# Restoration of Document Images using Bayesian Inference


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***Abstract*—Restoration of documents has critical applications in document understanding as well as in digital libraries (for example as in book readers). This paper presents a method for restoration of document images, using a Maximum a Posteriori formulation. The advantage of our method is that the prior need not be learned from the training images. The extraction of a single high-quality enhanced text image from a set of degraded images can benefit from a strong prior knowledge, typical of text images. The restoration process should allow for discontinuities but at the same time discourage oscillations. These properties were represented in a total variation based prior model. Results indicate that our method is appropriate for document image restoration, where resolution enhancement is an added gain.**


## I. Introduction

Document images are often obtained by digitizing paper documents like books or manuscripts. They could be poor in appearance due to degradation of paper quality, spreading and flaking of ink toner, imaging artifacts etc. All the above phenomena lead to different types of noise at the word level including boundary erosion, dilation, cuts/breaks and merges of characters. Restoration of such images has many applications in enhancing the performance of character recognizers as well as in book readers used in digital libraries. Often, along with the restoration, one also looks for enhancement of the resolution. Text observed from these sources is often low-resolution degraded images, and requires restoration and resolution expansion in order to improve OCR performance. Moreover, these imperfect images may be inadequate for subsequent human use. The visual and recognition ability fall due to these effects. The accuracy of today's document recognition algorithms falls abruptly when image quality degrades even slightly [1]. Significant improvement in accuracy on hard problems now depends as much, or more, on the size and quality of training sets as on algorithms and hardware [1].

Restoration and enhancement are well studied in image processing literature. The linear filters are based on the assumption of linear, space invariant degradation. The restoration technique can be carried out in the frequency domain. The linear filter is easy to design and analyze. Popular low pass noise removal filters do not make any significant assumption about the scene content. Inverse-filtering based restoration technique model the degradation (eg. motion blur) and recover the signal in a model-based framework. But document images have sharp edges. The restriction that the estimation rule be linear combination of observed values is not suitable. We exploit the properties of document images to develop a specific restoration technique, specially suited for the same. This paper presents a document restoration technique that takes advantage of the repetitive structural nature of a document image which is further enhanced by a document specific prior information. Both prior and likelihood distributions are then formulated as a maximum a posterior (MAP) solution, which is a special case in the Bayes framework.

The remainder of the paper is organized as follows: In Section II we present some related work. Section III describes our text restoration algorithm in detail and Section IIIA presents a discussion on the algorithm. We present experimental result in Section IV and conclusion and future work in Section V.

## II. Related Work

There has been significant amount of research in the field of document restoration. Text enhancement efforts focus on fixing broken or touching characters [3], [4]. Traditional methods for text image enhancement can be classified into four categories: filtering, contrast enhancement, model-based image restoration, and resolution expansion. Some of the restoration efforts are based on morphological filters [2], [5] which discuss a method for binary morphological filter design to restore document images degraded by subtractive or additive noise, given a constraint on the size of filters. Bern and Goldberg [6] assume a probabilistic model of the scanning process, and uses this model to cluster instances of the same letter and to compute super-resolved representatives of the clusters. Other methods [7] use similar model based approaches. A variety of methods have been proposed in order to improve contrast within text images. They include methods based on multi-resolution pyramid and fuzzy edge detectors [8] where document image to be enhanced is obtained from a scanner and is a blurred binary image that is corrupted by additive noise. A mixed approach using topological features and contour beautification [9] for restoring high-resolution binary images is presented to improve legibility of low-resolution document images. The initially restored image is generated by simple techniques, and is then improved by integrating a variety of features obtained through image analysis. Missing strokes of characters are complemented based on topographic features. Few of the resolution expansion approaches include text bitmap averaging [10] where the essence of the method is in finding and averaging bitmaps of the same symbol

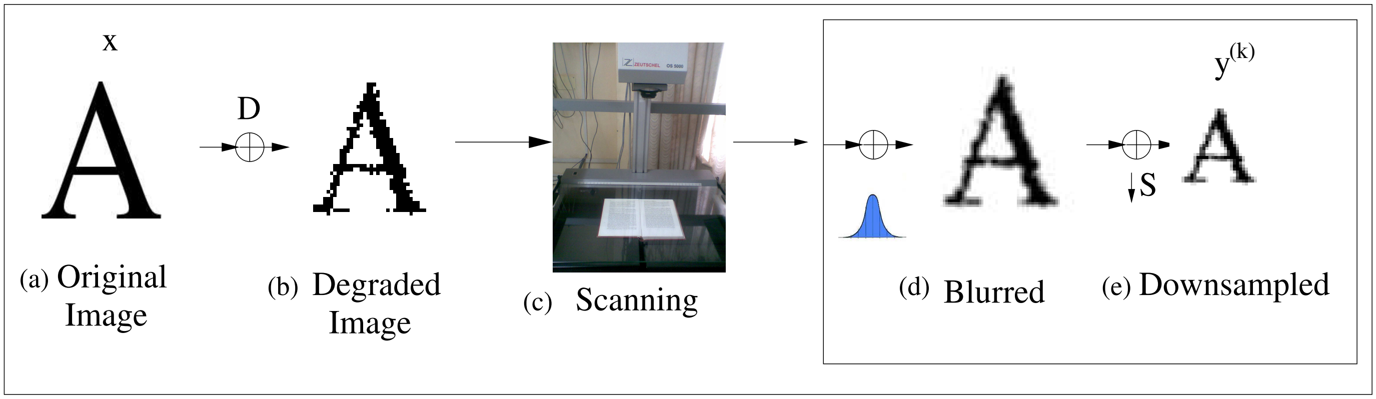



Fig. 1. Generative Model: (a) A typical ideal image with Serif font. (b) is the Degradation version of (a) with parameters $(\alpha_0, \alpha, \beta_0, \beta) = (0.6, 1.5, 0.8, 2.0)$ [2]. (c) is the scanning process. (d) and (e) are the Blurred version and then down-sampled versions of (b), respectively. Our problem is to rectify the low resolution degraded image to a high-quality magnified document image, making it suitable for further machine and human use.

that are scattered across a text page. Outline descriptions of the symbols are then obtained that can be rendered at arbitrary resolution. Shannon interpolation is performed with text separation from the image background in [11] to improve the OCR accuracy of digital video. Restoration of images is widely considered as an example of an ill-posed inverse problem. Such problems may be approached using regularization based methods, which constrain the feasible solution space by exploiting the *a priori* knowledge [12].

A number of research efforts investigated combining text enhancement with resolution expansion in order to improve low-resolution text images. Perhaps the most salient property of text is that it is generally bimodal. By its very nature, text characters must have some contrast with the background to make them human-readable. This constraint has been successfully applied to the resolution enhancement of text in single images [13], [14]. This technique creates a strongly bimodal image with smooth regions in both the foreground and background, while allowing for sharp discontinuities at the edges. The restored image, which is constrained by the given low-resolution image, is generated by iteratively solving a nonlinear optimization problem. Dalley *et al.* [15] adopt a training-based method, where a database is build to map the output high-resolution patch for a given input low-resolution patch. Given a single image of text scanned in at low resolution from a piece of paper, return the image that is mostly likely to be generated from a noiseless high-resolution scan of the same piece of paper. Though this method is efficient, it assumes that we have the font and script information, which is not always true.

This paper describes a restoration technique with enhancement for document images that mimics image sequences by clustering similar character components. Spatiotemporal observation constraints are additionally added to constrain the feasible solution space with *a priori* assumptions on the form of the solution. The prior information in our formulation is independent of script and font information which is hard to predict. Our method differs from the previous work [10] in the context that we have focused on the requirement of the prior information, further combining the prior and data distribution in a Bayesian framework.

We propose a method for restoring high-quality binary images from degraded gray-scale images in low resolution. An effective approach to tackle this problem is to utilize a Bayesian inference approach. The restored image is generated from a collection of similar images by estimating the likelihood, and it is then improved by integrating with a prior information, making it a *Maximum a Posteriori* estimate. Here, we present a new image prior model based on Total Variational (TV) energy minimization. The basic idea stems from the need for preserving sharp edges, while discouraging degradations. In this paper the performance of this method is demonstrated by showing the improvement in visual quality of the document image. Further, the results are quantitatively evaluated by running an OCR engine on the restored document images.

## III. Document Restoration by Bayesian Inference

Given an input page as a gray-scale image, we first perform skew detection and page layout analysis upto character segmentation. We need to find images of the same character symbol that are scattered on a document page. For restoring document images, we assume that the input image is obtained by digitizing and down-sampling a degraded character. A pictorial explanation of the imaging process is given in Figure 1, where we see that the image gets degraded on the paper as well as while imaging. Given input pages of a document as a binary image, we segment them to obtain the word images. Connected components within this word image are then extracted from all the segmented words. The bitmaps of the segmented character images are initially clustered using a correlation based method [6]. (An alternate method is also available in [10].) We say component $C_1$ is equivalent to component $C_2$ if:

$$r(C_1/C_2) > \theta_1 \text{ and } r(C_2/C_1) > \theta_2 \tag{1}$$

where $\theta_1$ and $\theta_2$ are the tight thresholds. For our experimentation we assume $\theta_1$ and $\theta_2$ to be 0.85. The value $r(C_1/C_2)$ is computed as:

$$r(C_1/C_2) = \frac{\max_{x_{i,j}} corr(C_1, C_2)}{\max_{x_{i,j}} corr(C_2, C_2)}$$

where $x_{i,j}$ is an element of the correlation matrix.

Document restoration problem can now be formulated as generation of good prototypes corresponding to each cluster, where in our case the clustering is done using the Equation 1. We exploit the simple fact that a textual region is generated by repetition of character images according to a language/script model. We assume that the document image being processed has enough repetitive characters to take advantage of their multiple occurrences. Since the whole page is from one book or collection, it is also in a single font.

The imaging model (Figure 1) specifies how the high-resolution text is transformed to generate a low-resolution degraded image. This typically involves blurring, spatial sampling and adding of noise. A high-resolution scene $\mathbf{x}$ with $N$ pixels, is assumed to have generated a set of $K$ low-resolution images $\mathbf{y}^{(k)}$, each with $M$ pixels. The generative model for the $k$th image is

$$\mathbf{y}^{(k)} = \mathbf{W}^{(k)}\mathbf{x} + \boldsymbol{\epsilon_G}^{(k)} \tag{2}$$

where $\boldsymbol{\epsilon_G}$ represents noise on the low-resolution image, and consists of *i.i.d.* samples from a zero-mean Gaussian with precision $\beta_G$ (equivalent to *standard deviation* $\sigma_N = {\beta_G}^{(-1/2)}$). For each image, the blurring and sub-sampling of the scene is modeled by an $M \times N$ sparse matrix $\mathbf{W}^{(k)}$ which is assumed to be parameterized by some vector $\boldsymbol{\theta}^{(k)}$. In other words, $\mathbf{W}^{(k)}$ is a function of $\boldsymbol{\theta}^{(k)}$. Given the sequence $\{\mathbf{y}^{(k)}\}$, the goal is to recover $\mathbf{x}$, without any explicit knowledge of the registration parameters $\{\boldsymbol{\theta}^{(k)}, \boldsymbol{\epsilon_G}^{(k)}\}$.

We argue that the image registration parameters may be determined *a priori*. For an individual low-resolution image, given registrations and $\mathbf{x}$, the likelihood is

$$p\Big(\mathbf{y}^{(k)}|\mathbf{x}, \boldsymbol{\theta}^{(k)}, \boldsymbol{\epsilon_G}^{(k)}\Big) = \left(\frac{\beta_G}{2\pi}\right)^{M/2} exp\left[-\frac{\beta_G}{2}\left\|\mathbf{y}^{(k)} - \mathbf{W}^{(k)}\mathbf{x}\right\|^2\right] \tag{3}$$

The vector $\mathbf{x}$ yielding the maximal value of Equation 3, would be the Maximum Likelihood (ML) estimation to the problem. But super-resolution images recovered in this way often tend to be dominated by a great deal of high-frequency noise [16]. Moreover, the super-resolution problem is almost always poorly conditioned, so a prior over $\mathbf{x}$ is usually required to avoid solutions that are subjectively implausible to the human viewer.

In real world applications, it is critical that we use an accurate prior model. The problem becomes more challenging when we deal with document images, because of its pseudo binary nature and the regularity of the patterns used in this "visual" language. Images of text are also usually smooth in both the foreground and background regions with sharp transitions only at the edges. In addition, expanded images are constrained so the average of a group of high-resolution pixels is close to the original value of the low-resolution pixel from which they were derived. The challenges of complex content, various types of structures (e.g., corners, edges or surfaces) has to be incorporated in the model accurately.

We present the prior over the high resolution image by employing a total variational energy minimization function. A major concern in designing image denoising models is to preserve important image features, such as those most easily detected by the human visual system, while removing noise. One such important image feature are the edges typical of a document image; these are places in an image where there is a sharp change in image properties, which happens for instance at object boundaries. Total variation (TV) based image restoration models were first introduced by Rudin, Osher, and Fatemi in their pioneering work [17] on edge preserving image denoising. It is one of the earliest and best known examples of PDE based edge preserving denoising. It is designed with the explicit goal of preserving sharp discontinuities (edges) in images while removing noise and other unwanted fine scale detail. The revolutionary aspect of this model is its regularization term that allows for discontinuities but at the same time discourages oscillations. This algorithm seeks an equilibrium state (minimal energy) of an energy functional comprised of the TV norm of the image $\mathbf{x}$ and the fidelity of this image to the noisy input image $\mathbf{x_0}$. The minimizing energy function is:

$$\mathbf{E_{TV}} = \int_{\Omega} (|\nabla\mathbf{x}|) + \frac{1}{2}\lambda(\mathbf{x} - \mathbf{x_0})^2 \mathrm{d}u\, \mathrm{d}v \tag{4}$$

Here, $\Omega$ denotes the image domain, and is usually a rectangle and $\lambda$ is a Lagrange multiplier.

If we assume a uniform prior over the input images, the *Maximum a Posteriori* (MAP) solution is found using the Bayes' rule. The posterior distribution over $\mathbf{x}$ is of the form

$$p\Big(\mathbf{x}|\mathbf{y}^{(k)}, \boldsymbol{\theta}^{(k)}, \boldsymbol{\epsilon_G}^{(k)}\Big) = p(\mathbf{x}) \prod_{k=1}^{K} p\Big(\mathbf{y}^{(k)}|\mathbf{x}, \boldsymbol{\theta}^{(k)}, \boldsymbol{\epsilon_G}^{(k)}\Big) \tag{5}$$

As the prior probability distribution on the super-resolution image is available, this information is used to "regularize" the estimation. Inserting this prior into Equation 5, the posterior over $\mathbf{x}$, and taking the negative log, the MAP *(maximum a posterior)* estimator has the form:

$$\mathbf{x}_{MAP} = \underset{x}{\operatorname{argmax}}(-\mathcal{L}) \tag{6}$$

where

$$\mathcal{L} = \beta\, \mathbf{E_{TV}} + \sum_{k=1}^{K} \left\|\mathbf{y}^{(k)} - \mathbf{W}^{(k)}\mathbf{x}\right\|^2$$

where the right-hand side has been scaled to leave a single unknown ratio $\beta$ between the data error term and the prior

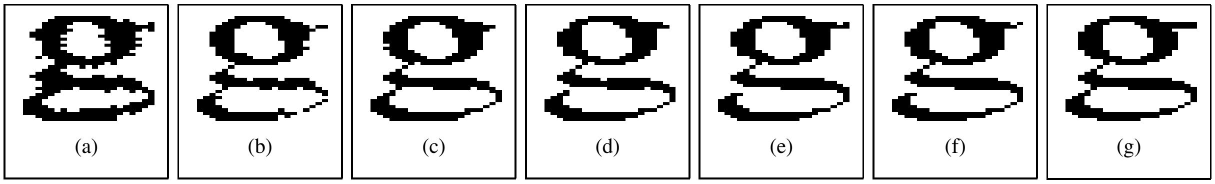



Fig. 2. Evolution of a character image. (a) Degraded Input (b)-(f) Intermediate restored images and (g) Final restored image.

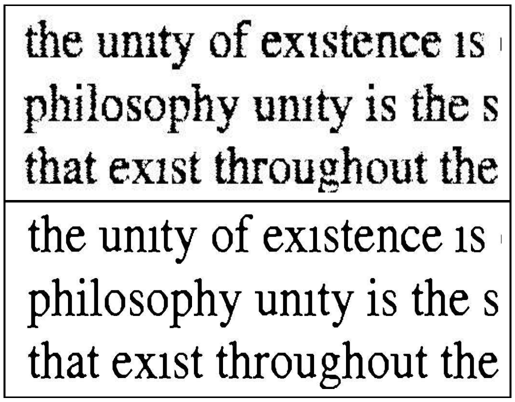



Fig. 3. (a) Original Image (b) Restored Image.

term. We optimize the objective function of Equation 6 using conjugate gradient method to obtain an approximation to our resultant image. Here, we assume that the matrix $\mathbf{W}^{(k)}$ is available. To estimate $\mathbf{W}^{(k)}$ we have used a method suggested by Tipping and Bishop [18]. These enhanced images form the high-quality representatives of their respective clusters.

Our restoration and enhancement algorithm is based on the basic Bayesian framework. The Algorithm 1 shows the flow of our procedure. It is an iterative procedure, where at every stage we infer a better estimate of restored image $\mathbf{x}$. Assuming a set of $K$ low-resolution degraded observation images, $\{\mathbf{y}^{(k)}\}$, the algorithm finds the corresponding high-quality image $\mathbf{x}$ such that the conditional probability of $\mathbf{x}$, given the observed images $\{\mathbf{y}^{(k)}\}$, $p(\mathbf{x}|\mathbf{y}^{(k)})$, is maximized. In our case this is difficult to calculate directly. Thus using Bayes' law, we obtain $p(\mathbf{x}|\mathbf{y}^{(k)}) \propto p(\mathbf{x})p(\mathbf{y}^{(k)}|\mathbf{x})$, which is the MAP estimator. Once $p(\mathbf{x})p(\mathbf{y}^{(k)}|\mathbf{x})$ are defined, the output image, $\mathbf{x}$, that maximizes $p(\mathbf{x}|\mathbf{y}^{(k)})$, is iteratively calculated by stepping down the gradient of the negative log likelihood of $p(\mathbf{x})p(\mathbf{y}^{(k)}|\mathbf{x})$ until a minimum is reached or a maximum number of iterations are executed. Finally, reassembling the output page by replacing each member of the cluster by its representative we restore the document.

**Algorithm 1**

**Require:** Given the document image and parameter $\mathbf{W}^{(k)}$ [18]. Perform a character level segmentation. Here $\mathbf{y}$ and $\mathbf{x}$ are the input and output images respectively.

**Ensure:**

Perform the initial clustering [Equation 1].
**for** each cluster bin **do**
  **for** each element of the bin **do**
    **repeat**
      1) Parameterize the posterior distribution as a function of $\mathbf{x}$ by substituting the values of $\mathbf{y}$ in Equation 3.
      2) The equation is then minimized using conjugate gradient algorithm to get a estimate of $\mathbf{x}$.
      3) Total energy minimization of $\mathbf{x}$ is then performed to get the next estimation on $\mathbf{x}$ [Equation 4].
    **until** the energy is minimized
  **end for**
**end for**

## A. Discussion

We make the following comments about our method and its implementation. Document image processing algorithms to detect text regions, and then segmenting them to obtain word and component images are not described here. There exists significant amount of material in this respect [19]. In real-life situations, character images could be split into multiple components or merged to form single component. They may affect the clustering process. In [10] a procedure is discussed to find images of the same character symbol that are scattered on a document page. They employ a sequence of different clustering techniques, each applied to a different set of shape features derived from the character images. The motivation is to progressively divide all characters on a page into groups of decreasing sizes, and delay the uses of more expensive techniques until later stages when the groups are sufficiently small. This method is experimentally verified to be quite effective. However, in our case by defining appropriate similarity measure in clustering, they are taken care of. It is important to classify the character images into as few clusters as possible, since this is how the algorithm achieves its benefits. Yet it is even more important to avoid clustering incompatible character images since this leads to "mistakes" in the output. The clustering results are important side products of the procedure and they have other potential uses that remain to be explored. The computational requirement of this algorithm is directly proportional to the number of similar components in the cluster and the conjugate gradient method used in the optimization process. Further, it is worth while mentioning that our method differs from the previous super-resolution methods in following three aspects: (i) we do not learn a low-resolution to high-resolution match to build up our output image; (ii) since we are using energy function (i.e., total variation minimizing process) to determine our prior, we need not have any font or script information; (iii) our approach of image restoration cum resolution expansion adopts a *Maximum a Posteriori* estimation approach as it provides a rigorous theoretical framework with several desirable mathematical properties.

## IV. Experimental results

The complete algorithm is implemented on degraded document images scanned at a specific resolution. We expect restored document images as output, at the end of our experimentation. We show the effectiveness of our algorithm by demonstrating the results using samples collected mainly from degraded books. We scan these books in 200dpi using a *ZEUTSCHEL OS 5000* scanner shown in Figure 1(c). The scanning device used here has a mounted camera on top of the flat bed where the book is kept. The focus of the camera has to be adjusted to get a sharp image. We have scanned 20 pages from four different variety of books containing different fonts and styles. The document books already contain degradations. After the scanning process the resultant image gets blurred and down-sampled. We proceed with binarizing and skew correcting the scanned images. After a character level segmentation, we cluster the components. For a character we get an approximate of 10-15 or more similar components.

Effectiveness is demonstrated for improving image quality. Fig. 3b shows the generated binary image with resolution enhanced by a factor of two, along with the original image in 200dpi shown in Fig.3. Image quality is improved as resolution increases; strokes are reconstructed more precisely, linearity and smoothness of contours are improved, stroke width is more uniform, and shape features of fonts are reconstructed more finely. The proposed method is effective for Latin scripts as well as oriental scripts. The plot in Fig. 4 depicts the evolution of the degraded character “g”. The *x-axis* shows the number of connected components used and the *y-axis* determines the Mean Square Error (MSE). The performance of our algorithm was evaluated with respect to the mean square error (MSE). The figure shows how the mean square error function decreases steadily as the number of collection of the similar components increases. We see that the number of similar components is directly related to the accuracy of the result. The step-by-step changes in the output of the image is shown in Fig. 2 where the image in the left is the degraded image and image in the extreme right is the restored image. If there are sufficient number of similar components then we get a high-quality restored image.

| Specification | Noisy Page | Restored Page |
|---|---|---|
| **Number of words** | 325 | 325 |
| **Recognized words** | 268 | 325 |
| **% Accuracy** | 82% | 100% |

TABLE I
OCR Evaluation of image restoration results.

We examined effectiveness of the proposed method for improving OCR accuracy. Binary images in 400dpi were generated from input images in 200dpi by the proposed method, and the OCR accuracy using these images as input was compared with the results using bilinear interpolation. Gray-scale images in 400dpi generated from input images in 200dpi by bilinear interpolation which gives around 82% accuracy as shown in Table I. Few of the words incorrectly recognized during the whole process are listed in Table II. Our method gives around 100% accuracy. The page level output to our algorithm is shown in Figure 5.

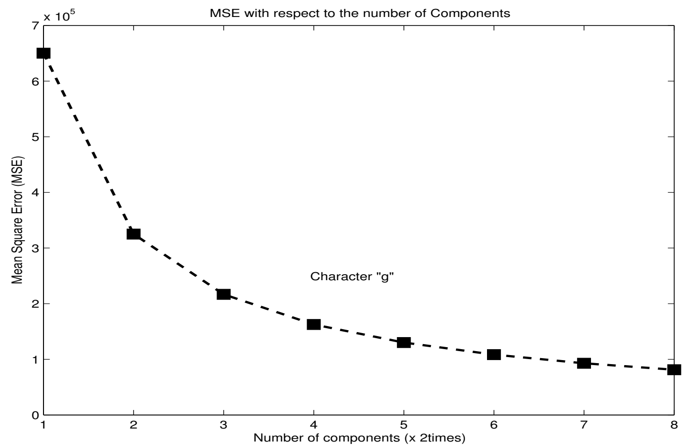


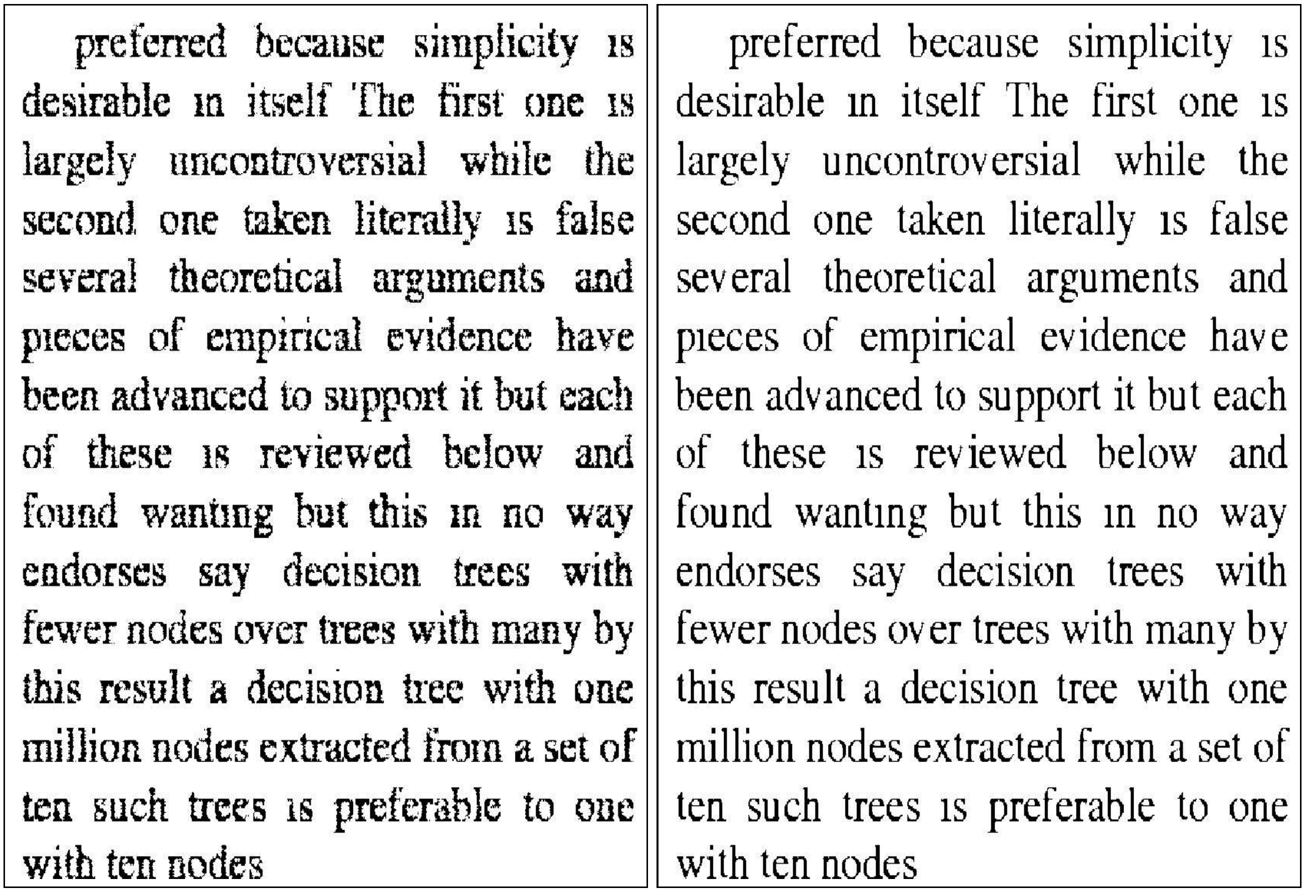

Fig. 4. MSE with ground truth for the character image ”g”.

| Word Images | OCR recognized as |
|---|---|
| unity | **urity** |
| pillar | **piHlar** |
| purity | **pur¿ty** |
| appearing | **appeanng** |
| egotism | **egottsrn** |

TABLE II
OCR recognition output for few of the degraded words using a commercial OCR(CuneiForm OCR).

## V. Conclusion and Future Work

To improve quality and OCR accuracy for degraded low-resolution text images, a new method has been presented for restoring high-quality binary text images from a set of low-resolution degraded image. The initially restored image is improved by MAP based approach where a suitable a priori information is used to guide the restoration, resolution enhancement being the byproduct. The proposed method can deal



preferred because simplicity is desirable in itself The first one is largely uncontroversial while the second one taken literally is false several theoretical arguments and pieces of empirical evidence have been advanced to support it but each of these is reviewed below and found wanting but this in no way endorses say decision trees with fewer nodes over trees with many by this result a decision tree with one million nodes extracted from a set of ten such trees is preferable to one with ten nodes

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Fig. 5. The document page on the left suffers from degradation and low-resolution. The second image on the right shows the content restored using the algorithm presented in Algorithm 1

with various scripts, and entails relatively simple computation. Through experiments, it has been validated that the proposed method improves both OCR accuracy and image quality. Our future work will include the investigation of the effect on ideal character when it is subjected to the scanning process. It would aim to develop the framework that would facilitate us to restore the document without considering its repetitive structural nature. We believe that as documents are "visual words", normal image processing approaches might not be adequate for the purpose.

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