CS 747, Autumn 2020: Week 12, Lecture 1

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Autumn 2020

Reinforcement Learning

1. Policy gradient methods
2. Variance reduction in policy gradient methods
3. Batch reinforcement learning

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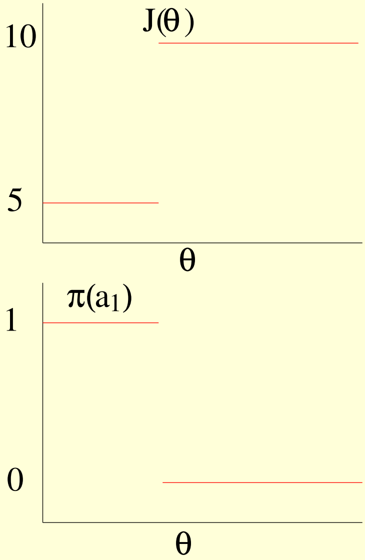
Stochastic Policies

- Single state; actions a_1, a_2.
- $R(a_1) = 5$; $R(a_2) = 10$.
- Policy π; parameter θ.

$$\pi(a_1) = \begin{cases} 1 & \text{if } \theta < 0.6, \\ 0 & \text{otherwise.} \end{cases}$$

$$J(\theta) = \pi(a_1) \cdot 5 + \pi(a_2) \cdot 10.$$



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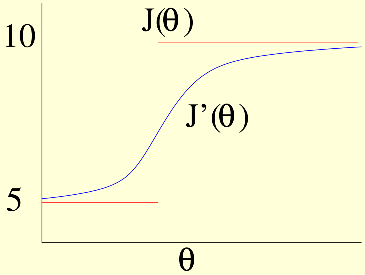

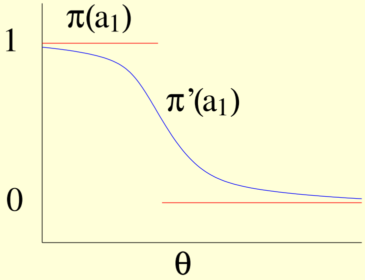

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- **Example.** If we have features $x(s, a) \in \mathbb{R}^d$ for $s \in S, a \in A$, a common template for π is:

$$\pi(s, a) = \frac{e^{\theta \cdot x(s,a)}}{\sum_{b \in A} e^{\theta \cdot x(s,b)}},$$

where $\theta \in \mathbb{R}^d$ is the vector of policy parameters.

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In this case, work out that

$$\nabla_\theta \pi(s, a) = \left(x(s, a) - \sum_{b \in B} \pi(s, b) x(s, b) \right) \pi(s, a).$$

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- But what's the connection between $\nabla_\theta J$ and $\nabla_\theta \pi(\cdot, \cdot)$?

Policy Gradient Theorem

- For simplicity assume episodic task with $\gamma = 1$.
- Assume there is a fixed start state s^0.
- We leave it implicit that π is fixed by parameter vector θ.
- $J(\theta) = V^\pi(s^0)$.
- We shall derive the connection between $\nabla_\theta J$ and $\nabla_\theta \pi(\cdot, \cdot)$.

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where $\mathbb{P}\{s \to x, k, \pi\}$ is the probability of reaching x from s in k steps if following π.

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- Recall that $J(\theta) = V^{\pi}(s^0)$.

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- But how to do gradient ascent? We do not know $\mathbb{P}\{s^0 \to s, k, \pi\}$ and $Q^{\pi}(s, a)$!

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- But how to do gradient ascent? We do not know $\mathbb{P}\{s^0 \to s, k, \pi\}$ and $Q^{\pi}(s, a)$!
- We perform stochastic gradient ascent.
- We use the following fact. For any discrete, real-valued random variable X with pmf $p : X \to [0, 1]$,

$$\sum_{x \in X} p(x) x = \mathbb{E}[X].$$

Towards Gradient Ascent

- Generate episode $s^0, a^0, r^0, s^1, a^1, r^1, s^2, \dots, s^T = s^\top$ by acting according to π, parameterised by θ.

Towards Gradient Ascent

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$$
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&= \mathbb{E}_\pi \left[\sum_{t=0}^{T-1} \left(\nabla_\theta \ln \pi(s^t, a^t) \right) G_{t:T} \right].
\end{aligned}
$$

REINFORCE Algorithm

- Reference: Williams (1992).
- For clarity we show explicit dependence of π on parameter vector $\theta \in \mathbb{R}^d$.
- Assume θ is initialised arbitrarily.

Repeat for ever:

$\quad \theta_{\text{new}} \leftarrow \theta$.

$\quad$ Generate episode $s^0, a^0, r^0, s^1, \ldots, s^T = s^\top$, following π_θ.

$\quad$ **For** $t = 0, 1, \ldots, T-1$:

$\quad\quad G \leftarrow \sum_{k=t}^{T-1} r^k$. //This is $G_{t:T}$.

$\quad\quad \theta_{\text{new}} \leftarrow \theta_{\text{new}} + \alpha G \nabla_\theta \ln \pi_\theta(s^t, a^t)$.

$\quad \theta \leftarrow \theta_{\text{new}}$.

Reinforcement Learning

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Baseline Subtraction

- Policy Gradient Theorem

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$$\nabla_\theta J(\theta) = \sum_{s \in S} \sum_{k=0}^{\infty} \mathbb{P}\{s^0 \to s, k, \pi\} \sum_{a \in A} \nabla_\theta \pi(s, a) (Q^\pi(s, a) - B(s)).$$

- How come?

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- How come? Observe that

$$\sum_{s \in S} \sum_{k=0}^{\infty} \mathbb{P}\{s^0 \to s, k, \pi\} \sum_{a \in A} \nabla_\theta \pi(s, a) B(s)$$

$$= \sum_{s \in S} \sum_{k=0}^{\infty} \mathbb{P}\{s^0 \to s, k, \pi\} B(s) \nabla_\theta \sum_{a \in A} \pi(s, a) = 0.$$

Baseline Subtraction

- The policy gradient estimate can have high variance.

s	$Q^\pi(s, a_1)$	$Q^\pi(s, a_2)$	$Q^\pi(s, a_3)$	$V^\pi(s)$
s_1	105	79	100	90
s_2	10	6	13	12
s_3	-50	-60	-50	-55

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- Common practice to subtract out $V^{\pi}(s)$—approximated independently as $\hat{V}(s)$.

- REINFORCE with baseline:

$$\theta_{\text{new}} \leftarrow \theta_{\text{new}} + \alpha \sum_{t=0}^{T-1} (G_{t:T} - \hat{V}(s^t)) \nabla_{\theta} \ln \pi_{\theta}(s^t, a^t).$$

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- Called the Actor-Critic architecture.

- Actor updates θ and hence π_θ.
- Critic evaluates π_θ (say using TD(0)) and provides input for the gradient ascent update.

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- Not always provably convergent, but widely used in practice.

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Batch Updates to $\hat{Q}$

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- On-line methods such as TD(0) “extract” very little information from each transition; are computationally lightweight.
- In many applications, samples are more expensive than computation; need to get more out of samples.
- Batch RL keeps transitions in memory, performs computationally heavier updates.

Batch RL outer loop

$\hat{Q} \leftarrow 0$, $D \rightarrow \emptyset$.

Repeat for ever: //Each iteration is a batch.

$\pi \leftarrow \epsilon$-greedy($\hat{Q}$).

Follow π for N episodes; gather data $D' = (s_i, a_i, r_i, s_{i+1})_{i=1}^{L}$.

$D \leftarrow D \cup D'$.

$\hat{Q} \leftarrow$ BatchUpdate(D, $\hat{Q}$).//$\hat{Q}$ optional in RHS.

Experience Replay

- Reference: Lin (1992).

BatchUpdateExperienceReplay$(D, \hat{Q})$

Repeat M times:
  Pick (s, a, r, s') uniformly at random from D.
  $\hat{Q}(s, a) \leftarrow \hat{Q}(s, a) + \alpha\{r + \gamma \max_{a' \in A} \hat{Q}(s', a') - \hat{Q}(s, a)\}$.
Return $\hat{Q}$.

- Sometimes $\hat{Q}$ reset/forgotten before the batch update.
- M usually large; hence multiple updates using each sample.

Fitted Q Iteration

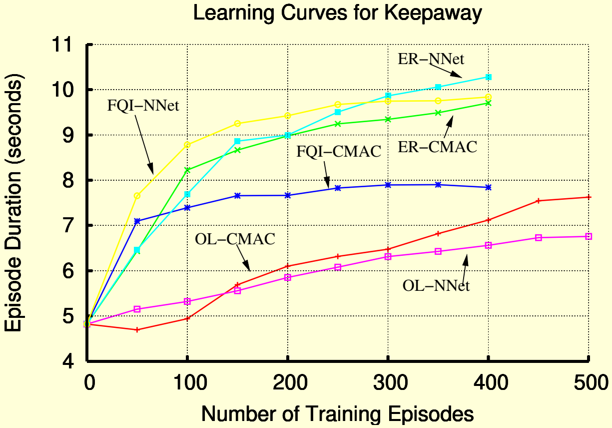
- Reference: Ernst, Geurts, Wehenkel (2005).
- Idea: obtain $\hat{Q}$ using supervised learning. Wait—labels?

BatchUpdateFittedQIteration(D)

$\hat{Q}_0 \leftarrow \mathbf{0}$.
For $i = 0, 1, \ldots, H - 1$:
  For $j \in \{1, 2, \ldots, L\}$: //Create a labeled data set.
    $x_j \leftarrow \text{FeatureVector}(s_j, a_j)$.
    $y_j \leftarrow r_j + \gamma \max_{a \in A} \hat{Q}_i(s_{j+1}, a)$.
  $\hat{Q}_{i+1} \leftarrow \text{SupervisedLearning}((x_j, y_j)_{j=1}^{L})$.
Return $\hat{Q}_H$.

- Will not diverge if the supervised learning model is an averager (nearest neighbour methods, decision trees, etc.).

Illustrative Graph



Batch Reinforcement Learning in a Complex Domain. Shivaram Kalyanakrishnan and Peter Stone, In Proceedings of the Sixth International Joint Conference on Autonomous Agents and Multiagent Systems (AAMAS 2007), pp.650–657, IFAAMAS, 2007.

Batch RL: Summary

- High computational complexity, low sample complexity.
- Can also be interpreted as a model-based approach (the data set D implicitly represents the model).
- Forms the basis for many modern neural network-based algorithms, such as DQN.
- Many variations possible
  - Gathering multiple batches of data in parallel.
  - Picking experience replay samples more intelligently.

Why So Many RL Methods?

- We have seen
  - Model-based RL,
  - On-line TD methods (Q-learning, Sarsa, Expected Sarsa),
  - Policy search (black box optimisation),
  - Policy gradient and actor-critic methods,
  - Batch RL methods.

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- Other topics we will cover:
  - Monte Carlo Tree search,
  - Multiagent RL,
  - Case studies: Atari games, AlphaGo.