

# Two-Dimensional Optimal Transform for Appearance Based Object Recognition

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**Abstract.** This paper proposes a new method of feature extraction called two-dimensional optimal transform (2D-OPT) useful for appearance based object recognition. The 2D-OPT method provides a better discrimination power between classes by maximizing the distance between class centers. We first argue that the proposed 2D-OPT method works in the row direction of images and subsequently we propose an alternate 2D-OPT which works in the column direction of images. To straighten out the problem of massive memory requirements of the 2D-OPT method and as well the alternate 2D-OPT method, we introduce bi-projection 2D-OPT. The introduced bi-projection 2D-OPT method has the advantage of higher recognition rate, lesser memory requirements and better computing performance than the standard PCA/2D-PCA/Generalized 2D-PCA method, and the same has been revealed through extensive experimentations conducted on COIL-20 dataset and AT&T face dataset.

**Keywords:** Principal Component Analysis; Optimal Transform; Appearance Based Model; Object Recognition; Face Recognition.

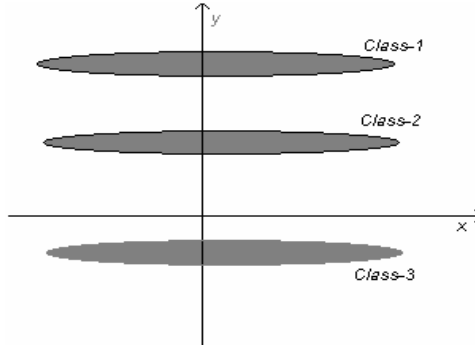
## 1 Introduction

Appearance based object recognition methods have demonstrated their success in various visual learning and recognition chores such as 3D object recognition, face recognition, ear recognition, palm recognition, and tracking. In particular, principal component analysis (PCA) [5, 7, 8, 9] based methods have been proposed and shown to have good performance. The PCA has also been exploited for accurate identification of faces [2, 10, 13], palms [11], and ears [2]. The drawback of the conventional PCA based approaches is the curse of dimensionality as the size of the covariance matrix is proportional to the size of images. In addition to this, specifically to the application of face recognition, an alternative model, *Fisherfaces* [1], a derivative of Fisher's Linear Discriminant (FLD) has been proposed. The objective of the FLD is to find the optimal projection for the samples such that the discrimination ratio of between-class scatter matrices to within-class scatter matrices reaches its

maximum. So far, the FLD method and its variants have been well received by the face recognition community. However, it should be noticed that the PCA/FLD methods [2, 5, 7, 8, 9, 10, 13] are based on the analysis of vectors. When dealing with images, we should firstly transform the image matrices into vectors. Then based on these vectors, the covariance matrix is calculated and the optimal projection is obtained. As object images are high-dimensional patterns, it is difficult to evaluate the covariance matrix in such a high-dimensional vector-space. To overcome this drawback, Yang et al. [15] proposed a image projection technique called 2D-PCA that is directly based on the analysis of original image matrices. The Generalized 2D-PCA method proposed by Kong et al. [4] overcomes the limitations of 2D-PCA.

On the other hand, we have also seen an evolutionary improvement in the use of PCA based approach for efficient representation and recognition of 3D objects. The major advantage of the approach is that both learning and recognition are performed using just two-dimensional intensity images without any low-level or intermediate-level processing. However, as noticed by many researchers [5, 8, 9, 10] the method in its standard form cannot handle problems such as occlusion, and of varying background. Pentland et al. suggested the use of modular eigenspaces [10] to alleviate the problem of occlusion. Ohba and Ikeuchi [9] proposed the eigen-window method to be able to recognize partially occluded objects. But, due to local windows, these methods lack the global aspect and usually require further processing. To eliminate the effects of varying background, Murase and Nayar [8] introduced the search window, which is the AND area of the object regions of all the images in the training image set. However, the assumption is too restrictive and fails for some class of object models as mentioned in their work itself. Moreover, the target object may be occluded by other target objects which are the images of the training set only, rather than some foreign object. In order to alleviate these problems, Leonardis and Bischof [5] proposed a robust and an efficient approach which is based on multiple eigenspaces. A novel self-organizing framework has been used in their work to construct multiple, low-dimensional eigenspace from a set of training images.

However, it is observed from [3, 12] that the idea of principal component transform is based on the reduction of the dimension of original image vectors using some linear mapping such that the resulting feature vectors show pairwise maximum distance. Besides, feature vectors resulting from PCA allow the reconstruction of images with minimal mean quadratic error. If the distribution of features is such that the principal axes of all classes are parallel to each other, the projected features will allow no discrimination of these classes. This problem is called as ADIDAS problem [12] and is illustrated in Fig. 1 considering a 2-D example, where we project the features onto the  $x$ -axis. Hence, an alternative objective function is introduced in [3] which eliminates the ADIDAS problem. However, as noticed by many researchers, the computational complexity in the evaluation of covariance matrix still exists in this approach. Motivated by [4, 15], we proposed 2D-OPT and its variants to eliminate the problem of massive memory requirements, higher computational complexity involved in covariance matrix computation, and as well to resolve the ADIDAS problem.



**Fig. 1.** The 2D-PCA and ADIDAS problem

The rest of the paper is organized as follows. In section 2, we discussed the working model of the 2D-PCA method along with its limitations. In section 3, we propose 2D-OPT transform and establish that the proposed 2D-OPT works in the row direction of images and hence an alternate 2D-OPT which works in the column direction of images is introduced and in sequel a combined model, bi-projection 2D-OPT is proposed. The results of the experiments are presented in section 4 and conclusions are given in section 5.

## 2 Problems in 2D-PCA

**Working model of 2D-PCA:** Training is a process of acquiring features from available training images and storing them in a knowledge base for the purpose of recognizing an unknown future scene image. Given a set of samples of each class, the 2D-PCA approach extracts most informative features which could establish a high degree of similarity between samples of the same class and a high degree of dissimilarity between samples of two different classes.

Formally, let there be  $T$  number of classes each with  $k_i, i=1...T$ , number of training images. Therefore, we have totally  $N = \sum_{i=1}^T k_i$  number of training images. Let  $A_i^j$  be an image of size  $m \times n$  representing the  $j^{\text{th}}$  sample in the  $i^{\text{th}}$  class. Let  $C$  be the average image of all  $N$  training images. In 2D-PCA, the scatter matrix  $G$  is computed as follows.

$$G = \frac{1}{N} \sum_{i=1}^T \sum_{j=1}^{k_i} (A_i^j - C)^T (A_i^j - C) \tag{1}$$

Once  $G$  is computed, it is recommended to find the optimal projection axis  $X$  so that the total scatter of the projected samples of the training images is maximized. For this purpose, the criterion used is,

$$J(X) = X^T G X \tag{2}$$

It is a well-known fact that the eigenvector corresponding to the maximum eigenvalue of  $G$  is the optimal projection axis which maximizes  $J(X)$ . Generally, as it is not enough to have only one optimal projection axis, we usually go for  $d$  number of projection axes, say  $X_1, X_2, \dots, X_d$ , which are the eigenvectors corresponding to the first  $d$  largest eigenvalues of  $G$ . In 2D-PCA, once these  $X_1, X_2, \dots, X_d$  are computed, each training image  $A_i^j$  is then projected onto these  $X$ 's to obtain the feature matrix  $W_i^j$  of size  $m \times d$  of the training image  $A_i^j$ . So, during training, for each training image  $A_i^j$ , a corresponding feature matrix of size  $m \times d$ ,  $d \ll n$ , is constructed and stored in the knowledge base for matching at the time of recognition.

**Limitations of 2D-PCA:** Albeit the above described 2D-PCA overcomes the limitations of standard PCA based approaches, still it has some shortcomings. As noticed by many researchers, 2D-PCA has massive memory requirements for feature representation and hence consumes much recognition time. Problems occur if the distribution of features is such that the principal axes of all classes are parallel to each other, resulting in ADIDAS problem (see Fig. 1). Hence, the projected features will allow no discrimination of these classes. For this situation, projection on the  $y$ -axis would provide discrimination among the classes. Hence, in general, it is necessary to have another plausible optimization criterion, which does not show the disadvantages of 2D-PCA.

### 3 Two-Dimensional Optimal Transform (2D-OPT)

The scatter matrix introduced in this work is in such a way that the features of the same class have minimum distance and possess maximum distance to other classes. However, knowledge of the classified sample set, like FLD, is required in this case.

#### 3.1 Learning Formulation in 2D-OPT

To devise 2D-OPT, we propose to compute the scatter matrix  $H_r$  as follows. Let  $\bar{C}_i$  be the average image of all  $k_i$  training images of the  $i^{\text{th}}$  class.

$$H_r = \frac{2}{T(T-1)} \sum_{p=2}^T \sum_{q=1}^{p-1} (\bar{C}_p - \bar{C}_q)^T (\bar{C}_p - \bar{C}_q) \tag{3}$$

Using this scatter matrix  $H_r$ , similar to the original 2D-PCA, in this proposed model also we find the optimal projection axis  $Y$  so that the total scatter of the projected samples is maximized using the same criterion introduced in 2D-PCA given by,

$$J(Y) = Y^T H_r Y \tag{4}$$

Thus, the eigenvectors of  $H_r$  are computed and then  $r$  numbers of eigenvectors corresponding to the first  $r$  largest eigenvalues of  $H_r$  are chosen. Finally projection of

a training image onto these optimal projection axes results with a feature matrix of the respective training image. That is, if  $Z_i^j$  represents the feature matrix of  $A_i^j$ , then

$$Z_i^j = A_i^j Y \tag{5}$$

It should be noted that the above described scatter matrix (Eq. (3)) in the feature space is equivalent to an optimal transform  $\psi$  given by,

$$\psi = \frac{2}{T(T-1)} \sum_{p=2}^T \sum_{q=1}^{T-1} (\mu_p - \mu_q)(\mu_p - \mu_q)^T, \tag{6}$$

where  $\mu_i, i=1\dots T$ , is the average feature matrix of all feature matrices of the  $i^{\text{th}}$  class. Hence, using an optimal transform, the distance between the class centers is maximized.

### 3.2 Recognition

Let  $I$  be an image given for recognition. Let  $I^l$  be its projected image onto the  $r$  number of optimal projection axes computed by  $I^l = IY$ . Given two images, say  $v_{i_1}$  and  $v_{i_2}$  of any object(s)/face(s), represented by feature matrices  $Z_1^{i_1} = [z_1^{i_1}, z_2^{i_1}, \dots, z_q^{i_1}]$  and  $Z_2^{i_2} = [z_1^{i_2}, z_2^{i_2}, \dots, z_q^{i_2}]$ , the similarity  $dist(Z_1^{i_1}, Z_2^{i_2})$  is defined as

$$dist(Z_1^{i_1}, Z_2^{i_2}) = \sum_{j=1}^q \|z_j^{i_1} - z_j^{i_2}\|_2 \tag{7}$$

where  $\|a - b\|_2$  denotes the Euclidean distance between the two vectors  $a$  and  $b$ . If the feature matrices of the training images are  $Z_1, Z_2, \dots, Z_N$ , and each image belongs to some object  $O_i$ , then for a given test image  $I^l$ , if  $dist(I^l, Z_i) = \min_j dist(I^l, Z_j)$  and  $Z_i \in O_i$ , then the resulting decision is  $I^l \in O_i$ .

**Theorem 1:** The 2D-OPT approach works in the row direction of images.

**Proof**

Let  $\bar{C}_i = \left[ \binom{(1)}{} \bar{C}_i \right]^T \binom{(2)}{} \bar{C}_i \dots \binom{(m)}{} \bar{C}_i \right]^T$ , where  $\binom{(l)}{} \bar{C}_i$  denotes the  $l^{\text{th}}$  row vector of  $\bar{C}_i$ .

The Eq. (3) can be written as

$$H_r = \frac{2}{T(T-1)} \sum_{p=2}^T \sum_{q=1}^{p-1} \left( \sum_{k=1}^m \binom{(k)}{} \bar{C}_p - \binom{(k)}{} \bar{C}_q \right)^T \left( \binom{(k)}{} \bar{C}_p - \binom{(k)}{} \bar{C}_q \right) \tag{8}$$

Equation (8) implies that the image covariance matrix  $H_r$  is obtained from the outer product of the row vectors of mean images. Therefore we claim that the 2D-OPT method works in the row direction of images.  $\square$

In the following section, we present a modified version of this 2D-OPT, called alternate 2D-OPT, which works in the column direction of images. It shall be noticed that both the 2D-OPT method and the alternate 2D-OPT method help us in reducing dimension only either in the row direction or in the column direction. Thus, a combined model called bi-projection 2D-OPT which works in both the directions is also presented in the next section. The advantage of this combined approach is that the reduction in dimensionality can be achieved in both row and column directions without any deterioration in recognition performance.

### 3.3 Alternate 2D-OPT

In alternate 2D-OPT, we propose to compute the scatter matrix  $H_c$  as follows.

$$H_c = \frac{2}{T(T-1)} \sum_{p=2}^T \sum_{q=1}^{p-1} (\bar{C}_p - \bar{C}_q)(\bar{C}_p - \bar{C}_q)^T \tag{9}$$

It shall be observed that  $H_c$  in Eq. (9) is obtained in this new formulation as outer products of column vectors unlike  $H_r$  (Eq. (3)) in the case of the 2D-OPT. Using this scatter matrix, similar to the original 2D-PCA, in this alternate 2D-OPT model also we find the optimal projection axes  $V$  ( $m \times s$ ) so that the total scatter of the projected samples is maximized using the same criterion given by

$$J(V) = VH_c V^T \tag{10}$$

Thus, the eigenvectors of  $H_c$  are computed and then  $s$  numbers of eigenvectors corresponding to the first  $s$  largest eigenvalues of  $H_c$  are chosen. Finally projection of a training image onto these optimal projection axes results with a feature matrix of the respective training image. That is if  $U_i^j$  represents the feature matrix of  $A_i^j$ , then

$$U_i^j = V^T A_i^j \tag{11}$$

**Recognition:** Let  $I$  be an image given for recognition. Let  $I^1$  be the feature matrix obtained by projecting  $I$  onto  $V$ , i.e.,  $I^1 = V^T I$ . By using a Euclidean distance based nearest neighbor classifier, the class label of  $I$  is obtained as explained in section 3.2.

**Theorem 2:** The alternate 2D-OPT approach works in the column direction of images.

**Proof**

Let  $\bar{C}_i = \left[ \binom{(1)}{\bar{C}_i} \binom{(2)}{\bar{C}_i} \dots \binom{(n)}{\bar{C}_i} \right]$ , where  $\binom{(l)}{\bar{C}_i}$  denotes the  $l^{th}$  column vector of  $\bar{C}_i$ .

The Eq. (9) can be written as

$$H_c = \frac{2}{T(T-1)} \sum_{p=2}^T \sum_{q=1}^{p-1} \left( \sum_{k=1}^n \binom{(k)}{\bar{C}_p} - \binom{(k)}{\bar{C}_q} \right) \left( \sum_{k=1}^n \binom{(k)}{\bar{C}_p} - \binom{(k)}{\bar{C}_q} \right)^T \tag{12}$$

Equation (12) implies that the image covariance matrix  $H_c$  is obtained from the outer product of the column vectors of mean images. Therefore we claim that the alternate 2D-OPT method works in the column direction of images.  $\square$

### 3.4 Bi-projection Two-Dimensional Optimal Transform (B2D-OPT)

In the preceding subsection, we proposed an alternate 2D-OPT concept which works in the column direction capturing information between columns of images. On the other hand, the 2D-OPT works in the row direction capturing information between rows of images. In this subsection, we recommend to project the images on both the directions simultaneously while extracting feature matrices.

Let  $Y$  denote  $n \times r$  optimal projection matrix obtained as explained in the 2D-OPT method (Section 3.1) and let  $V$  denote the  $m \times s$  matrix obtained by the alternate 2D-OPT method (Section 3.3). During training, each training image  $A_i^j$  is projected onto both  $Y$  and  $V$  simultaneously to obtain the respective feature matrix  $F_i^j$  which is of dimension  $s \times r$  as follows.

$$F_i^j = V^T A_i^j Y \quad (13)$$

**Recognition:** Let  $I$  be an image given for recognition. Let  $I^l$  be the feature matrix obtained by projecting  $I$  onto  $V$  and  $Y$  simultaneously, i.e.,  $I^l = V^T I Y$ . By using a Euclidean distance based nearest neighbor classifier, the class label of  $I$  is obtained as explained in section 3.2.

Thus, the B2D-OPT algorithm for training a system is as follows.

**Algorithm:** B2D-OPT [TRAINING PHASE]

**Input:** Set of images:  $\{A_i^j \mid i = 1 \dots T, j = 1 \dots k_i\}$

**Output:** Knowledge base:  $\mathcal{F} = \{F_i^j \mid i = 1 \dots T, j = 1 \dots k_i\}$

**Procedure:**

#### A. [Computation of optimal projection axes in the row direction: $Y$ ]

- a. Compute the image scatter matrix  $H_r$  as explained in section 3.1 (Eq. (3)).
- b. Find the eigenvectors of  $H_r$ .
- c. Choose  $r$  eigenvectors say  $Y_1, Y_2, \dots, Y_r$  associated with the first  $r$  largest eigenvalues of  $H_r$  and let  $Y = (Y_1, Y_2, \dots, Y_r)$ .

#### B. [Computation of optimal projection axes in the column direction: $V$ ]

- a. Compute the image scatter matrix  $H_c$  as explained in section 3.3 (Eq. (9)).
- b. Find the eigenvectors of  $H_c$ .
- c. Choose  $s$  eigenvectors, say  $V_1, V_2, \dots, V_s$  associated with the first  $s$  largest eigenvalues of  $H_c$  and let  $V = (V_1, V_2, \dots, V_s)$ .

### C. [Creation of Knowledge base: $\mathcal{F}$ ]

$$a. \quad \mathcal{F} = \{ F_i^j = V^T A_i^j Y \mid i = 1 \dots T, j = 1 \dots k_i \}$$

**Algorithm B2D-OPT Training ends.**

The corresponding recognition algorithm is as trivial as follows.

**Algorithm:** B2D-OPT Recognition

**Input:** Test image,  $I (m \times n)$   
 Knowledge base,  $\mathcal{F}$ ,  
 Optimal projection axes:  $Y$ ,  
 Optimal projection axes:  $V$

**Output:** Class label of  $I$

#### Procedure:

1. Obtain the feature matrix  $I^l$  of the input image  $I$  using  $Y$  and  $V$ ,  
 $I^l = V^T I Y$ .
2. Find  $F_p^q$  such that
 
$$\|I^l - F_p^q\|_2 = \arg \min \left( \|I^l - F_i^j\|_2, \forall i = 1 \dots T, j = 1 \dots k_i \right),$$
 where  $\|\cdot\|_2$  denotes Euclidean distance.
3. Classify the test image  $I$  as a member of  $p^{\text{th}}$  class.

**Algorithm B2D-OPT Recognition ends.**

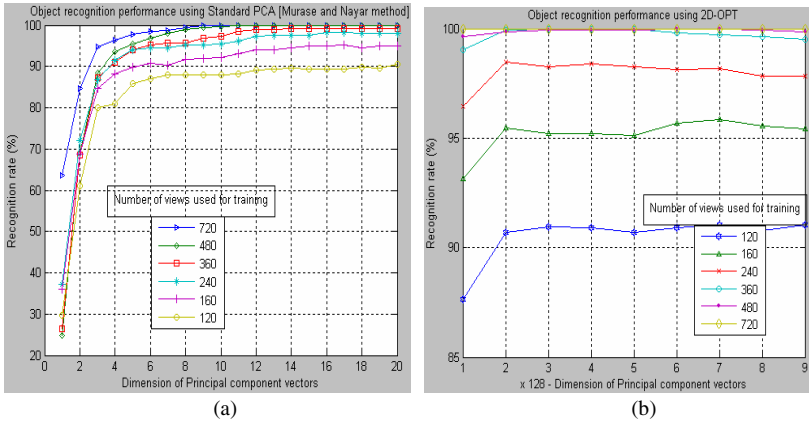
## 4 Experimental Results

**Experiments on COIL dataset:** In this section, we present several experiments conducted to demonstrate the performance of the proposed method for object recognition. We performed all experiments on the standard set of images, COIL-20 [<http://www1.cs.columbia.edu/CAVE/research/soflib/coil-20.html>] which is used by many researchers as a bench mark dataset to verify the validity of their proposed object recognition models. Each object is represented in the database by 72 views obtained by rotating the object in  $5^\circ$  intervals (1440 views in total).

We have conducted a series of experiments to compare the performances of the 2D-OPT, the alternate 2D-OPT, the B2D-OPT and the standard PCA (1D-PCA) [7] methods with varying number of training views. More specifically, we have considered alternate views and tested with the remaining views. Similarly, we have conducted experiments considering 480, 360, 240, 160 and 120 views as training views of the COIL-20 database choosing 24, 18, 12, 8 and 6 views respectively from each object and the recognition performances have been obtained considering the remaining views as test views. The computing time taken by each method (1D-PCA [7], 2D-OPT, alternate 2D-OPT and B2D-OPT) during feature extraction for different



training samples is given in Table-1. Table-1 also summarizes the recognition accuracy of each method. It should be noticed that the 2D-OPT method and its variants consume less time when compared to 1D-PCA method for feature extraction and in addition, they have relatively higher recognition rate. As the number of training samples per object set is increased, the relative gain among the 2D-OPT, alternate 2D-OPT, the B2D-OPT and the 1D-PCA becomes more apparent. Figures 2(a), 2(b), 2(c), and 2(d) show respectively the recognition performance of the 1D-PCA, the 2D-OPT, the alternate 2D-OPT and the B2D-OPT methods with varying number of dimensions of feature vector with varying number of training samples. This experiment is conducted to reveal the superiority of the proposed approach over a well accepted method (Murase and Nayar [7]) for 3D object recognition.



**Fig. 2.** Recognition performance with varying number of training samples and varying number of principal components- (a) Standard PCA (1D-PCA); (b) 2D-OPT; (c) Alternate 2D-OPT; (d) B2D-OPT, on COIL-20 database

**Table 1.** Object recognition performance of 1D-PCA, 2D-OPT, alternate 2D-OPT and B2D-OPT

No. of views used to train	No. of views used to test	Computing time for feature extraction (in secs.)				Percentage of recognition			
		1D-PCA (20-D PCs)	2D-OPT (128x10 PCVs)	Alternate 2D-OPT (10x128 PCVs)	B2D-OPT (10x10 PCVs)	1D-PCA (20-D PCVs)	2D-OPT (128x10 PCVs)	Alternate 2D-OPT (10x128 PCVs)	B2D-OPT (10x10 PCVs)
720	720	4080.68	<b>72.578</b>	<b>72.453</b>	<b>67.921</b>	100	<b>100</b>	<b>100</b>	<b>100</b>
480	960	1091.33	<b>48.812</b>	<b>47.296</b>	<b>47.218</b>	99.27	<b>99.79</b>	<b>99.90</b>	<b>100</b>
360	1080	167.98	<b>35.468</b>	<b>35.359</b>	<b>34.421</b>	96.48	<b>99.26</b>	<b>98.98</b>	<b>99.35</b>
240	1200	58.906	<b>23.609</b>	<b>23.765</b>	<b>24.015</b>	94.50	<b>97.42</b>	<b>97.58</b>	<b>98.00</b>
160	1280	35.953	<b>16.281</b>	<b>16.375</b>	<b>17.468</b>	90.86	<b>94.92</b>	<b>94.22</b>	<b>95.08</b>
120	1320	28.766	<b>12.375</b>	<b>12.671</b>	<b>13.609</b>	86.67	<b>90.23</b>	<b>90.83</b>	<b>91.67</b>

**Experiments on AT&T dataset:** We have also conducted experiments on the standard AT&T face database [<http://www.uk.research.att.com/facedatabase.html>] in order to corroborate the success of the proposed methodology even for face recognition. This face database contains images from 40 individuals, each providing 10 different images of size 112x92. In our experiment, we have considered alternate five samples per class during training and the remaining samples for testing. The recognition performances of the methods 2D-PCA [15], Generalized 2D-CPA [4], 2D-OPT, alternate 2D-OPT and B2D-OPT with varying dimension of feature vectors are given in Fig. 3. The running times of 2D-PCA [15], Generalized 2D-PCA [4], 2D-OPT, alternate 2D-OPT and B2D-OPT with varying dimension of feature vectors are given in Fig. 4. Table-2 gives a comparative analysis of the methods [4, 15] with respect to their running times and dimension of feature vectors. It can be observed from Table-2 that the proposed 2D-OPT and alternate 2D-OPT have better recognition rate with least running time when compared to the 2D-PCA method. The proposed B2D-OPT method achieves the best recognition rate with reduced dimension of feature vector among all the approaches. Nevertheless, it has relatively better running time.

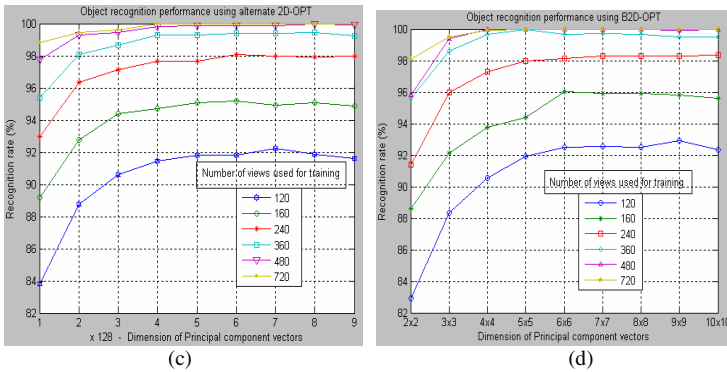


Fig. 3. Contd.

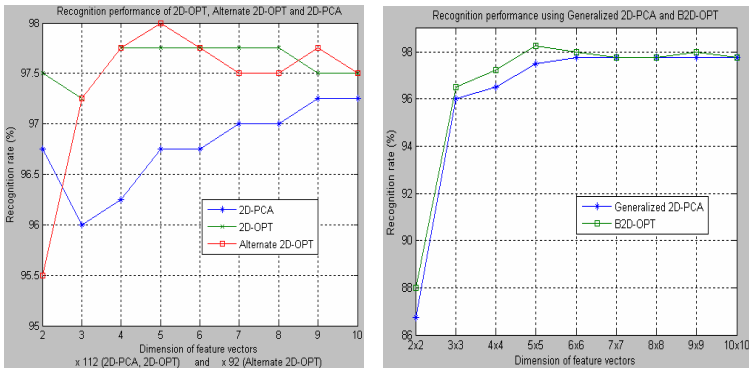
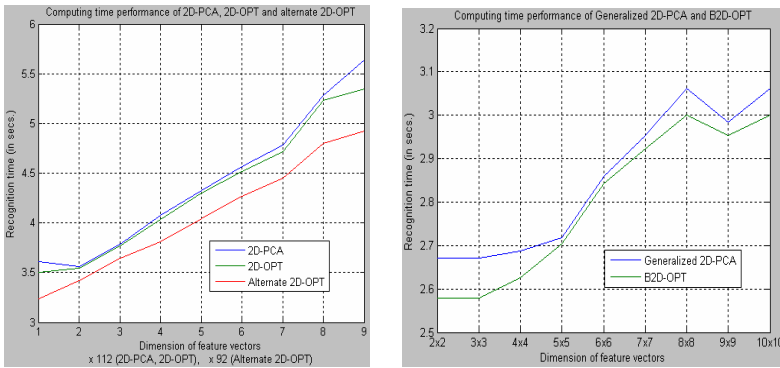


Fig. 4. Recognition performance of different approaches with varying dimension of feature vectors on AT&T face database

**Table 2.** Running times, Dimension of feature vectors and Recognition rate

Method	Running time (in secs.)	Dimension of feature vector	Best Recognition rate (%)
2DPCA [15]	5.265	112x9	97.25
Generalized 2DPCA [4]	2.719	6x6	97.75
<b>2D-OPT (Proposed method)</b>	<b>3.766</b>	<b>112x4</b>	<b>97.75</b>
<b>Alternate 2D-OPT (Proposed method)</b>	<b>3.812</b>	<b>92x5</b>	<b>98.00</b>
<b>B2D-OPT (Proposed method)</b>	<b>2.703</b>	<b>5x5</b>	<b>98.25</b>



**Fig. 5.** Running time of different approaches with varying dimension of feature vectors on AT&T face database

## 5 Conclusions

In this paper, an efficient appearance based object representation and recognition method called 2D-OPT and its variants are introduced. The major advantage of the proposed method, B2D-OPT, is that it requires fewer coefficients for object/face image representation unlike the standard PCA/2D-PCA as it works simultaneously on both row and column directions. Experimental results reveal that the proposed approach is relatively faster and has better recognition rate when compared to the other standard approaches available in the literature for 3D object recognition and face recognition, and, hence, is suitable for real-time recognition applications.

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