National Conference on Computer Vision, Pattern Recognition, Image Processing & Graphics (NCVPRIPG -2013)

Image Denoising: Recent Trends

Tutorial

Session 2-b

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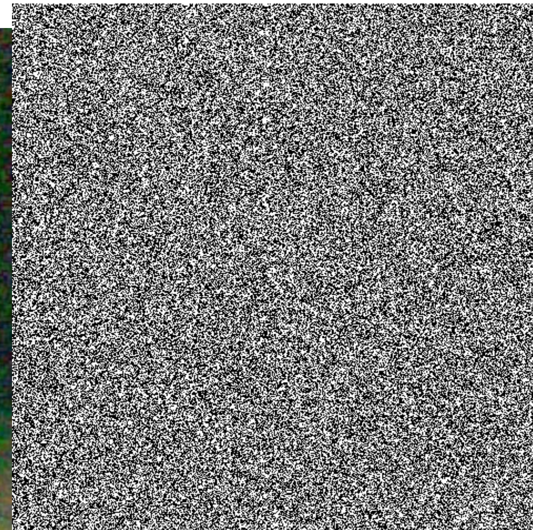
Noisy Image Formation

$$I(x) + \eta(x) = I_n(x)$$

Original Image



Noise (σ=10)

Noisy Image

Real Image captured from mobile camera

In all the experiments, the noise is assumed to be IID Gaussian with zero mean and σ standard deviation.

Performance (Image Quality) Measures

Peak Signal to Noise Ratio (PSNR)

- Used to compare various image processing algorithms.
- Most commonly know measures are
  - Root Mean Squared Error (RMSE)/ Mean Squared Error (MSE)
  - Peak Signal to Noise Ratio (PSNR)

Mean Squared Error $$MSE = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}\left(I(i,j) - J(i,j)\right)^2$$

where I and J are images of size MxN

Peak Signal to Noise Ratio $$PSNR = 10\log_{10}\left(\frac{L^2}{MSE}\right)$$

where L is the dynamic range of the image, eg. For 8bit image, L=255

- Very simple in calculation
- But does not involve the characteristics of the human visual system (HVS)

- Wang, Z.. & Bovik, A. C., (2009),'Mean Squared Error: Love it or Leave it?', *IEEE Signal Processing Magazine*, pp. 98-117, Jan.2009.

Q-Index

- Universal Image Quality Index (Q Index)

  Let $x=\{x_i \mid i=1,2,\ldots,N\}$ and $y=\{y_i \mid i=1,2,\ldots,N\}$ be the original and the test image signals, respectively. The index is defined as

$$\boxed{Q=\frac{4\sigma_{xy}\mu_x\mu_y}{(\sigma_x^2+\sigma_y^2)[\mu_x^2+\mu_y^2]}}$$

  - The dynamic range of Q is [-1, 1]
  - The best value 1 is achieved *iff* $y_i = x_i \forall i=1,2,\ldots,N$

$$Q=\underbrace{\frac{\sigma_{xy}}{\sigma_x\sigma_y}}_{\text{Correlation Coefficient}}\cdot\underbrace{\frac{2\mu_x\mu_y}{\mu_x^2+\mu_y^2}}_{\text{Luminance Distortion}}\cdot\underbrace{\frac{2\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}}_{\text{Contrast Distortion}}$$

- Local quality index can also be define in sliding window fashion

$$Q=\frac{1}{M}\sum_{j=1}^{M}Q_j$$

where M is the total number of blocks in image

- Wang, Z. & Bovik, A. C., (2002), ‘ A Universal Image Quality Index‘, *IEEE Signal Processing Letters*, vol. 9(3), pp. 81-84.

Structure Similarity Index Measure (SSIM)

- Generalization of Universal Quality Index
- The luminance of the surface of an object being observed is the product of the illumination and the reflectance
- Structures of the objects in the scene are independent of the illumination.
- Similarity measure should satisfy the following condition
  1. Symmetry: $SSIM(x,y) = SSIM(y,x)$
  2. Boundedness: $SSIM(x,y) \le 1$
  3. Unique Maximum: $SSIM(x,y) = 1 \text{ iff } x_i = y_i, i = 1,2,\ldots,N$

$$SSIM(x,y) = [l(x,y)]^{\alpha}[c(x,y)]^{\beta}[s(x,y)]^{\gamma} \qquad \alpha > 0, \beta > 0, \gamma > 0$$

$$l(x,y) = \frac{2\mu_x\mu_y + C_1}{\mu_x^2 + \mu_y^2 + C_1} \quad c(x,y) = \frac{2\sigma_x\sigma_y + C_2}{\sigma_x^2 + \sigma_y^2 + C_2} \quad s(x,y) = \frac{\sigma_{xy} + C_3}{\sigma_x\sigma_y + C_3}$$

$$C_1 = (K_1 L)^2, K_1 << 1 \quad C_2 = (K_2 L)^2, K_2 << 1$$

$$\boxed{SSIM(x,y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)}}$$

Taking $\alpha = \beta = \gamma = 1$, $C_3 = C_2 / 2$

Mean SSIM Index

$$MSSIM(X,Y) = \frac{1}{M}\sum_{j=1}^{M} SSIM(x_j, y_j)$$

- Wang , Z.., Bovik, A. C., Sheikh, H. R. & Simoncelli, E. P., (2004), 'Image Quality Assessment: From Error Visibility to Structural Similarity', *IEEE Trans. On Image Processing*, vol. 13(4), pp. 600-612.

Feature Similarity Index Measure (FSIM)

- Combination of Phase Congruency (PC) and Gradient Magnitude (GM)
- Computation of overall FSIM index consists of two stages
  - First stage: compute local similarity map
  - Second stage: pool the similarity map into a single similarity score

$$S_L(x) = [S_{PC}(x)]^{\alpha} [S_G(x)]^{\beta}$$

$$S_{PC}(x) = \frac{2PC_1(x)PC_2(x) + T_1}{PC_1^2(x) + PC_2^2(x) + T_1} \qquad S_{GM}(x) = \frac{2GM_1(x)GM_2(x) + T_2}{GM_1^2(x) + GM_2^2(x) + T_2}$$

$$\boxed{FSIM = \frac{\sum_{x \in \Omega} S_L(x) PC_m(x)}{\sum_{x \in \Omega} PC_m(x)}} \qquad PC_m(x) = \max(PC_1(x), PC_2(x))$$

- Zhang., L., Zhang. L., Mou, X. & Zhang, D., (2011), 'FSIM: A Feature Similarity Index for Image Quality Assessment', *IEEE Trans. On Image Processing*, vol. 20(8), pp. 2378-2386.

Some more approaches

- Method Noise: (Noisy Image – Denoised Image)
  - The residual image (also called “method noise”) – defined as the difference between the noisy image and the denoised image – should look like (and have all the properties of) a pure noise image.

- A denoising algorithm should transform a pure noise image into another noise image (of lower variance).

- A competent denoising algorithm should find for any pixel ‘i’, **<u>all and only</u>** those pixels ‘j’ that have the same model as ‘i’ (i.e. those pixels whose intensity would have most likely been the same as that of ‘i’, if there were no noise).

- A. Buades, B. Coll and J. M. Morel, A review of denoising algorithms, with a new one, Multiscale Model, Simul, Vol. 4, No. 2, pp. 490-530, 2005.
- Dominique B., Edward R.V. and Zhou W., The use of Residuals in Image Denoising, International Conference on Image Analysis and Representation, July 2009.

Early Age Denoising (Spatial Location based Filters)

Early Age Denoising (Spatial Location based Filters)

- Simple Averaging (mean filter) :

$$\hat{I}(x,y) = \frac{1}{|N(x,y)|} \sum_{j \in N(x,y)} I(x_j, y_j)$$

- Median Filtering:

$$\hat{I}(x,y) = median\,(N(x,y))$$

Mean Filter

- Lee Filter: Weighing according to the distance between Spatial locations of the pixels:

$$\hat{I}(x,y) = \frac{\sum_{j \in N(x,y)} I(x_j, y_j) e^{-((x-x_j)^2 + (y-y_j)^2)/(2\sigma^2)}}{\sum_{j \in N(x,y)} e^{-((x-x_j)^2 + (y-y_j)^2)/(2\sigma^2)}}$$

Lee Filter

Diffusion Processes

Isotropic Diffusion

$$\frac{\partial I}{\partial t} = div(\nabla I) = I_{xx} + I_{yy}$$

$$I^{(t+1)} = I^{(t)} + \Delta t(I_{xx}^{(t)} + I_{yy}^{(t)}); \; I^{(0)} = \text{noisy image}$$

- This process smoothes the image in order to remove noise. But also blurs out the fine details and edges.

Isotropic Filter

Anisotropic Diffusion [Perona-Malik diffusion]

- Anisotropic Diffusion is a technique aiming at reducing image noise without removing significant parts of the image content, typically edges, lines or other details that are important for the interpretation of the image .

$$\frac{\partial I}{\partial t} = div[g(\|\nabla I\|)\nabla I]$$

- Here, g(·) is an edge stopping function (Decreasing function of gradient magnitude) which is chosen such that the diffusion is stopped across edges.

$$g(s^2) = \frac{1}{1 + s^2/\lambda^2}; (\lambda > 0)$$

Perona Malik Filter

- P. Perona and J. Malik, Scale-Space and Edge Detection Using Anisotropic Diffusion, IEEE Trans. On PAMI, Vol. 12, No. 7, pp. 629-639, 1990.

Weickert's Anisotropic Diffusion

- Anisotropic diffusion inhibits diffusion at edges, hence noise at edges cannot be eliminated successfully.
- Weickert suggested process that allows diffusion along edges and inhibits diffusion perpendicular to them.
- The eigenvalues of the diffusion tenser are chosen as:

$$\lambda_1(\nabla I) = g\left(\left|\nabla I\right|^2\right),\ \lambda_2(\nabla I) = 1$$

Total Variation (Rudin et. at 1992)

$$TVF_\rho(I) = argmin_J TV(J) + \rho \int |I(\mathbf{x}) - J(\mathbf{x})|^2 dx$$

I : Noisy Image
J : Original Image
x: Image Space

Lagrange's Multiplier

Total Variation of J

$$TV(J) = \int |\nabla J| \, dx$$

Tikhonov Regularization

$$TR(J) = \int |\nabla J|^2 \, dx$$

- J. Weickert, Theoretical foundations of anisotropic diffusion in image processing, Computing Suppl. 11, 221-236, 1996.
- L. Rudin, S. Osher, and E. Fatemi. Nonlinear total variation based noise removal algorithms. Physica D, 60:259–268, 1992

Neighbourhood Filters

Neighbourhood Filters (Intensity value based)

- Yaroslavsky Filter:

$$I(x,y)=\frac{\sum_{j\in N(x,y)} I(x_j,y_j)e^{-(I(x,y)-I(x_j,y_j))^2/(2\sigma^2)}}{\sum_{j\in N(x,y)} e^{-(I(x,y)-I(x_j,y_j))^2/(2\sigma^2)}}$$

- L. P. Yaroslavsky, Digital Picture Processing. An Introduction, Springer-Verlag, Berlin, 1985.
- L. Yaroslavsky and M. Eden, Fundamentals of Digital Optics, Birkhauser Boston, Boston, MA, 1996.

Bilateral Filtering

- It combines gray levels or colors based on both their *geometric closeness* and their *photometric similarity*, and prefers near values to distant values in both domain and range.
- Its *non-iterative*, *local*, *edge preserving* and simple.
- Two pixels can be *close* to one another, i.e., occupy nearby spatial location or they can be similar possibly in a perceptually meaningful fashion.

$$h_1(x) = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\xi)c(\xi,x)d\xi}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c(\xi,x)d\xi}$$

Geometric Closeness

$$h_2(x) = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\xi)s(f(\xi), f(x))d\xi}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s(f(\xi), f(x))d\xi}$$

Photometric Similarity

$$H(x) = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\xi)c(\xi,x)s(f(\xi), f(x))d\xi}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c(\xi,x)s(f(\xi), f(x))d\xi}$$

Bilateral Filter

- Tomasi, C. & Manduchi R., (1998), ‘Bilateral Filtering for Gray and Color Images‘, *Proc. Of IEEE ICCV*, pp. 839-846.
- Paris, S., Kornprobst, P., Tumblin, J. & Durand, F., (2008), ‘Bilateral Filterning: Theory and Applications‘, *Foundataions and Trends in Computer Graphics and Vision*, vol. 4(1), pp. 1-73.

Scaled Bilateral Filtering

$$H(x) = \frac{\sum_{\xi \in \mathrm{N}(x)} c(\xi, x)\, s\big(f_G(\xi), f(x)\big) f(\xi)}{\sum_{\xi \in \mathrm{N}(x)} c(\xi, x)\, s\big(f_G(\xi), f(x)\big)}$$

$$f_G(x) = \sum_{\xi \in \mathrm{N}(x)} w_G(\xi, x) f(\xi)$$

- Aswatha, S. M., Mukhopadhyay, J. & Bhowmick, P., (2011), 'Image Denoising by Scaled Bilateral Filtering', *Proc. 3rd National conference on Computer Vision, Pattern Recognition, Image Processing and Graphics*, pp. 122-125.

Trilateral Filtering

- Rank Ordered Absolute Differences (ROAD) Statistics
  - Consider a window of size 3x3 at pixel location x with intensity $f(x)$
  - Define $d_{x,\xi}$ as the absolute difference in intensity of pixels between x and $\xi \in N(x)$

$$d_{x,\xi} = \left| f(x) - f(\xi) \right|$$

  - Finally, sort these values in increasing order and define

$$ROAD_m(x) = \sum_{i=1}^{m} r_i(x), \ \ 2 \le m \le 7$$

$$r_i(x) = i\text{th smallest } d_{x,\xi}$$

Impulsive weight w_I at a point x is given by $w_I(x) = \exp\left[-\frac{ROAD(x)^2}{2\sigma_I{}^2} \right]$

Joint Impulsivity, J $J(x,\xi) = 1 - \exp\left[- \frac{\left(\frac{ROAD(x) + ROAD(\xi)}{2} \right)^2}{2\sigma_J^2} \right]$

Trilateral weights $h(x,\xi) = c(x,\xi) s(x,\xi)^{1-J(x,\xi)} w_I(\xi)^{J(x,\xi)}$

- Garnett., R., Huegerich, T., Chui, C. & He, W., (2005), 'A Universal Noise Removal Algorithm with an Impulse Detector', *IEEE Trans. On Image Processing*, vol. 14(11), pp. 1747-1754.

Trilateral Filtering

- Trilateral Weights are defined as follows

$$h(\xi, x) = (1 - a(x))c(\xi, x) + a(x)c(\xi, x)s(f(\xi), f(x))\sum_{i=1}^{D-1} d_i(\xi, x)$$

where $a(x) \in [0,1]$ is regularized local signal amplitude of the pixel at x

$a \to 0$ in the homogeneous regions.

D is the dimensionality of the image

d_i measures the similarity of the rank i local structural orientation between the pixels at ξ and x.

$$d_i(\xi, x) = \exp\left[-\frac{\delta^2(\xi - x, \hat{e}_i)}{2\sigma^2}\right]$$

$\hat{e}_i$ is the direction of rank i orientation

$$\delta(u, v) = 1 - \left|\frac{u.v}{\|u\|\|v\|}\right|$$

Local structure information is obtained from the Eigen value decomposition of orientation tensor.

• Wong, W. C. K., Chung, A. C. S. & Yu, S. C. H., (2004), 'Trilateral Filtering for Biomedical Images', *Proc. of IEEE International Symposium on Biomedical Imaging: Nano to Macro*, vol. 1, pp. 820-823.

Rough Set Theory

- Introduced by Z. Pawlak during early 1980s .
- Another approach to vagueness (Not an alternative to classical set theory like fuzzy set).
- Imprecision in this approach is expressed by boundary region of the set.

Let $\zeta = \langle U, A \rangle$ be an information system,and let $B \subseteq A$ and $X \subseteq U$.

B-lower Approximation $\quad \hat{B}X = \{x \in U : [x]_B \subseteq X\}$

B-upper Approximation $\quad \tilde{B}X = \{x \in U : [x]_B \cap X = \phi\}$

Boundary Region $\quad \tilde{B}X - \hat{B}X$

Roughness of X w.r.t. B $\quad R = 1 - \dfrac{|\hat{B}X|}{|\tilde{B}X|}$

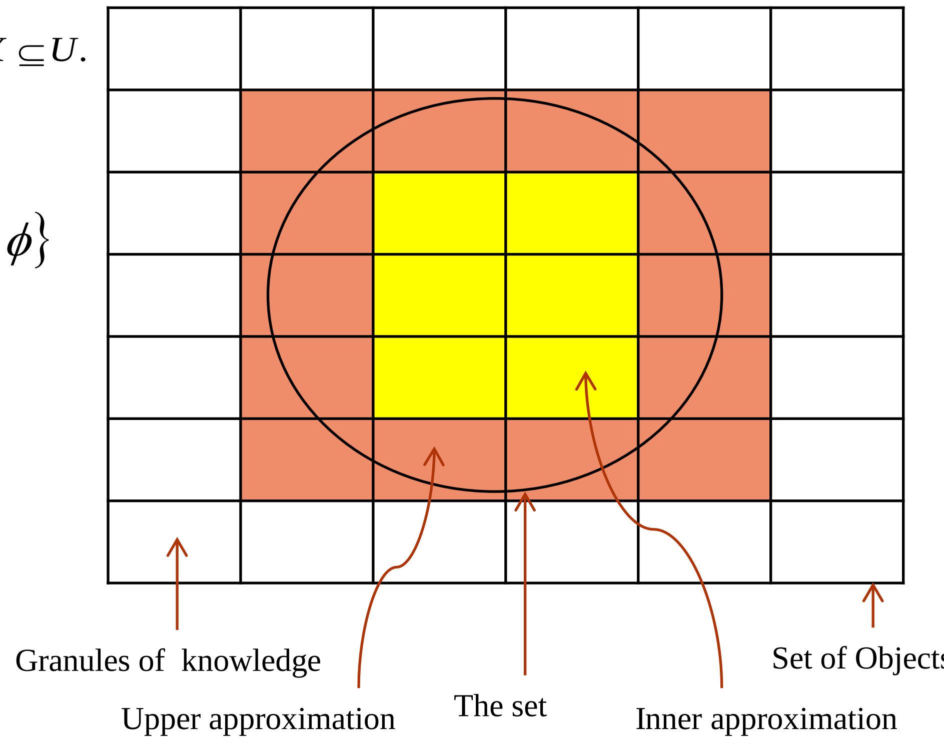


Figure 1: Rough Set Theory

- Pawlak, Z. & Skowron, A., (2007), ‘Rudiments of Rough Sets’, *Information Sciences*, vol. 177 (1), pp. 3-27.
- A. Phophalia. S. K. Mitra, A. Rajwade, A new Denoising filter for brain MR image, In Proc. Of ICVGIP, 2012.

Rough Set based Trilateral like Filter

Derivation of Class Information

- The goal is to assign a class label to each pixel of the image.
- Rough set is used to capture impreciseness present due to noise.
- The concept of Rough Set based binarization (two class problem) is utilized successively.
- As an initial step, approximate thresholds (valleys in the histogram) are obtained.
- In the next step, the approximate thresholds are optimized separately using Rough Entropy measure.

$$RE_T = -\frac{1}{2}\left[R_{O_T}\log_e\left(\frac{R_{O_T}}{e}\right) + R_{B_T}\log_e\left(\frac{R_{B_T}}{e}\right)\right], \qquad T = Threshold$$

- The image under consideration is binarized for each threshold and a pixel is given a symbol either 0 or 1 for each threshold.
- Thus, a pixel will get K such symbols for K thresholds present in the image. Each pixel could then be represented by a binary string of length K.

- Pal, S. K., Uma Shankar, B. & Mitra, P., (2005), 'Granular Computing, Rough Entropy and Object Extraction', *Pattern Recognition Letters*, vol. 26, pp. 2509-2517.

Proposed Method

Derivation of Class Information

1. Find Threshold from noisy image histogram
2. Optimize each threshold separately using Rough Entropy Criteria
3. Binarize the image w.r.t each optimized thresholds
4. Combine assigned symbols as binary string of length K for each pixel
5. Classify all binary string and there by pixels

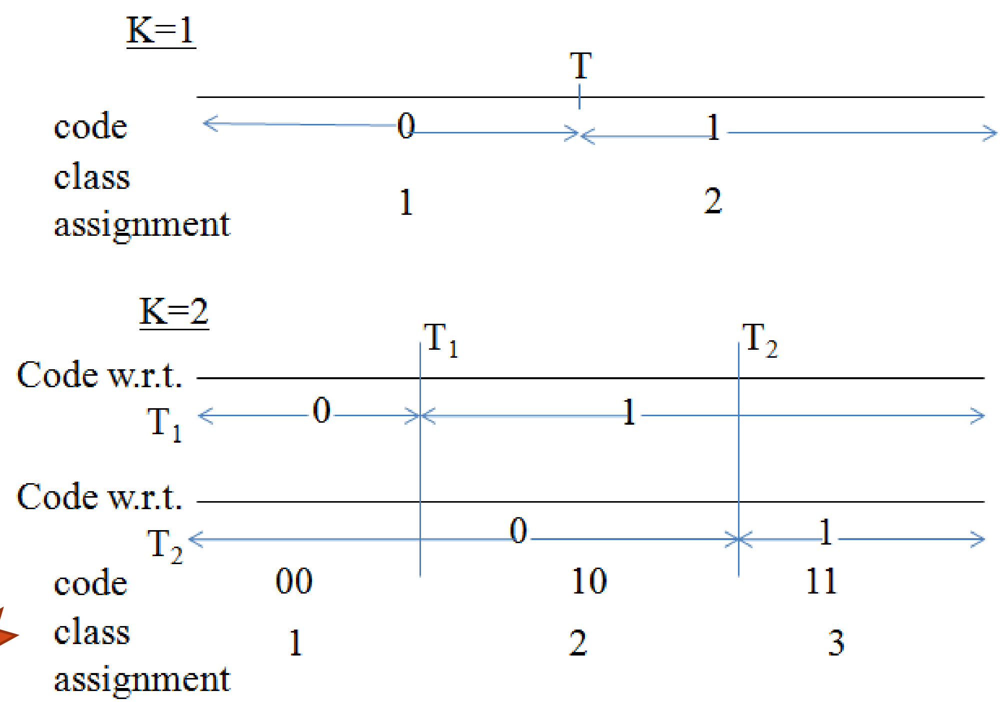


Figure 2: (a) Flow chart, (b) Code and Class assignment for different number of thresholds. Here K is number of thresholds.

Proposed Method

Derivation of Rough Edge Map (REM)

- To obtain the cardinality of lower and upper approximation of the object, a granule based method is adopted. The minimum granule size could be 1x2, 2x1 or 2x2.
- This leads to a situation where some granules will not be counted entirely in lower approximation or entirely in upper approximation. These granules, thus could be considered as thick rough edge of the object.
- Each optimized threshold will generate a thick (e.g. 2x2 blocks) rough edge map for the object under consideration.
- The union of all such rough edge maps is expected to fetch the rough edge map of the entire image.

Proposed Method

Denoising Filter

- The denoised pixel is given by

$$\hat{Y}_{i,j} = \frac{\sum_{j \in N(i)} \varphi(i, j) Y(i, j)}{\sum_{j \in N(i)} \varphi(i, j)}$$

where *Y(i,j)* is the noisy image pixel and *N(x)* denotes neighborhood of x

$$\varphi(i, j) = \gamma(i, j)\alpha(i, j)\beta(i, j)$$

$$\gamma(i, j) = \text{either } \gamma_s, \gamma_m \text{ or } \gamma_l \quad \alpha(i, j) = \exp\left(-\frac{i^2 + j^2}{2\sigma}\right) \quad \beta(i, j) = \exp\left(-\frac{1}{2\sigma_\beta^2 Blk\_diff(i, j)}\right)$$

$$\gamma_s(i, j) \in [0, 0.4]$$
$$\gamma_m(i, j) \in (0.4, 0.7]$$
$$\gamma_l(i, j) \in (0.7, 1]$$

Derived based on neighborhood relation from REM and RCL

Results

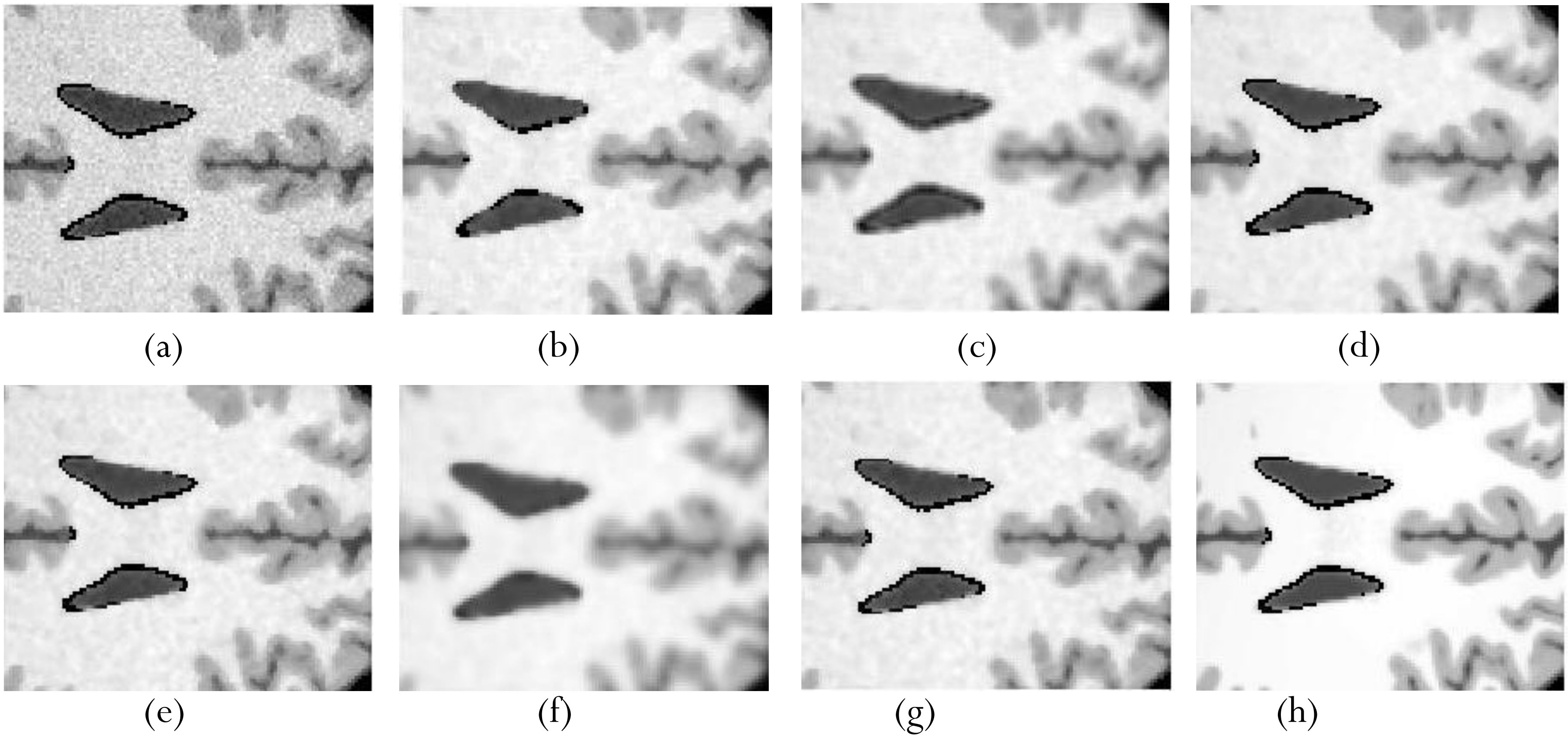


Figure: (a) Noisy Image, (b) Median filter (3x3), (c) Mean filter (3x3), (d) Bilateral filter, (e) Trilateral filter (Garnett et al.), (f) Scaled Bilateral filter, (g) Rough Set based Trilateral like filter, (h) Original Image

- Simulated Brain Database http://mouldy.bic.mni.mcgill.ca/brainweb/

Transformed Domain Filtering

Transformed Domain Filtering

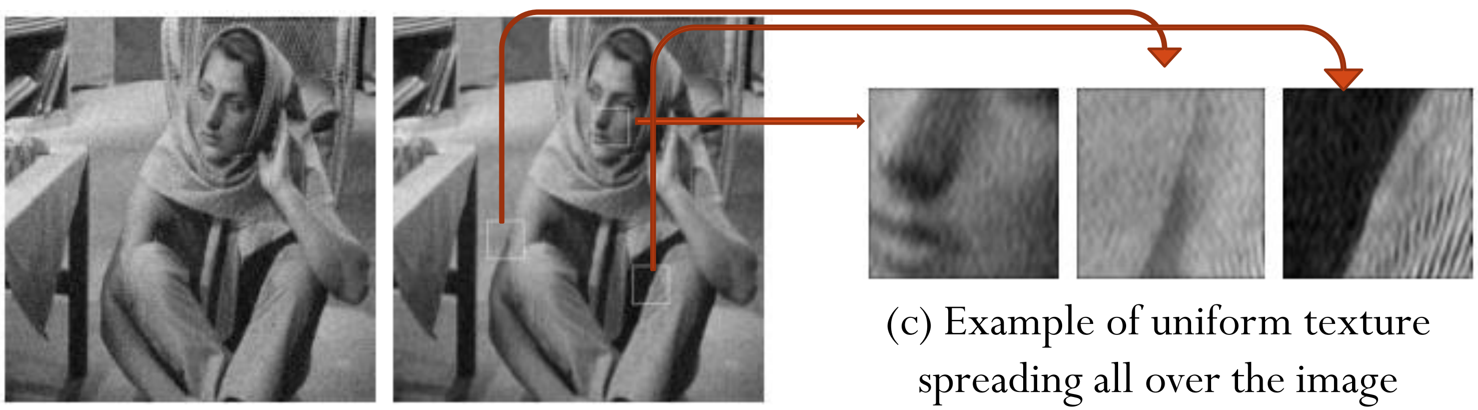
- Noisy image formation model: $I_n(x) = I(x) + \eta(x)$
- β be the orthonormal basis, hence the model is transformed as:

$$I_{n\beta}(\alpha) = I_\beta(\alpha) + \eta_\beta(\alpha)$$

- Various transformed domain filters are applied independently to every transform coefficient $I_{n\beta}(\alpha)$.
- The denoised image is estimated by inverse transform of the modified coefficients.
- Transformation domains:
  - **Fourier domain**
  - **DCT domain**
  - **Wavelet domain**
  - **Any other orthonormal basis**

Global vs. Local

- In case of Global denoising, whole image is processed at once.
- Here, global image characteristics may prevail over local ones and create spurious periodic patterns.



(a) Noisy Image (b) Denoised image in Fourier Domain

(c) Example of uniform texture spreading all over the image

- To over come this issue, local adaptive filters are used.
- The noisy image is analyzed in a moving window fashion. Each position of the window is processed and modified independently.

Various methods for modifying the coefficients

- Model of an image in transformed domain: $I_{n\beta}(\alpha) = I_{\beta}(\alpha) + \eta_{\beta}(\alpha)$
- Noisy coefficients $I_{\eta\beta}(\alpha)$ are modified to $a(\alpha)\, I_{\eta\beta}(\alpha)$, $0 \le a(\alpha) \le 1$.
- Some of the ways of finding $a(\alpha)$ are:
- **Hard Thresholding:**

$$a(\alpha) = \begin{cases} 1\ ; & |I_{\eta\beta}(\alpha)| \ge \tau \\ 0\ ; & \text{otherwise} \end{cases}$$

- **Soft Thresholding:**

$$a(\alpha) = \begin{cases} \dfrac{I_{\eta\beta}(\alpha) - sign(I_{\eta\beta}(\alpha))\tau}{I_{\eta\beta}(\alpha)}\ ; & |I_{\eta\beta}(\alpha)| \ge \tau \\ 0\ ; & \text{otherwise} \end{cases}$$

$\tau = \sigma\sqrt{2\log(N)}$

N is the window size

- **Weiner Filter Update:**

$$a(\alpha) = \frac{\sigma_I^2}{\sigma_I^2 + \sigma^2};$$

σ is standard deviation of noise

σ_I is standard deviation of original image I

- Note – these thresholding rules cannot be applied in the spatial domain directly, as neighboring pixels values are strongly correlated.

Translation invariance (TI)

- Usual practice in these local adaptive techniques is of processing all the overlapping patches of the image independently.
- In such cases, a pixel undergoes the denoising process many times.
- Considering the latest update for a particular pixel may loose some information.
- Further improvement is achieved by averaging the different results that appear at each pixel.

Hard thresholding results on Barbara

Without TI

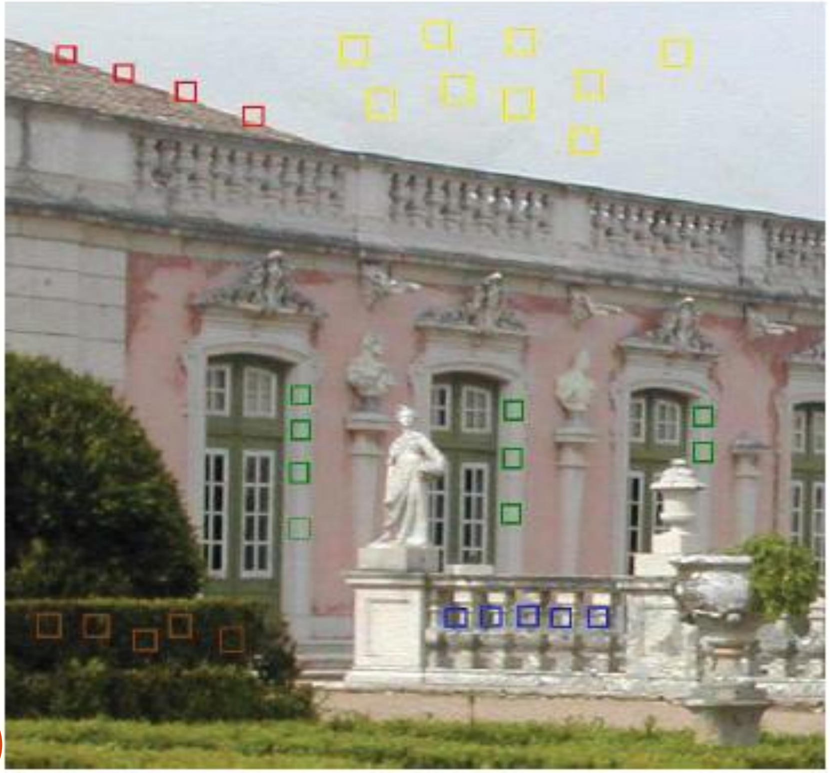
With TI

Haar Wavelets DB2 Wavelets DCT

Non-Local Self Similar Approach

Non-local Self Similarity

- The methods discussed so far aim at noise reduction and reconstruction of the main geometrical configurations, but not at the preservation of the fine structure, details and texture as they are based on the principle of piece-wise constant intensity.
- Such fine details are smoothed out as they behave in all functional aspects as noise.



- In order to overcome this issue, the high degree of redundancy present in the natural image is exploited.
- Every small window (patch) in a natural image has many similar windows in the same image.

Non-Local Means

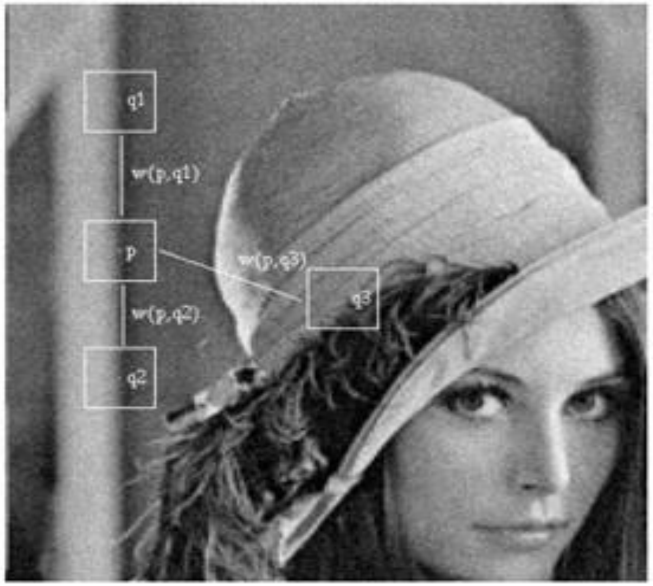
- Non-local means compares entire patches (not individual pixel intensity values) to compute weights for denoising pixel intensities.
- Comparison of entire patches is more robust, i.e. if two patches are similar in a noisy image, they will be similar in the underlying clean image with very high probability.

$$NL(I_\eta)(i) = \sum_{j \in I_\eta} w(i, j) I_\eta(j)$$

- Here, the weight *w(i,j)* depends on similarity between the neighborhood window *N(i)* and *N(j)* of both the pixels respectively.

$$w(i, j) = \frac{1}{Z(i)} e^{-\frac{\left\| I_\eta(N(i)) - I_\eta(N(j)) \right\|^2}{h^2}}$$

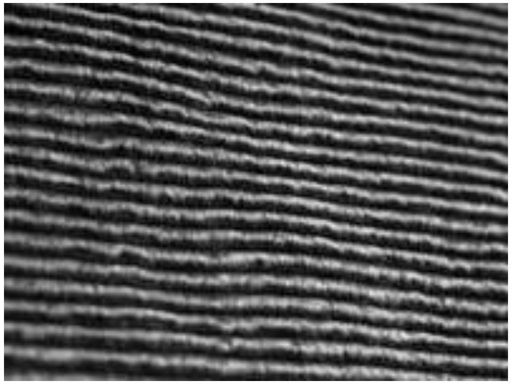
$Z(i) = \sum_j e^{-\frac{\left\| I_\eta(N(i)) - I_\eta(N(j)) \right\|^2}{h^2}}$ is the normalizing factor and h controls the decay of the function.



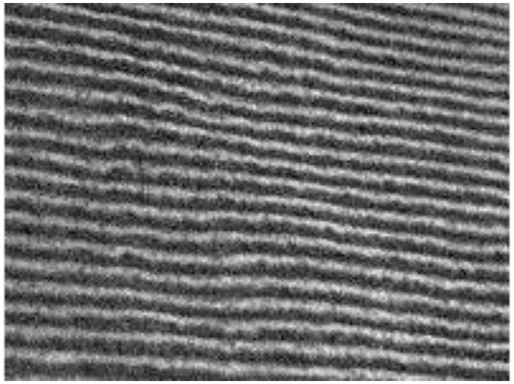
- Due to fast decay of the exponential kernel, large Euclidean distances lead to nearly zero weights, acting as an automatic threshold.
- Too-small patch size tends to create fake edges and patterns in the constant intensity regions whereas too large patch size over smooth the fine details.

- A. Buades, B. Coll and J. M. Morel, A review of denoising algorithms, with a new one, Multiscale Model, Simul, Vol. 4, No. 2, pp. 490-530, 2005.

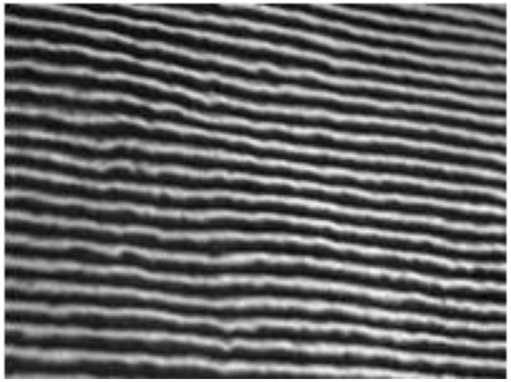
Non-Local Means Algorithm Results



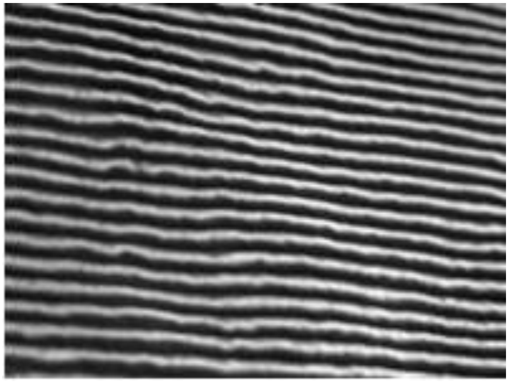
Original Periodic Image



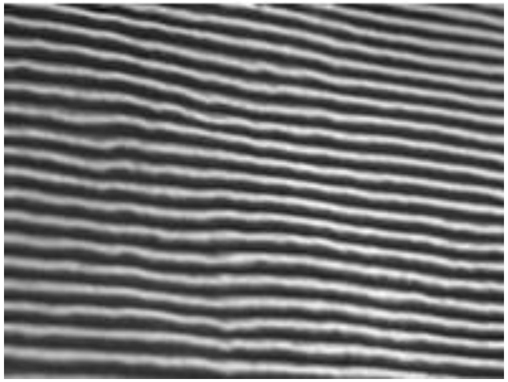
Noisy Image (σ=20)



Patch Size= 3 x 3
(PSNR: 26.894, SSIM: 0.9698)



Patch size= 5 x 5
(PSNR : 26.35, SSIM: 0.9655)



Patch size = 8 x 8
(PSNR: 25.77, SSIM: 0.9600)

Non-Local Means Algorithm Results

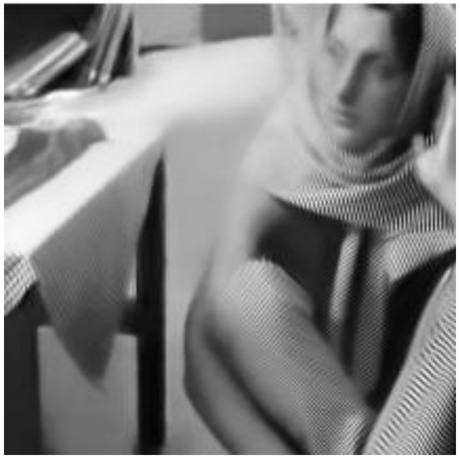
Original Image

Noisy Image (σ=20)

Patch size= 3 x 3
(PSNR : 26.95, SSIM: 0.8194)

Patch size = 5 x 5
(PSNR: 26.51, SSIM: 0.8091)

Patch Size= 8 x 8
(PSNR: 25.58, SSIM: 0.7739)



Patch size = 16 x 16
(PSNR: 25.09, SSIM: 0.7513)

Comparison of different denoising methods

Original Image

Noisy Image (σ=20)

Gaussian Smoothing:
(PSNR: 24.22, SSIM:0.7343)

Anisotropic Diffusion:
(PSNR: 28.06, SSIM: 0.8099)

Bilateral Filtering:
(PSNR: 27.47, SSIM: 0.7664)

NL-Means:
(PSNR: 28.08, SSIM: 0.812)

Comparison of different denoising methods

Original Image

Noisy Image (σ=20)

Gaussian Smoothing:
(PSNR: 24.09, SSIM:0.8118)

Anisotropic Diffusion:
(PSNR: 25.27, SSIM: 0.8237)

Bilateral Filtering:
(PSNR: 24.67, SSIM: 0.8198)

NL-Means:
(PSNR: 25.98, SSIM: 0.8359)

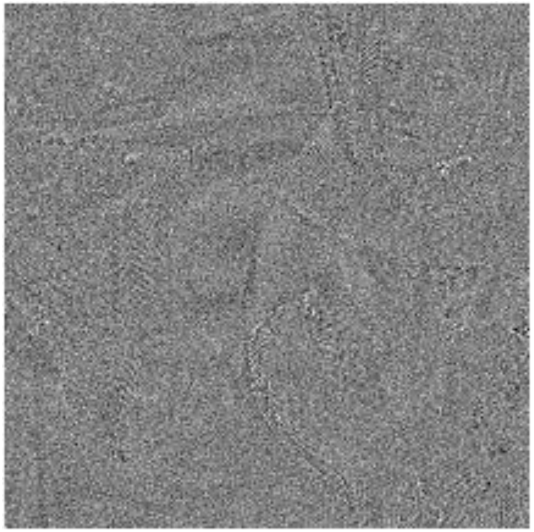
Method Noise

Original Image

Gaussian Smoothing

Anisotropic Diffusion

Bilateral Filtering



NL-Means filtering

Method Noise

Original Image

Gaussian Smoothing

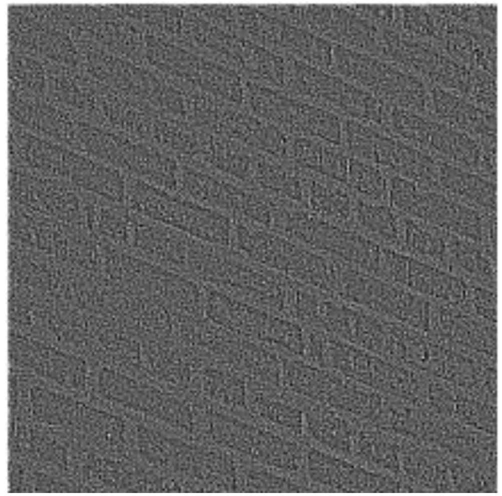
Anisotropic Diffusion

Bilateral Filtering

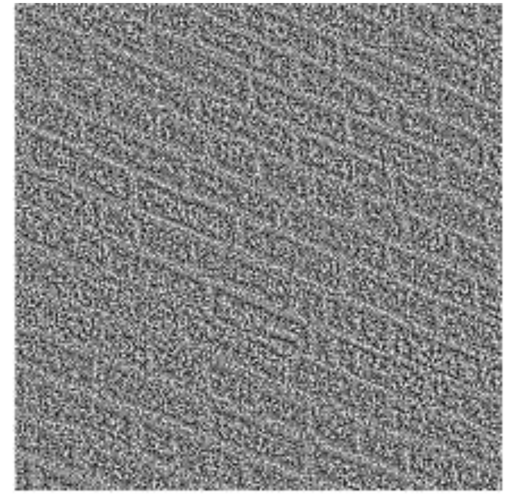
NL-Means filtering

Method Noise

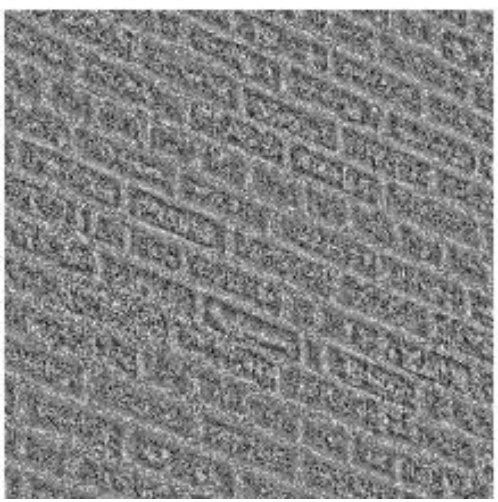
Original Image



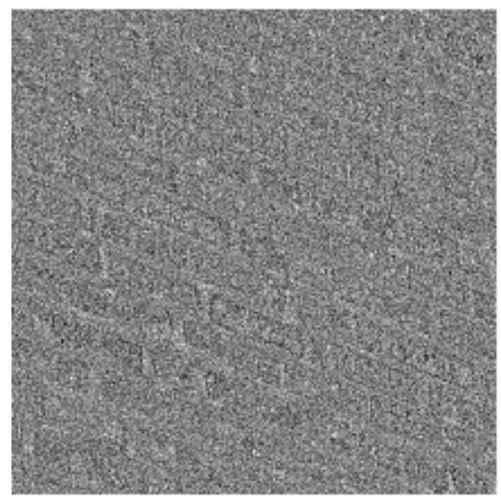
Gaussian Smoothing



Anisotropic Diffusion



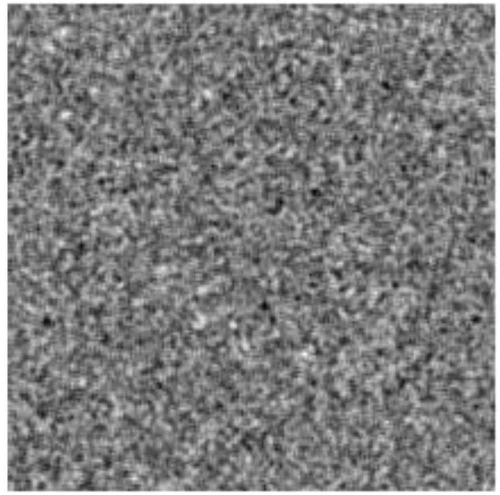
Bilateral Filtering



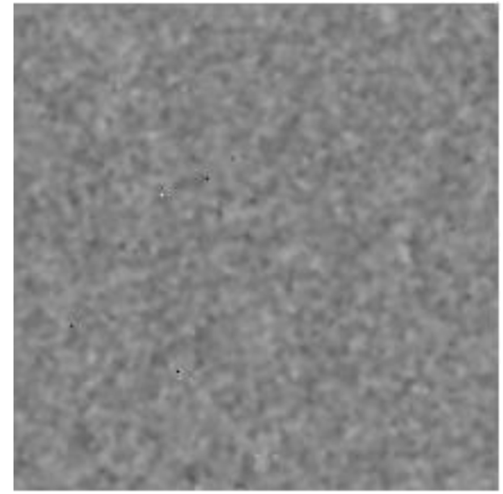
NL-Means filtering

Noise to noise

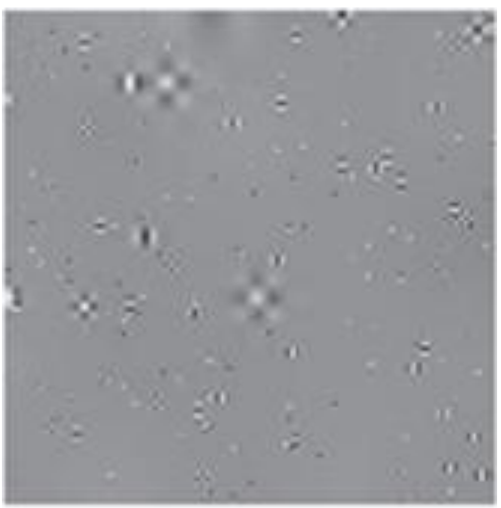
Noise Sample



Filtered Noise: Gaussian Smoothing



Filtered Noise: Anisotropic Diffusion



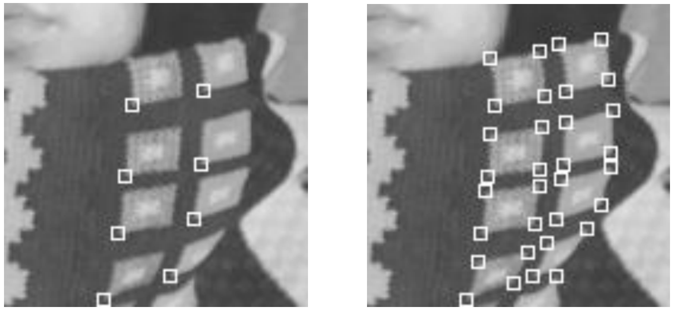
Filtered Noise: Wavelet Thresholding

Filtered Noise: Bilateral Filtering

Filtered Noise: NL-Means

Variants of NLM Method

- **Rotationally Invariant Block Matching (RIBM):** (Zimmer et al. 2008)
  - Estimate the angle of rotation between blocks
  - To each pixel in the first block, find the position of the corresponding pixel in the second block by rotating its vector by estimated angle.



- **Moment Invariant based Measure** (Zimmer et al. 2011)
  - 7 Moment Invariants proposed by Hu using Theory of Algebraic Invariants in 1962.
  - Grewenig et al. used Zernike Moments as features of each block and used it in block matching to classical framework of NL Means method.

- **SSIM based NLM method** (Rahman et al. 2011)
  - Patches are compared using SSIM measure (As discussed earlier)
  - It provides an inherent advantage of comparing structurally similar patches.

- Zimmer, Sebastian, Stephan Didas, and Joachim Weickert. "A rotationally invariant block matching strategy improving image denoising with non-local means." Proc. 2008 International Workshop on Local and Non-Local Approximation in Image Processing. 2008.
- Grewenig, Sven, Sebastian Zimmer, and Joachim Weickert. "Rotationally invariant similarity measures for nonlocal image denoising." Journal of Visual Communication and Image Representation 22.2 (2011): 117-130.
- Rehman, Abdul, and Zhou Wang. "SSIM-based non-local means image denoising." Image Processing (ICIP), 2011 18th IEEE International Conference on. IEEE, 2011.

Combination of NL Principle and Transform Domain Approaches

Non-Local PCA and its variants

- So far, fixed orthonormal basis are used for the transformed domain approaches for denoising.
- New transform bases can be learnt by using the non-local similarity of the patches.
- The most basic transform learnt is Principle Component Analysis (PCA).
- Similar patches x_is are grouped together in the data matrix : $X = [x_1 \ x_2 \ \ldots\ldots\ldots\ldots \ x_M]$

$x_1, x_2, \ldots., x_m$ are patches in vector fromat.

- Normalize the data to have zero mean.

$$A = X - \overline{X} \qquad \text{where,} \qquad \overline{X} = \frac{1}{M}\sum_{i=1}^{M} x_i$$

- Calculate the Covariance matrix: $C = AA^T$
- Basis matrix W can be found by solving the eigenvalue problem: $CW = \lambda W$

$W = [w_1 \ w_2 \ \ldots\ldots\ldots\ldots \ w_k]$, where each w_i is the eigenvector of C.

- Coefficients of patches in the transformed (PCA) domain are:

$$Y = W^T X$$

Non-Local PCA and its variants

- **Adaptive PCA** [Muresan et. al.]**:** The bases are learnt for each patch from the collection of similar patches for that particular patch.
- The patches are considered to be similar if their squared distance is less than or equal to $3n\sigma^2$. (n : patch size, σ: standard deviation of noise)
- The coefficients of patch in the transformed PCA space are manipulated using soft or hard thresholding or wiener filter update.
- Inverse the transformation, repeat the procedure for each patch and reconstruct the image.
- **LPG-PCA**[Zhang et. al.] works in 2-stages. Local Pixel Grouping (LPG) refers to finding similar patches to the reference patch.
- Denoised image using first stage is again denoised in the same way after estimating the noise level: $\sigma_s = c_s\sqrt{\sigma^2 - E[\tilde{I}^2]}$

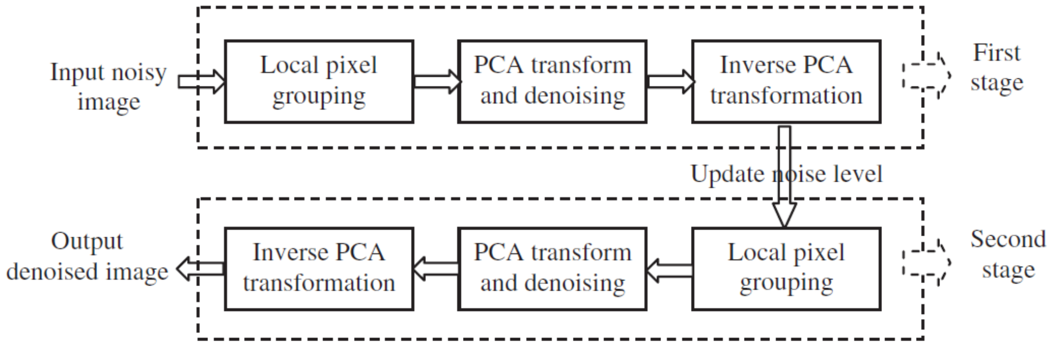


Fig. 1. Flowchart of the proposed two-stage LPG-PCA denoising scheme.

- Muresan and Parks, Adaptive principal components for image denoising, ICIP , Vol. 1. pp. 101-104, 2003.
- L. Zhang, W. Dong, D. Zhang, and G. Shi. Two-stage image denoising by principal component analysis with local pixel grouping. Pattern Recognition, 43(4):1531–1549, Apr. 2010.

Non-Local PCA and its variants

- **From Gaussian Mixture Models to Structured Sparsity** [Yu et. Al.]: The approach uses Gaussian Mixture Models to perform image denoising. MAP-EM algorithm is used to estimate original image *x* from the degraded image *y* is:
  - E-step: Assuming the estimates of $\{\mu_k, \Sigma_k\}_{1 \le k \le K}$ - Gaussian parameters are known, calculate MAP estimate for each patch x_i with all the gaussians and select the best Gaussian model k_i to obtain the estimate of the patch.
  - M-step: Update the Gaussian parameters using the information obtained in the previous step.
- Each of the K component will have a covariance matrix which can be defined in terms of PCA basis$\Sigma_k = W_k S_k W_k^T$. Here, W_k is the PCA basis and S_k is the diagonal matrix consisting of the eigenvalues of the covariance matrix. For each component, these basis are linear in nature.
- The patch x_i can be transformed to the PCA basis $a_i^k = W_k^T x_i$ So if the PCA coefficients a_i 's are known, $x_i^k = W_k a_i^k$.

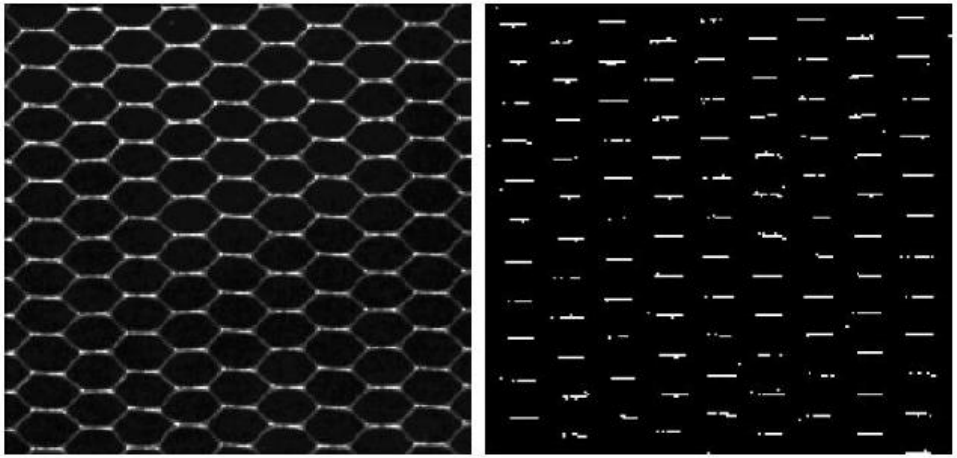
$$a_i^k = \arg\max_{a_i} \left(\left\| W_k a_i - y_i \right\|^2 + \sigma^2 \sum_{m=1}^{N} \frac{|a_i[m]|^2}{\lambda_m^k} \right)$$

- For each patch, only one of the a_i's is active which makes the selection of the basis very sparse in nature.

- Yu, Guoshen, Guillermo Sapiro, and Stéphane Mallat. "Solving inverse problems with piecewise linear estimators: from Gaussian mixture models to structured sparsity." *Image Processing, IEEE Transactions on*, vol 21, no. 5 (2012): 2481-2499.

Non-Local PCA and its variants

- **Clustering based Sparse Representation (CSR)** : The work performs a clustering operation on the noisy patches, learns separate PCA bases W for each cluster, and then minimizes a criterion for the proximity of the coefficients of patches belonging to any given cluster.
- Any patch of the image can be represented as $x = W\alpha$, where α represents the coefficients of the patch.
  - Observation of these coefficients suggests that they are not randomly distributed and their location uncertainty is related to non-local self similarity of the image, which implies the possibility of achieving higher sparsity by exploiting such location related constraint.
  - The coefficients are updated using the following optimization problem:



Left to right: Image of regular texture. Distribution of coefficients corresponding to 6th basis vector.

$$(\alpha, \mu) = \arg\min_{\alpha, \mu_k} \frac{1}{2}\|Y - W\alpha\|_2^2 + \lambda_1\|\alpha\|_1 + \lambda_2 \sum_{k=1}^{K} \sum_{i \in C_k} \|W\alpha_i - \mu_k\|_1$$

Y : noisy image, W : reconstruction operator

μ_k : centroid of the k^{th} cluster C_k of α

- The shrinkage solution of the above problem is found iteratively.

• Dong, Weisheng, et al. "Sparsity-based image denoising via dictionary learning and structural clustering." *Computer Vision and Pattern Recognition (CVPR), 2011 IEEE Conference on*. IEEE, 2011.

Non-local collaborative filtering: Block Matching in 3D (BM3D)

- BM3D is the state of the art method today which again is based on the idea of non-local similarity at the patch-level and works in two-stages.

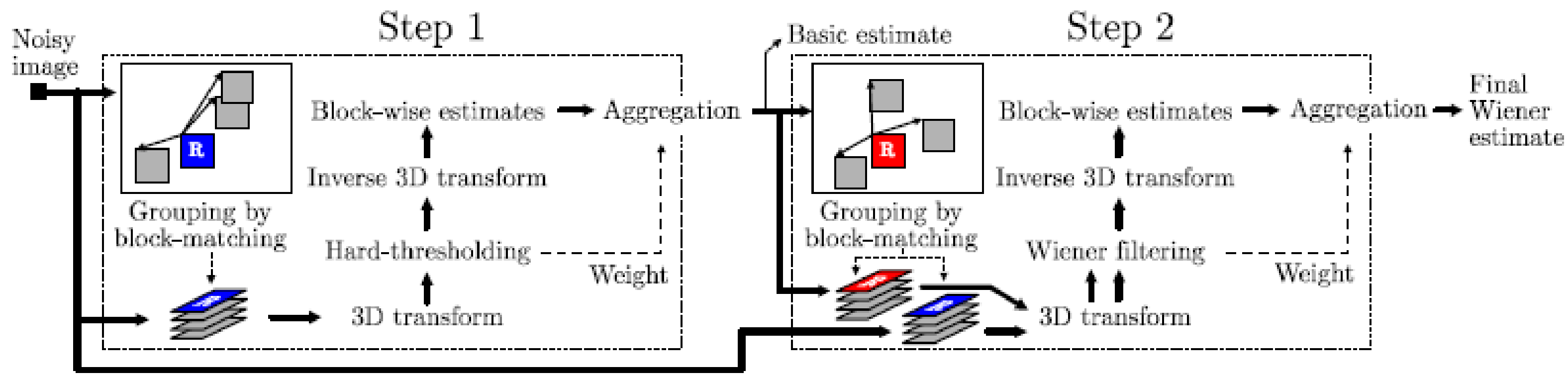


Fig. 3. Flowchart of the proposed image denoising algorithm. The operations surrounded by dashed lines are repeated for each processed block (marked with "R").

- Given a reference patch in the noisy image, this method again collects similar patches.
- But this time, the similar patches and the reference patch are arranged in the form of a 3D stack (of say some K patches in all).
- The stack is projected onto 3D transform bases (typically 3D DCT, or tensor product of 2D DCT and 1D Haar wavelets).
- The 3D transform coefficients are manipulated – usually by hard thresholding.
- All the patches in the entire stack are reconstructed using an inverse 3D transform.
- This is repeated for every patch in the image. The multiple answers appearing at any pixel are averaged.

- K. Dabov, A. Foi, V. Katkovnik, and K. Egiazarian, Image denoising by sparse 3d transform domain collaborative filtering, IEEE Trans on Image Processing, 16(8), 2080-2095, 2007.

Non-local collaborative filtering: BM3D – second stage

- In the second stage, the output image of the first step is used to compute patch similarities (this will be more robust than computing the similarities in the noisy image).
- Patches from the first-stage image are then appropriately assembled into a stack.
- Corresponding patches from the noisy image are assembled into a second stack.
- 3D transform coefficients of both the stacks are computed.
- The second stack is denoised using Wiener filtering as follows:

$$c_{\text{second-stage}} = \frac{c^2_{\text{stage-one}}}{c^2_{\text{stage-one}} + \sigma^2} c_{\text{noisy}} \qquad c_i : \text{coefficient in 3D trasformed domain}$$

- This is again repeated in sliding-window fashion with averaging.

Image denoising using New Orthogonal Locality Preserving Projection

- Orthogonal Locality Preserving Projection finds an embedding that preserves local information, and obtains a subspace that best detects the essential data manifold structure.
- Objective function of LPP is $\boldsymbol{min}\sum_{ij}(y_i - y_j)^2 S_{ij}$, to obtain orthonormal basis, constraint $w^T w = I$ is imposed.

  where Weight Matrix S is:

$$S_{ij} = \begin{cases} 1; \text{ if } x \le a \\ 1 - 2\left(\dfrac{x-a}{b-a}\right)^2; \text{ if } a \le x \le \dfrac{a+b}{2} \\ 2\left(b - \dfrac{x}{b-a}\right)^2; \text{ if } \dfrac{a+b}{2} \le x \le b \\ 0; \text{ if } x \ge b \end{cases}$$

Here, $x = \left\| x_i - x_j \right\|$ and $[a, b]$ is the distnace range that can be controlled.

- The objective function with this choice of symmetric weights incurs a heavy penalty if neighbouring points are mapped far apart.
- The minimization problem now reduces to $XLX^T w = \lambda w$ where w contains the orthogonal basis vectors.

- X. He and P. Niyogi, Locality Preserving Projections, Proc. Conf. Advances in Neural Information Processing Systems, 2003.
- G. Shikkenawis, S.K. Mitra, and A. Rajwade,A New Orthogonalization of Locality Preserving Projection and Applications, LNCS 8251, PReMI 2013.

Image denoising using New Orthogonal Locality Preserving Projection

- A new application of OLPP has been explored for Image Denoising, here the procedure of removing noise is carried out in the projection domain of New OLPP (NOLPP).
- Given the noisy image, it is divided into overlapping patches, and a set of global basis is learnt for the whole image/window.
- Each noisy patch is then projected on the corresponding NOLPP basis, and the coefficients are modified using Weiner filter update as follows:

$$\frac{\sigma_{I\eta w}^2 - \sigma^2}{\sigma_{I\eta w}^2} I_{\eta w}$$

σ : Standard deviation of noise, $\sigma_{I\eta w}$: Computed from the degraded image

$I_{\eta w}$: Coefficients of the noisy image in transformed domain

- Modified patches in the NOLPP domain are transformed back to the spatial domain.
- As opposed to the methods discussed before, the basis learnt here are global.

Original Image

Noisy Image (σ=30)

LPG-PCA
(PSNR: 24.70, SSIM: 0.68)

CSR
(PSNR: 26.15, SSIM: 0.75)

BM3D
(PSNR: 25.15, SSIM: 0.71)

NOLPP
(PSNR: 26.16, SSIM: 0.78)


Original Image


Noisy Image (σ=30)


LPG-PCA
(PSNR: 24.08, SSIM: 0.69)


CSR
(PSNR:24.23, SSIM: 0.72)


BM3D
(PSNR: 24.17, SSIM: 0.72)


NOLPP
(PSNR: 24.43 , SSIM: 0.71)

Special Mention to Other State-of-the-art Method

- BLS-GSM: Bayes Least Square Gaussian Scale Mixture
- Mean Shift Method
- K-SVD
- HOSVD: High Order Singular Value Decomposition

Beyond Gaussianity

Beyond Gaussianity

$$I(x)+\eta(x)=I_n(x)$$

Additive Noise

Gaussian PDF

What if these assumption does not hold???

- Multiplicative Noise
- Signal Dependent Noise
- Spatially Correlated Noise

- Poisson PDF
- Rice PDF
- Binomial and Other PDF

Beyond Gaussianity

- Multiplicative Noise $$I_n(x) = I(x) * \eta(x)$$
  - Denoising use Log function to make it Additive

$$\log I_n(x) = \log I(x) + \log \eta(x)$$

- Variance Stabilization Method (Anacombe's Transformation)
  - Transform data into approximately standard Gaussian distributed data.
  - For Poisson Noise

$$f : x \to 2\sqrt{x + \frac{3}{8}}$$

  - For Rician Noise (Foi 2011)

$$f : x \to \sqrt{\frac{x^2}{\sigma^2} + \frac{3}{8}} + c$$

- Signal Dependent Noise
  - Poisson PDF

- Anscombe, Francis J. "The transformation of Poisson, binomial and negative-binomial data." Biometrika 35.3/4 (1948): 246-254.
- Foi, Alessandro. "Noise estimation and removal in MR imaging: the variance-stabilization approach." *Biomedical Imaging: From Nano to Macro, 2011 IEEE International Symposium on*. IEEE, 2011.

Medical Image Denoising

Rice Distribution

If the real and imaginary data, with mean values A_R and A_I, respectively, are corrupted by Gaussian, zero mean, stationary noise with standard deviation σ, the magnitude data will be Rician distributed with PDF

$$p(M_i, A) = \frac{M_i}{\sigma^2} e^{-\left(\frac{M_i{}^2 + A^2}{2\sigma^2}\right)} I_0\left(\frac{AM_i}{\sigma^2}\right) u(M_i)$$

I_0 is the modified zeroth order Bessel function of the first kind, M_i denotes the ith data point of the magnitude image. The unit step function u is used to indicate that the expression for the PDF of M_i is valid for nonnegative values of M_i only.

Furthermore, A is given by $A = \sqrt{A_R{}^2 + A_I{}^2}$

An unbiased estimator of A^2 is given by $\hat{A}^2 = M^2 - 2\sigma^2$

Rician Noise Generation

The Rician noise was built from white Gaussian noise in the complex domain. First, two images are computed

$$I_{\mathrm{Real}}(x_i) = I_0(x_i) + \eta_1(x_i),\ \ \eta_1(x_i) \sim N(0,\sigma)$$
$$I_{\mathrm{Im}}(x_i) = \eta_2(x_i),\ \ \eta_2(x_i) \sim N(0,\sigma)$$

where I_0 is the "ground truth" and σ is the standard deviation of the added white Gaussian noise.

Then, the noisy image is computed as

$$I_{RN}(x_i) = \sqrt{I_{\mathrm{Real}}(x_i)^2 + I_{\mathrm{Im}}(x_i)^2}$$

The convention of 3% Rician noise defines the Gaussian noise used in complex domain is equivalent to $N(0,\upsilon(3/100))$, where υ is the value of the brightest tissue in the image.

For a same level of noise, the Rician noise is stronger than the Gaussian noise.

Rician Noise- Bias Correction

The bias correction is defined as

$$\hat{I}_0^{Unbiased} = \sqrt{\max(\hat{I}_0 - 2\sigma^2, 0)}$$

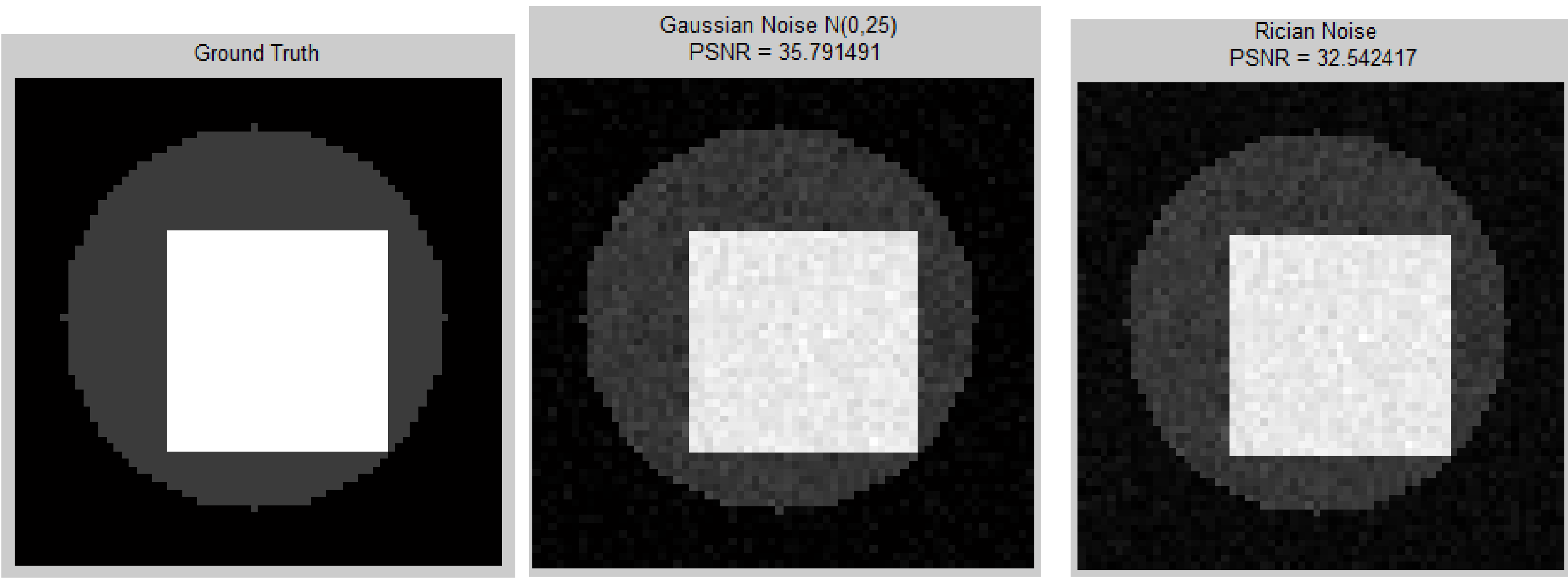
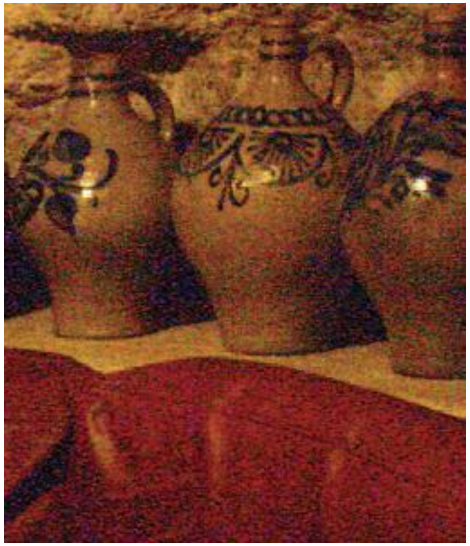


Figure: (a) Synthetic image of size 64x64 with three intensity level, 0, 35 and 150, (b) Corrupted image with Gaussian Noise, N(0, 5), (c) Corrupted with Rician with sd=5 of white Gaussian Noise

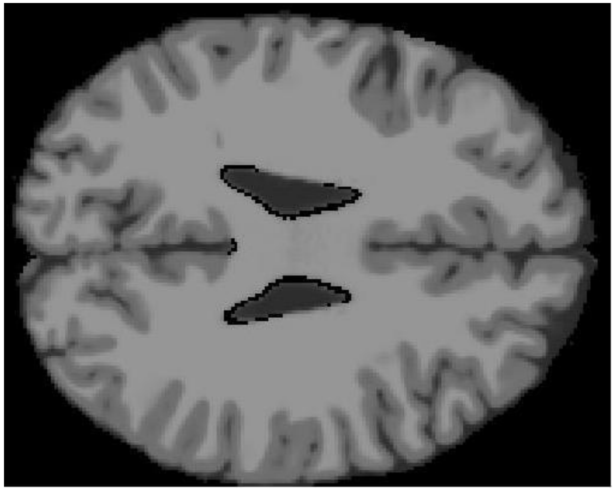
Multimodel Image Denoising

Multi Modal Imaging

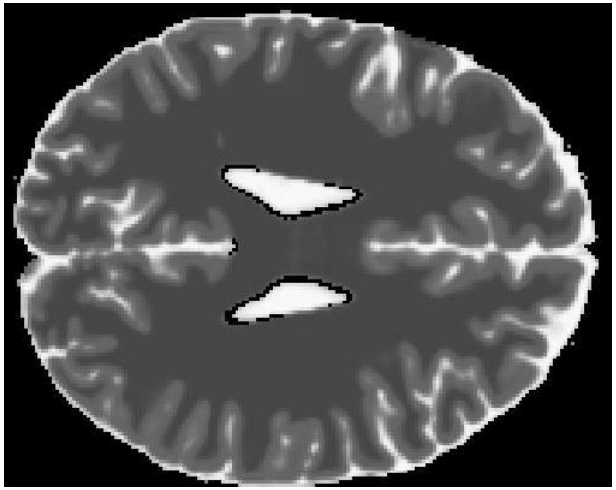
Flash Image



No Flash Image



T1 Image



T2 Image

Multi Modal Imaging

- Cross/ Joint Bilateral Filter
  - Decouple the notion of edges to preserve from the image to smooth.

$$CBF(I,E)_{\boldsymbol{p}} = \frac{1}{W_{\boldsymbol{p}}} \sum_{\boldsymbol{q} \in S} G_{\sigma_x}(\|\boldsymbol{p} - \boldsymbol{q}\|) G_{\sigma_y}(E_{\boldsymbol{p}} - E_{\boldsymbol{q}}) I_{\boldsymbol{q}}$$

- Dual Bilateral Filter
  - Consider any edge visible in either of the images.

$$DBF(I)_{\boldsymbol{p}} = \frac{1}{W_{\boldsymbol{p}}} \sum_{\boldsymbol{q} \in S} G_{\sigma_x}(\|\boldsymbol{p} - \boldsymbol{q}\|) G_{\sigma_I}(I_{\boldsymbol{p}} - I_{\boldsymbol{q}}) G_{\sigma_E}(\|E_{\boldsymbol{p}} - E_{\boldsymbol{q}}\|) I_{\boldsymbol{q}}$$

- E. Eisemann and F. Durand, "Flash photography enhancement via intrinsic relighting," *ACM Transactions on Graphics*, vol. 23, no. 3, pp. 673–678, Proceedings of the ACM SIGGRAPH conference, July, 2004.
- G. Petschnigg, M. Agrawala, H. Hoppe, R. Szeliski, M. Cohen, and K. Toyama, "Digital photography with flash and no-flash image pairs," *ACM Transactions on Graphics*, vol. 23, no. 3, pp. 664–672, Proceedings of the ACM SIGGRAPH Conference, 2004.
- E. P. Bennett, J. L. Mason, and L. McMillan, "Multispectral bilateral video fusion," IEEE Transactions on Image Processing, vol. 16, no. 5, pp. 1185–1194, May 2007.

Multi Modal Imaging with NLM

- Multicomponent NLM in Medical Images (Manjon et al. 2009)

$$w(N_p, N_q) = \frac{1}{W_p} e^{-\frac{1}{C}\left(\sum_{i=1}^{C} \frac{d(N^i{}_p, N^i{}_q)}{\sigma_i{}^2}\right)}$$

- Issues:
  - All images are registered???
  - All images are having same noise model???
  - All images are having same noise quantity???
  - How to transfer information from one component to another???

- Manjón, J. V., Thacker, N. A., Lull, J. J., Garcia-Martí, G., Martí-Bonmatí, L., & Robles, M. (2009). Multicomponent MR image denoising. Journal of Biomedical Imaging, 2009, 18.
- Buades, T., Lou, Y., Morel, J. M., & Tang, Z. (2009, August). A note on multi-image denoising. In Local and Non-Local Approximation in Image Processing, 2009. LNLA 2009. International Workshop on (pp. 1-15). IEEE.

Image Statistics & Understanding

Image Statistics & Understanding

Patch Comparison & Number of Patches Required

- Kumar, Neeraj, Li Zhang, and Shree Nayar. "What is a good nearest neighbors algorithm for finding similar patches in images?." Computer Vision–ECCV 2008. Springer Berlin Heidelberg, 2008. 364-378.
- Barnes, C., Shechtman, E., Goldman, D. B., & Finkelstein, A. (2010). The generalized patchmatch correspondence algorithm. In *Computer Vision–ECCV 2010* (pp. 29-43). Springer Berlin Heidelberg.
- Sureka, Harshit, and P. J. Narayanan. "Mixed-resolution patch-matching." Computer Vision–ECCV 2012. Springer Berlin Heidelberg, 2012. 187-198.
- Deledalle, Charles-Alban, Loïc Denis, and Florence Tupin. "How to compare noisy patches? Patch similarity beyond Gaussian noise." International journal of computer vision 99.1 (2012): 86-102.

Image Statistics & Understanding

Image Statistics

- Zoran, Daniel, and Yair Weiss. "Scale invariance and noise in natural images."*Computer Vision, 2009 IEEE 12th International Conference on*. IEEE, 2009.
- Zontak, Maria, and Michal Irani. "Internal statistics of a single natural image."*Computer Vision and Pattern Recognition (CVPR), 2011 IEEE Conference on*. IEEE, 2011.

Optimal Bound for Performance

- P. Chatterjee and P. Milanfar, "Is denoising dead?" IEEE Trans. on Image Proc., vol. 19, no. 4, pp. 895–911, April 2010
- P. Chatterjee and P. Milanfar, "Practical bounds on image denoising: From estimation to information," IEEE Trans. On Image Proc., vol. 20, no. 5, pp. 1221–1233, May 2011.
- P. Chatterjee and P. Milanfar, "Patch-based near-optimal denoising," IEEE Trans. on Image Proc., vol. 21, no. 4, pp. 1635–1649, April 2012
- A. Levin, B. Nadler, F. Durand, and W. T. Freeman, "Patch complexity, finite pixel correlations and optimal denoising," ECCV, October 2012
- H. Talebi, X. Xhu P. Milanfar, "How to SAIF-ly Boost Denoising Performance", IEEE Transactions on Image Processing, vol 22, No. 4, pp. 1470-1485, April 2013.

Image Statistics & Understanding

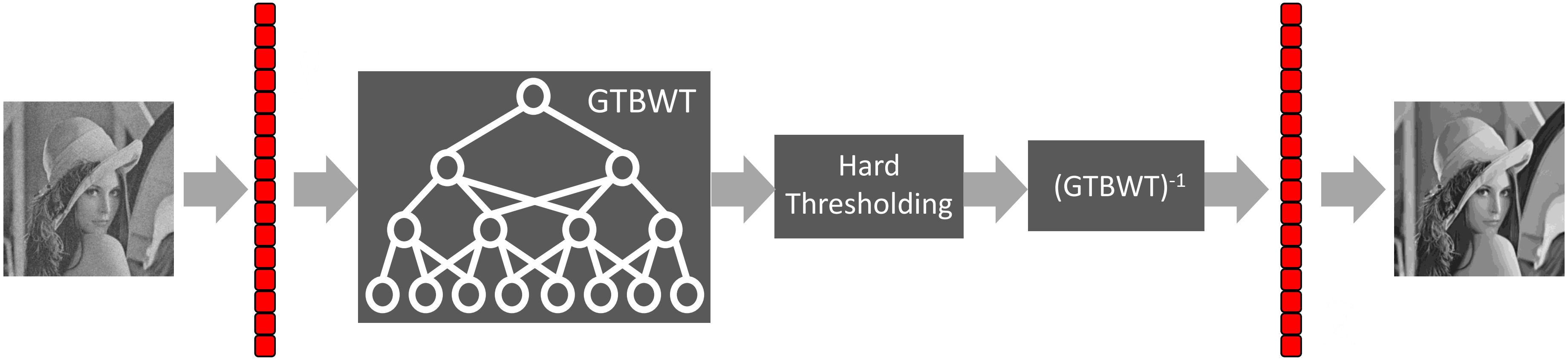


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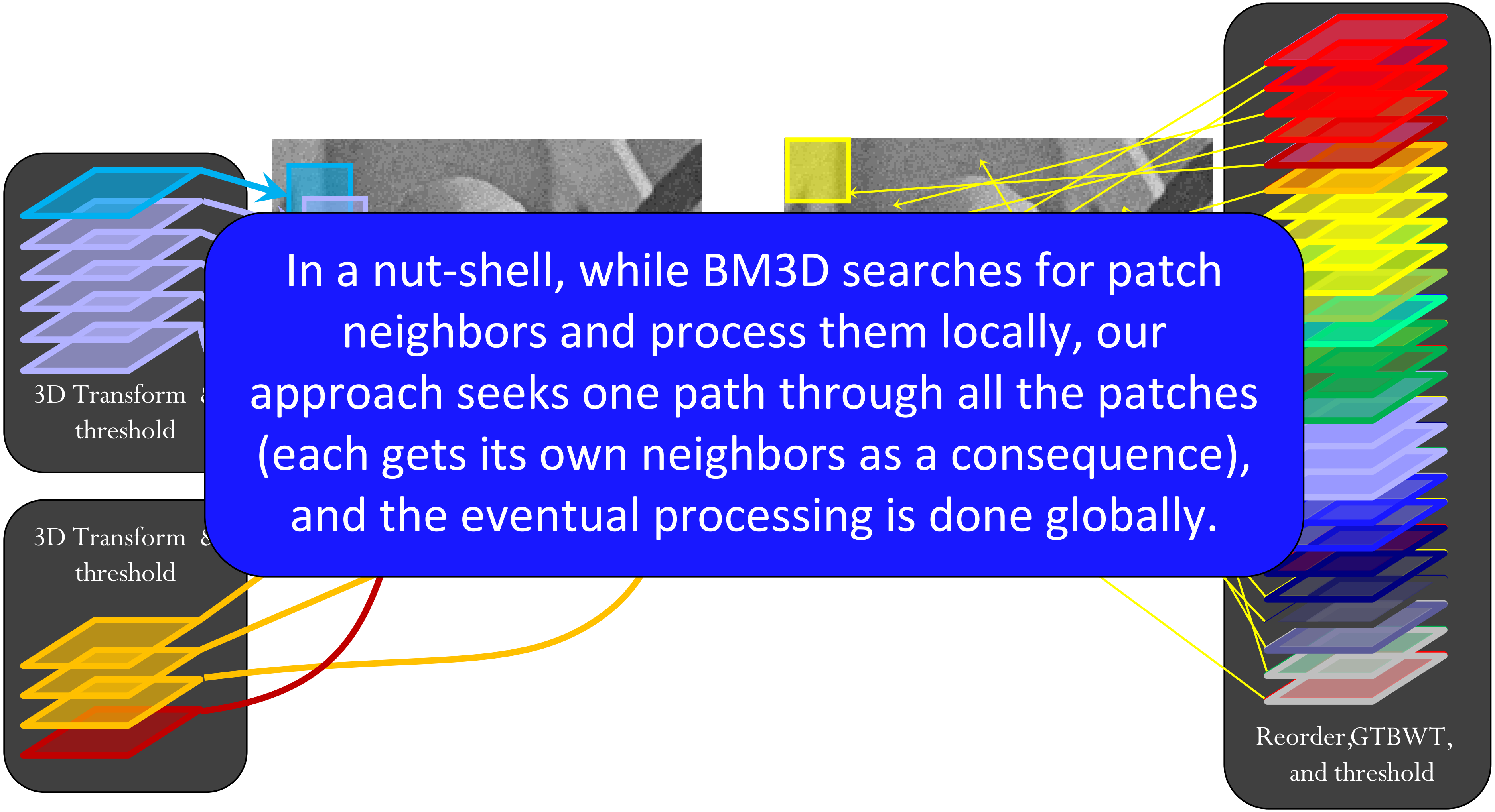
Image Statistics & Understanding



In a nut-shell, while BM3D searches for patch neighbors and process them locally, our approach seeks one path through all the patches (each gets its own neighbors as a consequence), and the eventual processing is done globally.

- Note: Slide is taken from Michael Elad presented in Workshop SIGMA 2012.

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AND MANY MORE ARE THERE…

Resources

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- And may be many more…

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Thank You