Combined Scale and Translation Invariants of Krawtchouk Moments

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***Abstract*— This paper derives scale, translation invariants as well as combined scale and translation invariants of Krawtchouk moments. Unmodified Krawtchouk moments are used for scale invariants whereas modified Krawtchouk moments are used for obtaining translation invariants because to retain the orthogonality property of discrete weighted Krawtchouk polynomials that is lost due to translation. Scale invariants are obtained from the Krawtchouk moments using the standard procedure of expressing the scaled moments independent of scale factors. In order to obtain the modified moments for translation invariants, the discrete weighted Krawtchouk polynomials are assumed to be periodic with period equal to the given number of data points and these are shifted to the centroid of the image. It is shown that the modified moments are invariant to translation. Further, a mathematical expression is derived for combined scale and translation invariance. In order to test the derived invariants, invariance and face recognition problems are attempted. First problem was tested using two standard images, where as the second problem was tested using ORL database images.**

***Index Terms*— Krawtchouk moments, Combined scale and translation invariants.**

I. INTRODUCTION

One of the important tasks of pattern recognition is to recognize the images irrespective of their size and position. Moments and functions of moments have been used as invariant pattern features in a number of applications. During the last few years, many moments such as Geometric, Legendre, Zernike, Tchebichef, Krawtchouk etc. are proposed and used them for image processing and computer vision problems. Hu[1] introduced geometric moments, which are projections of image function on to the monomials. He derived a set of moments which are invariant to rotation, scale and translation from algebraic invariants using the fundamental theorem of moment invariants. These invariants are widely used in pattern recognition, ship identification, aircraft identification, pattern matching and scene matching etc. But the geometric moments are not orthogonal. Hence it is difficult to reconstruct the image using these moments. In order to solve the above problem, Teague [2] reported image reconstruction results using Legendre and Zernike moments which are based on continuous Legendre and Zernike polynomials.

In practice, there are three methods namely image normalization method (INM), indirect method (IDM) and direct method (DM) for achieving the moment invariants. INM makes use of geometric or complex moments for obtaining the corresponding invariants of moments. Whereas in IDM, moment invariants are obtained by using the invariants of geometric moments. In DM, the derivation of these invariants is based on the polynomials used in defining the moments. Chee-Way Chong et al. [3] proposed translation and scale invariants of Legendre moments based on Legendre polynomials. They obtained translation invariants using Legendre central moments. In their work, the scale invariants are achieved by algebraically eliminating the scale factor contained in the scaled Legendre moments. A.Khotanzad et al.[5] used the invariants of Zernike moments for image recognition. Chee-Way Chong et al.[4] proposed translation invariants of Zernike moments. They first derived the translation invariants of radial moments and then these invariants are used to derive the translation invariants of Zernike moments. But there are some problems associated with the continuous moments such as Geometric, Legendre, Zernike etc. They are i) Numerical approximation of continuous integrals ii)Large variation in the dynamic range of moment values and iii) Coordinate space transformations. Hence, in order to solve the above problems, Mukundan et al.[6] proposed Tchebichef moments based on discrete Tchebichef polynomials. They experimentally verified that the reconstruction error is minimum using these moments as compared to other moments like Legendre and Zernike moments. Another problem associated with the continuous moments is that perfect invariance cannot be achieved for digital images since these moments are defined based on continuous polynomials.

Recently, Pew-Thian Yap et al.[7] proposed Krawtchouk moments based on discrete Krawtchouk polynomials. Their experimental results show that Krawtchouk moments are better in terms of reconstruction error when compared with Zernike, Legendre and Tchebichef moments. More recently, Bo Fu et al.[9] proposed translation invariants of Tchebichef moments by modifying the discrete Tchebichef polynomials. Krawtchouk moment invariants are derived in [7] by using

IDM. In their work, Krawtchouk moments are expressed in terms of the Geometric moments and Krawtchouk moment invariants are derived using the linear combination of Geometric moment invariants. Hence by using these moment invariants, perfect invariance cannot be achieved for digital images since the derivation of these invariants is not based on Krawtchouk polynomials rather it is based on continuous geometric moment invariants. Hence in this paper we derive scale and translation invariants of Krawtchouk moments using direct method. The scale invariants are obtained by algebraically eliminating the scale factor contained in the scaled Krawtchouk moments to make them invariant to scaling. C.W Chong's method [3,4] cannot be applied for obtaining the translation invariants of Krawtchouk moments because the central Krawtchouk polynomials are not orthogonal in the domain [0, N-1]. Hence, we propose modified Krawtchouk moments which preserve the translation invariant property by assuming that discrete weighted Krawtchouk polynomials are periodic with period equal to the number of data points and these polynomials are shifted to the centroid. We adopt the procedure given [9] for obtaining translation invariants of Krawtchouk moments. Simulation results are reported to test the proposed invariants.

This paper is organized into seven sections. Section II presents the details about Krawtchouk moments. Scale invariants are derived in section III. Section IV presents the modified Krawtchouk moments, which are translation invariant. In section V, combined scale and translation invariants are presented. Simulation results are reported in section VI in order to verify the proposed invariants. Finally, the last section presents the conclusions about the work.

II. KRAWTCHOUK MOMENTS

In this section, we present a brief introduction on Krawtchouk moments. Krawtchouk moments Q_{nm} of order (n, m) for a digital image $f(x,y)$ of size N x M are defined [7] as

$$Q_{nm} = \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} \bar{K}_n(x;p_1,N-1)\bar{K}_m(y;p_2,M-1)f(x,y) \quad (1)$$

Where $\bar{K}_n(x;p,N-1)$ is the nth order weighted Krawtchouk polynomial [7], which is defined as

$$\bar{K}_n(x;p,N-1) = K_n(x;p,N-1)\sqrt{\frac{w(x;p,N-1)}{\rho(n;p,N-1)}} \quad (2)$$

where $K_n(x;p,N-1)$ is the nth order discrete Krawtchouk polynomial defined as

$$K_n(x;p,N-1) = \sum_{k=0}^{n} a_{k,n,p}x^k = {}_2F_1(-n,-x;-(N-1);\frac{1}{p}) \quad (3)$$

for x, n=0,1,2,….N-1, The parameter p ∈ (0,1), ${}_2F_1$ is the hypergeometric function, defined as

$${}_2F_1(a,b;c;z) = \sum_{k=0}^{\infty}\frac{(a)_k(b)_k z^k}{(c)_k k!} \quad (4)$$

and $(a)_k$ is the Pochhammer symbol given by

$$(a)_k = a(a+1)....(a+k-1)$$

The weight function $w(x;p,N-1)$ is given by

$$w(x;p,N-1) = \binom{N-1}{x} p^x (1-p)^{N-1-x} \quad (5)$$

The weight function $w(x;p,N-1)$ can be recursively calculated using

$$w(x+1;p,N-1) = (\frac{N-1-x}{x+1})\frac{p}{1-p}w(x;p,N-1) \quad (6)$$

$$with \;\; w(0;p,N-1) = (1-p)^{N-1}$$

and $\rho(n;p,N-1)$ is the squared norm, which is given by

$$\rho(n;p,N-1) = (-1)^n(\frac{1-p}{p})^n\frac{n!}{(-N+1)_n} \quad (7)$$

The three term recursive relation for the weighted Krawtchouk polynomials is given by

$$\bar{K}_{n+1}(x;p,N-1) = \frac{\sqrt{\frac{p(N-1-n)}{(1-p)(n+1)}}((N-1)p-2np+n-x)}{p(N-1-n)}\bar{K}_n(x;p,N-1) - \frac{\sqrt{\frac{p^2(N-1-n)(N-n)}{(1-p)^2(n+1)n}}(n(1-p))}{p(N-1-n)}\bar{K}_{n-1}(x;p,N-1)$$

$$for \;\; n=1,2,........N-2 \quad (8)$$

$$with \;\; \bar{K}_0(x;p,N-1) = \sqrt{w(x;p,N-1)}$$

$$and \;\; \bar{K}_1(x;p,N-1) = (1-\frac{x}{p(N-1)})(\sqrt{\frac{p(N-1)}{(1-p)}})\sqrt{w(x;p,N-1)}$$

Krawtchouk moments of an image $\tilde{f}(x,y) = (\sqrt{w(x;p_1,N-1)w(y;p_2M-1)})^{(-\frac{1}{2})}f(x,y)$ can be written as

$$Q_{nm} = \sum_{x=0}^{N-1}\sum_{y=0}^{M-1}\bar{K}_n(x;p_1,N-1)\bar{K}_m(y;p_2,M-1)\tilde{f}(x,y)$$

$$= \frac{1}{\sqrt{\rho(n;p_1,N-1)\rho(m,p_2,M-1)}}\sum_{x=0}^{N-1}\sum_{y=0}^{M-1}K_n(x;p_1,N-1)K_m(y;p_2,M-1)f(x,y)$$

$$= \frac{1}{\sqrt{\rho(n;p_1,N-1)\rho(m;p_2,M-1)}}\sum_{i=0}^{n}\sum_{j=0}^{m}a_{i,n,p_1}a_{j,m,p_2}\sum_{x=0}^{N-1}\sum_{y=0}^{M-1}x^i y^j f(x,y) \quad (9)$$

where a_{i,n,p_1} *and* a_{j,m,p_2} are the coefficients obtained from (3)

III. SCALE INVARIANTS OF KRAWTCHOUK MOMENTS

In this section, scale invariants of Krawtchouk moments are derived. Let α and β be the scale factors along x and y directions respectively. Then the scaled Krawtchouk moments are given by

$$\dot{Q}_{nm} = \frac{1}{\sqrt{\rho(n;p_1,N-1)\rho(m;p_2,M-1)}}\sum_{i=0}^{n}\sum_{j=0}^{m} a_{i,n,p_1} a_{j,m,p_2} \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} (\alpha x)^i (\beta y)^j f(x,y)$$

Let $R_{ij} = \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} (\alpha x)^i (\beta y)^j f(x,y)$

The above expression can be written as $R_{ij} = \alpha^i \beta^j \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} x^i y^j f(x,y)$

Then $R_{10} = \alpha \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} x f(x,y)$ and $R_{01} = \beta \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} y f(x,y)$

The scale factors α *and* β can be cancelled out by considering $\frac{R_{ij}}{(R_{10})^i (R_{01})^j}$. The scale invariants can be obtained by algebraically eliminating the scale factor contained in the scaled Krawtchouk moments. Hence the scale invariants of Krawtchouk moments are expressed as

$$\dot{Q}_{nm} = \frac{1}{\sqrt{\rho(n;p_1,N-1)\rho(m;p_2,M-1)}}\sum_{i=0}^{n}\sum_{j=0}^{m} a_{i,n,p_1} a_{j,m,p_2} \frac{R_{ij}}{(R_{10})^i (R_{01})^j} \quad (10)$$

IV. TRANSLATION INVARIANTS OF MODIFIED KRAWTCHOUK MOMENTS

In this section, we propose a modified Krawtchouk moments which are invariant to translation by assuming that the discrete weighted Krawtchouk polynomials are periodic with period equal to number of data points(N) and shifting these polynomials to the centroid of the image. We expand the domain of discrete weighted Krawtchouk polynomials to a set of all integers with a period N. The modified Krawtchouk polynomials are given by

$$K_n^*(x;p,N-1) = \bar{K}_n((dN+x);p,N-1) \quad where\ x \in [0, N-1],\ d \in \mathrm{Z}$$

For simplicity, we denote $\bar{K}_n(x;p,N-1)$ as $\bar{K}_n(x)$ and $K_n^*(x;p,N-1)$ as $K_n^*(x)$ since p, N does not change. The modified discrete weighted Krawtchouk polynomials with a translation α in 'x' direction can be expressed as

$$K_n^*(x)\,|\,\alpha = \begin{cases} \bar{K}_n(x-\alpha) & for\ x \ge \alpha \\ \bar{K}_n(x+N-\alpha) & for\ x \prec \alpha \end{cases} \quad (11)$$

Similarly, the modified discrete weighted Krawtchouk polynomials with a translation β in 'y' direction can be expressed as

$$K_m^*(y)\,|\,\beta = \begin{cases} \bar{K}_m(y-\beta) & for\ y \ge \beta \\ \bar{K}_m(y+M-\beta) & for\ y \prec \beta \end{cases} \quad (12)$$

The mathematical proof to show that (11) satisfies the orthogonality property like the original weighted Krawtchouk polynomials is given in appendix.

If the modified discrete weighted Krawtchouk polynomials are translated to the image centroid, then the change of location of an image on moments calculation can be cancelled out thus translation invariance can be achieved. The modified Krawtchouk moments of an image $f(x,y)$ which satisfy the translation invariant property can be defined using the modified discrete weighted Krawtchouk polynomials as

$$\psi_{nm} = \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} K_n^*(x)\,|\,x_0\, K_m^*(y)\,|\,y_0\, f(x,y) \quad (13)$$

where the centroid of the binary image (x_c, y_c) is given by [9]

$$x_c = \frac{\sum_{x=0}^{N-1}\sum_{y=0}^{M-1} x(1-f(x,y))}{\sum_{x=0}^{N-1}\sum_{y=0}^{M-1}(1-f(x,y))} \quad \text{and} \quad y_c = \frac{\sum_{x=0}^{N-1}\sum_{y=0}^{M-1} y(1-f(x,y))}{\sum_{x=0}^{N-1}\sum_{y=0}^{M-1}(1-f(x,y))}$$

The intensity centroid can be given in terms of integers as $x_0 = [x_c] - \frac{N}{2} + 1$ *and* $y_0 = [y_c] - \frac{N}{2} + 1$ where [.] is an integer function.

Next we show that modified Krawtchouk moments are translation invariant.

Consider $\psi_{nm} = \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} K_n^*(x)\,|\,x_0\, K_m^*(y)\,|\,y_0\, f(x,y)$ and expressing it into sum of four terms using (11) and (12)

$$= \sum_{x=0}^{x_0-1}\sum_{y=0}^{y_0-1} \bar{K}_n(x+N-x_0)\bar{K}_m(y+M-y_0) f(x,y) + \sum_{x=0}^{x_0-1}\sum_{y=y_0}^{M-1} \bar{K}_n(x+N-x_0)\bar{K}_m(y-y_0) f(x,y)$$

$$+ \sum_{x=x_0}^{N-1}\sum_{y=0}^{y_0-1} \bar{K}_n(x-x_0)\bar{K}_m(y+M-y_0) f(x,y) + \sum_{x=x_0}^{N-1}\sum_{y=y_0}^{M-1} \bar{K}_n(x-x_0)\bar{K}_m(y-y_0) f(x,y)$$

Let $x_1 = x + N$ *and* $y_1 = y + M$, then

$$\psi_{nm} = \sum_{x_1=N}^{x_0+N-1}\sum_{y_1=M}^{y_0+M-1} \bar{K}_n(x_1-x_0)\bar{K}_m(y_1-y_0) f(x,y)$$

$$+ \sum_{x_1=N}^{x_0+N-1}\sum_{y=y_0}^{M-1} \bar{K}_n(x_1-x_0)\bar{K}_m(y-y_0) f(x,y)$$

$$+ \sum_{x=x_0}^{N-1}\sum_{y_1=M}^{y_0+M-1} \bar{K}_n(x-x_0)\bar{K}_m(y_1-y_0) f(x,y)$$

$$+ \sum_{x=x_0}^{N-1}\sum_{y=y_0}^{M-1} \bar{K}_n(x-x_0)\bar{K}_m(y-y_0) f(x,y)$$

Let $x = x_1 - x_0$ *and* $y = y_1 - y_0$, then

$$\psi_{nm} = \sum_{x=N-x_0}^{N-1} \sum_{y=M-y_0}^{M-1} \bar{K}_n(x)\bar{K}_m(y) f(x,y)$$
$$+ \sum_{x=N-x_0}^{N-1} \sum_{y=0}^{M-y_0-1} \bar{K}_n(x)\bar{K}_m(y) f(x,y)$$
$$+ \sum_{x=0}^{N-x_0-1} \sum_{y=M-y_0}^{M-1} \bar{K}_n(x)\bar{K}_m(y) f(x,y)$$
$$+ \sum_{x=0}^{N-x_0-1} \sum_{y=0}^{M-y_0-1} \bar{K}_n(x)\bar{K}_m(y) f(x,y)$$
$$= \sum_{x=N-x_0}^{N-1} \sum_{y=0}^{M-1} \bar{K}_n(x)\bar{K}_m(y) f(x,y)$$
$$+ \sum_{x=0}^{N-x_0-1} \sum_{y=0}^{M-1} \bar{K}_n(x)\bar{K}_m(y) f(x,y)$$
$$= \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} \bar{K}_n(x)\bar{K}_m(y) f(x,y)$$
$$= Q_{nm}$$

Since $\psi_{nm} = Q_{nm}$, we say that modified Krawtchouk moments are translation invariant.

The modified Krawtchouk moments are applicable for both symmetrical and non symmetrical images but a small modification is required for symmetrical images since all odd ordered moments of symmetrical images gives zero values due to the pair of pixels that are equidistant from its center. This problem is due to the image centroid used in the calculation of moments. In order to solve the above problem, two shift terms x_s *and* y_s are introduced in 'x' and 'y' axes respectively which are given by $x_s = \pm N/4$ *and* $y_s = \pm M/4$. Then the new reference point becomes $x_{new} = x_0 + x_s$ *and* $y_{new} = y_0 + y_s$. This enables the modified Krawtchouk moments computed from a center other than the centroid of the translated image.

V. COMBINED SCALE AND TRANSLATION INVARIANTS

By making use of the concept adopted in sections III and IV, we obtain combined scale and translation invariants in this section. By consider the scale factors and translations combinely, (9) can be written as

$$\tilde{Q}_{nm} = \frac{1}{\sqrt{\rho(n,p_1,N-1)\rho(m,p_2,M-1)}} \sum_{i=0}^{n}\sum_{j=0}^{m} a_{i,n,p_1} a_{j,m,p_2} \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} (\alpha(x^* \mid x_0))^i (\beta(y^* \mid y_0))^j f(x,y) \quad (14)$$

Let $R_{ij} = \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} (\alpha(x^* \mid x_0))^i (\beta(y^* \mid y_0))^j f(x,y)$ where

$$x^* \mid x_0 = \begin{cases} x - x_0 & for\ x \geq x_0 \\ x + N - x_0 & for\ x \prec x_0 \end{cases}$$

By considering the procedure adopted in sections 3 and 4, the combined scale and translation invariants are obtained as

$$\tilde{Q}_{nm} = \frac{1}{\sqrt{\rho(n,p_1,N-1)\rho(m,p_2,M-1)}} \sum_{i=0}^{n}\sum_{j=0}^{m} a_{i,n,p_1} a_{j,m,p_2} \frac{R_{ij}}{(R_{10})^i (R_{01})^j} \quad (15)$$

VI. SIMULATION RESULTS

In order to verify the derived invariants, we have selected two images namely Lena and Cameraman each of size 256 x 256 with in an image frame 300 x 300. These images are scaled with different scale factors in x and y directions($-3 \leq \alpha, \beta \leq 3$) in steps of 0.25. Only few of the scale invariants are computed for each of these images. Then mean and variance of scale invariants are calculated and the results are entered in table I. Since the variance is zero, the invariants remain unchanged for all scale factors. In order to verify the translation invariant property of Krawtchouk moments, the selected image is shifted up, down, left and right with in the image frame. Selected order of modified Krawtchouk moments are computed for different translations ($-8 \leq x_0, y_0 \leq 8$) in steps of 1. The mean and variance of translation invariants are calculated and they are entered in table II. These results shows that modified Krawtchouk moments remains same for all translations. Finally, mean and variance of selected order of combined scale and translation invariants are computed for different scale factors ($-3 \leq \alpha, \beta \leq 3$) as well as different translations ($-8 \leq x_0, y_0 \leq 8$) and they are same as scale invariants. These results also show that combined scale and translation invariants remain same for all scale factors and translations.

To test the suitability of these invariants for face recognition problem, we have considered 4 subjects each with 4 positions (total of 16 faces) from ORL database. From the ORL database, each image is cut to a size of 50 x 50 dimension and the resulted images are shown in figure1. Combined scale and translation invariants upto orders 3 (a total of 16 invariants) are computed for each of these images as well as given test images. Euclidean distance between the combined invariants of test image and the combined invariants of database images are calculated. Euclidean distance corresponds to the minimum matches the test image with the database image. The recognition accuracy is good because of the small database used in the simulation. It was observed that recognition accuracy decreases as the size of the database increases. This problem can be solved by considering more number of invariants.

VII. CONCLUSIONS

Combined scale and translation invariants of Krawtchouk moments are derived in this paper. The standard procedure is adopted to derive the scale invariants. Modified Krawtchouk moments which are invariant for translated images are obtained with the assumption that the discrete weighted Krawtchouk polynomials are periodic and shifted to the centroid. Simulation results are carried out to verify the derived invariants.

APPENDIX

In order to prove that (11) satisfies the orthogonality property, let us consider $\sum_{x=0}^{N-1} K_n^*(x)|\alpha\ K_m^*(x)|\alpha$

$$= \sum_{x=\alpha}^{N-1} \bar{K}_n(x-\alpha)\bar{K}_m(x-\alpha) + \sum_{x=0}^{\alpha-1} \bar{K}_n(x+N-\alpha)\bar{K}_m(x+N-\alpha)$$

Let $x_1 = x-\alpha \quad and \quad x_2 = x+N-\alpha$ then

$$\sum_{x=0}^{N-1} K_n^*(x)|\alpha\ K_m^*(x)|\alpha$$

$$= \sum_{x_1=0}^{N-1-\alpha} \bar{K}_n(x_1)\bar{K}_m(x_1) + \sum_{x_2=N-\alpha}^{N-1} \bar{K}_n(x_2)\bar{K}_m(x_2)$$

since x_1 *and* x_2 are dummy variables, we replace them by x and y respectively. Hence the above equation can be written as

$$\sum_{x=0}^{N-1} K_n^*(x)|\alpha\ K_m^*(x)|\alpha$$

$$= \sum_{x=0}^{N-1-\alpha} \bar{K}_n(x)\bar{K}_m(x) + \sum_{x=N-\alpha}^{N-1} \bar{K}_n(x)\bar{K}_m(x)$$

$$= \sum_{x=0}^{N-1} \bar{K}_n(x)\bar{K}_m(x) = \delta_{nm}$$

Hence we say that modified discrete weighted Krawtchouk polynomials with a translation α has the same orthogonal property as the original weighted Krawtchouk polynomials.

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Table I. Mean and variance of scale invariants of Krawtchouk moments

Scale invariants for $-3 \le \alpha, \beta \le 3$	Lena		Cameraman	
	Mean	Variance	Mean	Variance
Q'_{10}	2.4843e+8	0	2.4284e+8	0
Q'_{02}	3.0325e+9	0	2.9642e+9	0
Q'_{11}	4.2958e+e9	0	4.1990e+9	0
Q'_{30}	3.0173e+10	0	2.9493e+10	0
Q'_{03}	3.0173e+10	0	2.9493e+10	0
Q'_{21}	5.2437e+10	0	5.1255e+10	0
Q'_{32}	6.3687e+12	0	6.2252e+12	0

Table II. Mean and variance of translation invariants of Krawtchouk moments

Translation invariants for $-8 \le x_0, y_0 \le 8$	Lena		Cameraman	
	Mean	Variance	Mean	Variance
ψ_{10}	-584.8970	0	-1.7529e+3	0
ψ_{20}	3.4417e+3	0	2.9246e+3	0
ψ_{11}	-694.3731	0	-85.0959	0
ψ_{02}	2.9236e+3	0	2.2597e+3	0
ψ_{12}	238.6759	0	-624.2416	0
ψ_{22}	3.2933e+3	0	1.8267e+3	0
ψ_{32}	347.9082	0	-434.5257	0

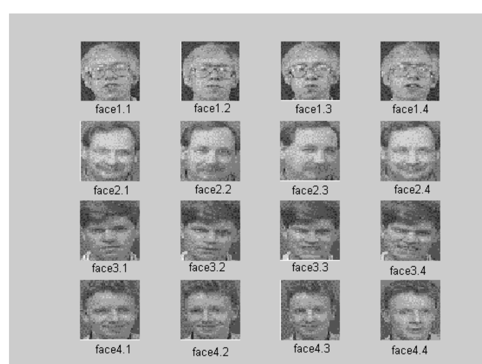

Fig.1. Images used in the face recognition experiment