2} \tag{5}$$

Considering equation 5, it is clear that the first derivative of the curvature can vanish under four different conditions. These are:

$$\textbf{C1} :: \frac{dy}{dx} = 0, \frac{d^2y}{dx^2} \neq 0, \frac{d^3y}{dx^3} = 0; \quad \textbf{C2} :: \frac{dy}{dx} = 0, \frac{d^2y}{dx^2} = 0, \frac{d^3y}{dx^3} = 0$$

$$\textbf{C3} :: \frac{dy}{dx} \neq 0, \frac{d^2y}{dx^2} = 0, \frac{d^3y}{dx^3} = 0; \quad \textbf{C4} :: \frac{dy}{dx} \neq 0, \frac{d^2y}{dx^2} \neq 0, \frac{d^3y}{dx^3} \neq 0$$

In C4, the numerator as a whole, of the expression on the right hand side of equation 5 goes to zero.

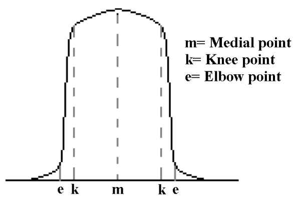

Fig. 2. Cross- section of a ridge and the various points of extremal curvature

The second derivative of the profile function is zero in C2 and C3. Hence, by equation 3, the STD measure of the profile function also goes to zero at such points. Therefore, a point satisfying C2 or C3 cannot be a PMMAX of the STD function. A medial point is either the top of a ridge profile, or the bottom of a valley profile. In other words, the medial points are points of the extremal image intensities. Hence, a PMMAX which satisfies C4 cannot be a medial point of a ridge/valley profile. Such PMMAX occur as 'knee/elbow' points of the ridge profiles as shown in fig. 2. In practice, it is either rejected by setting a threshold or in few rare cases, is wrongly classified as a ridge/valley pixel. Therefore, medial points which satisfy the criterion in Definition 1 should satisfy only C1. However, a point satisfying C1 need not satisfy the criterion in Definition 1. The condition under which a point satisfies C1 is also a PMMAX of the STD function can be found by applying Lemma 1 to the STD expression in 3. This condition is:

$$\left[\frac{d^2y}{dx^2}\left\{\frac{d^4y}{dx^4} - 4\left(\frac{d^2y}{dx^2}\right)^3\right\}\right]_{x=a} < 0 \tag{6}$$

If at some points on the profile, the fourth-derivative is non-zero in addition to C1 being satisfied and the curvature function has a PMMAX, then, the profile function has to satisfy the inequality in 6. This can be proven by expanding the STD expression using the Taylor's series. A detailed proof is given in [8].

We can now identify the classes of ridge/valley profiles detectable using the STD. C1 requires the third-derivative to be zero while the second-derivative is non-zero. There are five different possibilities for the second derivative function:

Class 1: It has a PMMAX where the first derivative vanishes.
Class 2: It is non-zero and is a point of inflection where the first-derivative vanishes.
Class 3: It is a non-zero constant function and there exists a point where the first-derivative vanishes.
Class 4: It has a non-zero PMMIN where the first and fourth-derivatives vanish.
Class 5: It has a PMMIN where the fourth-derivative is nonzero, and condition C1 and the inequality 6 are satisfied at that point.

These correspond to five classes of ridge/valley profiles that can be detected using the STD. The STD measure rejects PMMINs of the curvature function and non-extrema which have also been shown to be rejected by the true curvature measure $k(x)$ [8]. Thus in terms of scope, the STD is identical to the true curvature measure. However, due to the difference (lower for STD) in the power of the denominator of $\Upsilon(x)$ and $k(x)$ we can expect a lower specificity for the STD, which is not a serious problem in many applications. In the next section, we present a efficient method for computing the proposed STD measure.

4 Calculation of STD

The proposed scheme calculates the STD measure at a pixel location (which corresponds to a point on the image surface) for a particular direction α. Let the STD measure at a pixel location (n, m), along a direction α, be denoted by $\Gamma(n, m, \alpha)$. Using equation 3, the angle made by the surface tangent with the image plane at a pixel (n, m), along the direction α in the base-plane, is calculated as:

$$\Psi(n, m, \alpha) = tan^{-1}[I_\alpha(n, m)] \tag{7}$$

where $I(n, m)$ is the image function, $I_\alpha(n, m)$ is the first directional derivative along the direction α. The STD is the derivative of the angle Ψ and given as:

$$\Gamma(n, m, \alpha) = \Psi(n, m, \alpha) \tag{8}$$

In the above equation, $\Psi(n, m, \alpha)$ is the directional derivative of the surface tangent angle Ψ, in the direction α. We can use an efficient numerical technique to estimate these derivatives. In theory, at any given point on the surface, an STD measure can be obtained for every possible direction measuring the bend in the surface along that particular direction. However, for a function defined over a discrete grid (such as digital images), it is possible to evaluate the STD measure only along a finite number of directions. Thus, the STD can be calculated in four directions at each point, and the results be combined to obtain the two principle curvatures. The four different directions correspond to the 8 neighbours of a pixel. These four directions are specified in the set $\Omega = \{-45\,^\circ, 0\,^\circ, 45\,^\circ, 90\,^\circ\}$.

5 Topographical Feature Detection Using STD

Topographical features are characterised by high values of STD measure. In this section, we present algorithms for detection of two different kinds of features namely ridges/valleys which are characterised by points of maximum curvature in a *particular* direction and hills/craters which are characterised by maximum curvature in *all* directions.

5.1 Ridge Detection Algorithm

Let I(n,m) be the image function.Calculate the STD for four different directions as $\Gamma(n,m,a)$, $\alpha \in \Omega$ *. Let* t_Γ *be the threshold for ridge strength. For every pixel location* (n,m)*, do the following:*

1. *Evaluate* $|\Gamma_{max}| = max\{\Gamma(n,m,\alpha); \alpha \in \Omega\}$ *and the corresponding orientation* α_{max}.
2. *If* $|\Gamma_{max}| > t_\Gamma$ *and* $\Gamma(n,m,\alpha_{max}) < 0$*, then:*
   – *Check if* $|\Gamma_{max}|$ *is greater than* $|\Gamma(n,m,\alpha)|$ *of the neighbouring pixels corresponding to the direction* α_{max}*. If yes, then mark the pixel* (n,m) *as a ridge pixel. Else,do nothing.*
   Else: Do nothing.

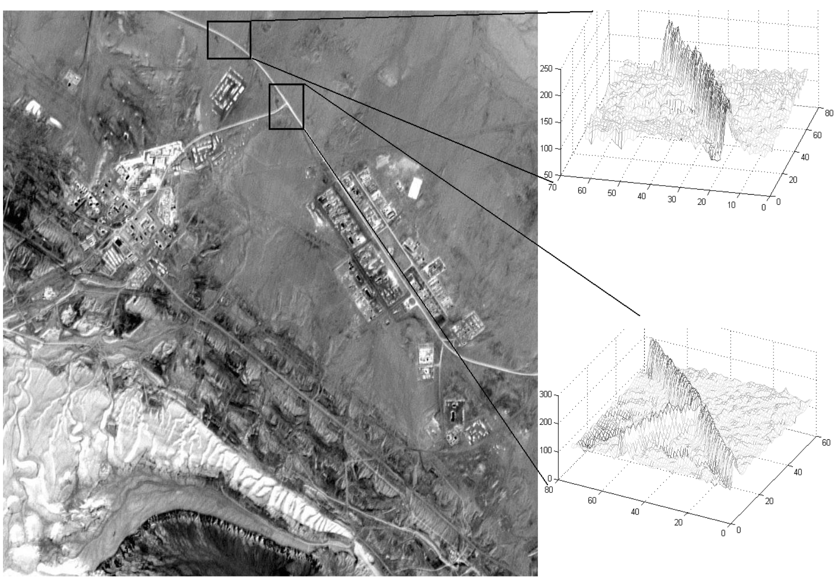

Fig. 3. Satellite test image and 3D profiles of ridges within sub regions

5.2 Hill Detection Algorithm

The ridge detection algorithm can be modified to label a pixel as a hill pixel if the value of $|\Gamma(n,m)|$ is a maximum in *all* directions.

6 Case Studies

We have chosen two different applications to test feature (ridges and hills) detection using the STD. The purpose of these case studies is to validate the working of STD measure and not to evaluate it against existing solutions to these problems.

6.1 Road Detection in Satellite Images

Roads in satellite images can be seen as narrow ridges or valleys in the intensity plane (shown in fig. 3). Curvature-based approaches have been used to extract road structures [9]. The ridge detection algorithm was used to extract roads from the satellite image using the STD measure computed using a fixed sized mask of 5×5. Fig. 4 shows the obtained output image.

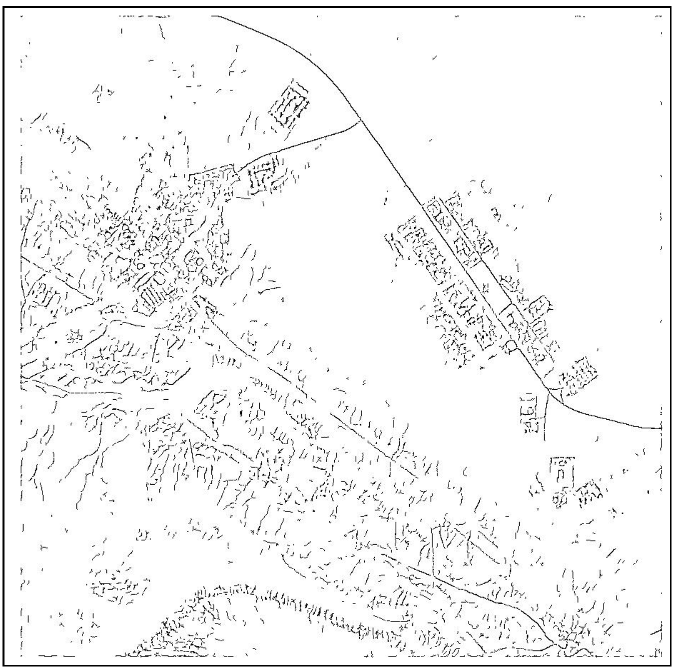

Fig. 4. Output image showing ridges (in white)

The presented scheme for ridge detection is able to extract all ridge profiles in the given input image. The image contains different profiles of the ridges which vary in their shape and intensity. The shape of a ridge is more important in extraction rather than the intensity values. In general, road detection methods involve two processes: road detection, post-processing. Post-processing uses domain knowledge to reduce the outliers from the road extraction module. In our result, we only present the results of road extraction process. Standard criteria to evaluate any road detection method include continuity in roads' structure, no under- and over-detection. The presented detection result appear to meet the given criteria. The medial lines of the salient roads are continuous in the output image. The profiles which are similar to ridges are extracted very well. STD's capability to extract different types of ridges profiles (in equation 3) are well illustrated in this case. However, the result seems to be noisy which is due to the presence of structures whose profiles are similar to the ridge profiles. As of now, our scheme does not use any domain knowledge of the structures such as their location, surround regions, characteristics of roads, etc. These information can be used to improve the results.

6.2 Detection of Micro-Aneurysms (MAs) in Fundus Fluorescein Angiograms (FFA)

Most diseases of the retina alter the structure and the functionality of the vasculature in the retina. One such disease, the Diabetic Retinopathy (DR), leads to the occurance of neo-vascularisation. Neo-vascularisation is the growth of new blood vessels branching out of the existing vessels. The early sprouts of these new vessels are called microaneurysms (MA). They are bulb-like microscopic structures occuring as small and bright circular disks in FFA images. MAs are therefore an important lesion in any retinopathy screening programme. Computer automated detection of MAs offers a fast, objective and reproducible method for quantifying DR, which also reduces the manual workload.

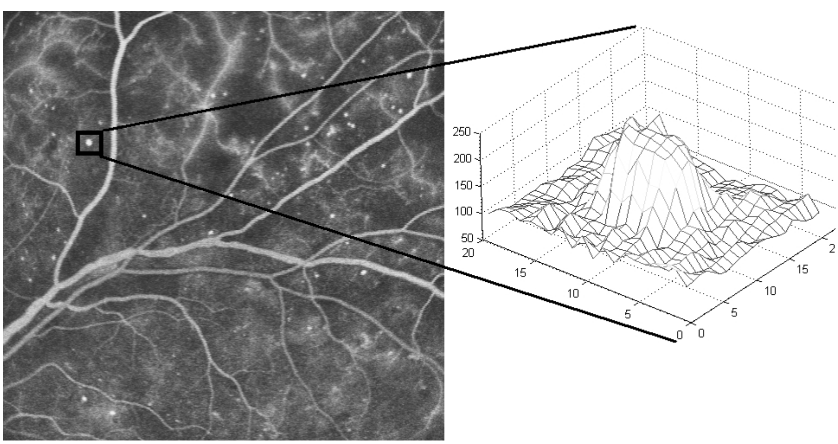

Fig. 5. A sample FFA image and the 3D profile of an MA

MAs appear as bright tiny spots in FFA images. If the image is visualised as a surface in 3D space, MAs form hill-like features (shown in Fig. 5). Hence, a hill detection algorithm can be used to detect MAs. A typical cross-sectional profile of a hill-like feature along some direction, is similar to profile shown in Fig. 2. The medial point of such a profile is a hill point and characterised by maximum curvature in *all* directions.

The algorithm given in section 5.2 is used to extract hill-like features (MAs). A threshold t_{Γ} for the strength of the hill-ness of a pixel is set so that the hills detected have a certain minimum strength. This helps in filtering out noise pixels which are otherwise prone to detection as hill pixels. Noise is further reduced by smoothing the image using 5×5 gaussian mask before the STD computation. The MAs vary in their size and appear as hills of different shapes (narrow to wide). As a fixed size mask is inadequate to capture all these features, we used multiple scales to compute the STD measures and collated (using a simple logical OR operation) the results across scales. Computation at 4 scales was used with Sobel masks of sizes (from 5×5 to 11×11) were used.

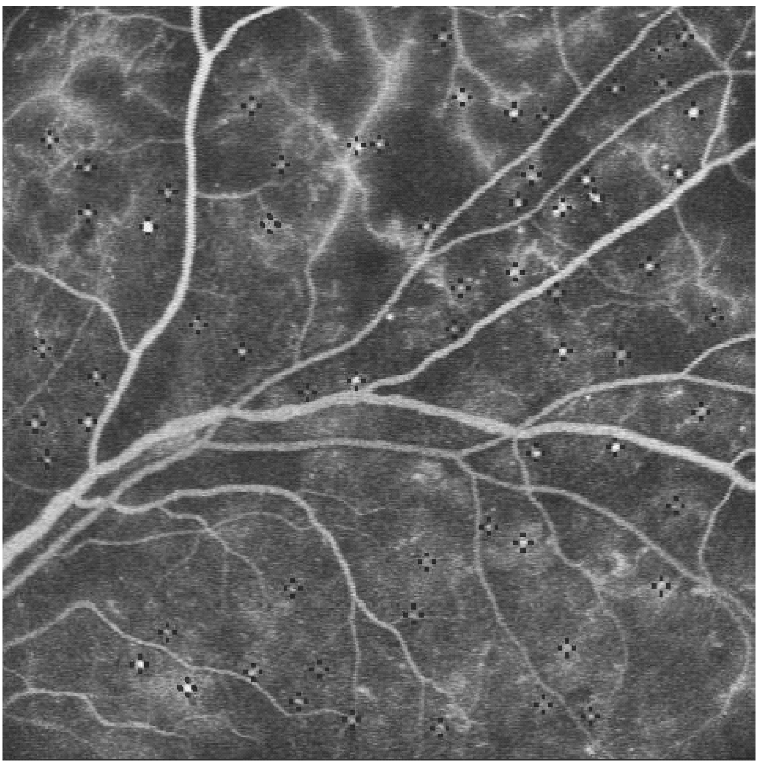

Fig. 6. Output of multi-scale detection of MAs shown superimposed on the original image

The results of MA detection are shown in Figure. 6 superimposed on the original image for convenience. MAs are indicated by enclosing with 4 small lines. The STD-based hill detection algorithm is able to detect all MAs present in the given

FFA image. However, it also gives false responses close to the veins which have similar characteristics as MAs. It is difficult to avoid such false responses in the detection process. These false responses can be removed using post-processing steps. It is noteworthy that our tests done over 25 FFA images (and visually verified by a medical expert) have shown that the detection rarely resulted in any false negatives [10]. In summary, we can state that STD based method provides a reliable extraction of hill profiles which helps to develop solutions for various image analysis problems.

7 Discussion and Conclusion

Topographical feature detection using curvature information is a widely used technique. We have re-examined the definition for extrinsic curvature and proposed a simpler curvature measure (STD) based on the derivative of the surface tangent to an image surface. A theoretical analysis of the the proposed measure has also been presented which shows its scope to be equal to that of the true curvature measure. STD-based algorithms were used to detect roads and MAs and the obtained results demonstrated the ability of STD to capture a variety of ridge/hill profiles. The results also illustrated that continuous ridge lines can be obtained using the STD-based approach. Theoretically, the STD has a lower specificity compared to the true curvature measure. The conducted experiments on ridge/hill detection using the STD did not focus on this point. Analytical and empirical determination of the degree of specificity of STD vs. true curvature is currently being investigated.

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