CS 747, Autumn 2022: Lecture 7

Shivaram Kalyanakrishnan

Department of Computer Science and Engineering
Indian Institute of Technology Bombay

Autumn 2022
Markov Decision Problems

1. Alternative formulations of MDPs

2. Some applications of MDPs
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2. Some applications of MDPs
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It is relatively straightforward to handle all these variations.
Episodic Tasks

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- **Episodic tasks** have a special **sink/terminal state** $s_T$ from which there are no outgoing transitions on rewards.

Additionally, from every non-terminal state and for every policy, there is a non-zero probability of reaching the terminal state in a finite number of steps. Hence, trajectories or episodes almost surely terminate after a finite number of steps.
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Definition of Values

- We defined $V^\pi(s)$ as **Infinite discounted reward**:
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  V^\pi(s) \overset{\text{def}}{=} \mathbb{E}_\pi [r^0 + \gamma r^1 + \gamma^2 r^2 + \ldots | s^0 = s].
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  Horizon $H \geq 1$ specified, rather than $\gamma$.
  Optimal policies for this setting need not be stationary.
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- **Average reward** (withholding some technical details):
  $$V^\pi(s) \overset{\text{def}}{=} \mathbb{E}_\pi[\lim_{m \to \infty} \frac{r^0 + r^1 + \ldots + r^{m-1}}{m} | s^0 = s].$$
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Controlling a Helicopter (Ng et al., 2003)

- Episodic or continuing task? What are S, A, T, R, γ?

[1]

Winning at Chess

Episodic or continuing task? What are $S, A, T, R, \gamma$?

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Preventing Forest Fires (Lauer et al., 2017)

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A Familiar MDP?

- Single state. $k$ actions.
- For $a \in A$, treat reward of $(s, a, s')$ as a random variable.

Annotation: "probability, reward distribution".
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\[ \gamma = 0.5 \]

Such an MDP is called a multi-armed bandit! 

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Shivaram Kalyanakrishnan (2022)
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Markov Decision Problems

- MDP, policy, value function
- MDP planning problem
- Policy evaluation

- Alternative formulations of MDPs
- Some applications of MDPs

- Banach’s fixed point theorem
- Bellman optimality operator
- Value iteration
- Linear Programming
- Policy iteration