An Introduction to Reinforcement Learning

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What is Reinforcement Learning?

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[Video[1] of toddler learning to walk]

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Learning by trial and error to perform *sequential* decision making.

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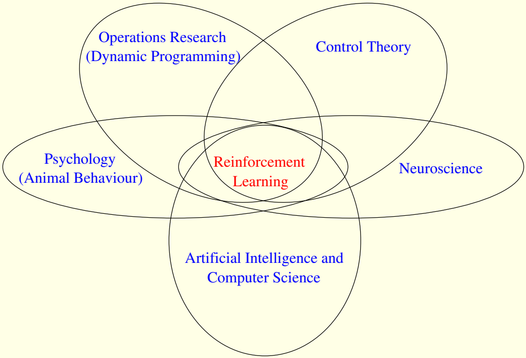
Our View of Reinforcement Learning



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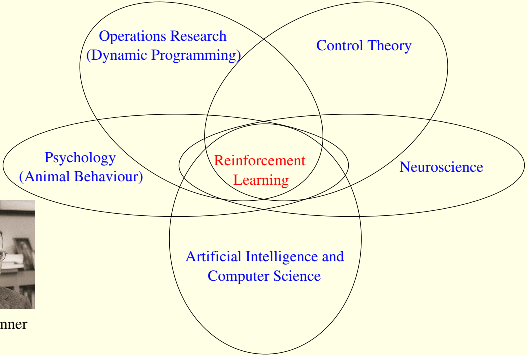

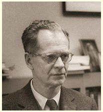
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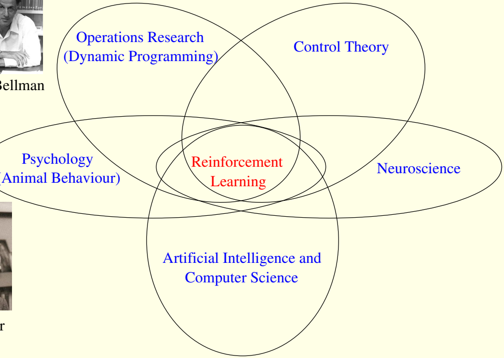

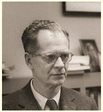

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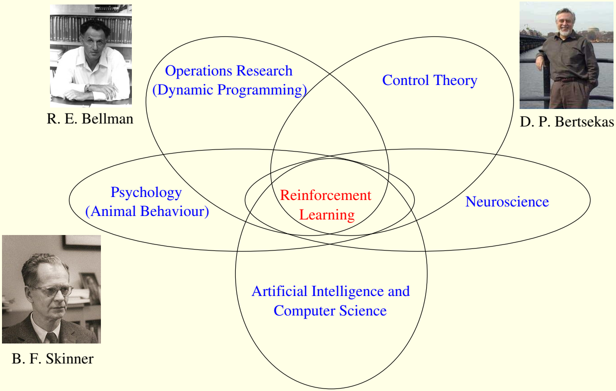

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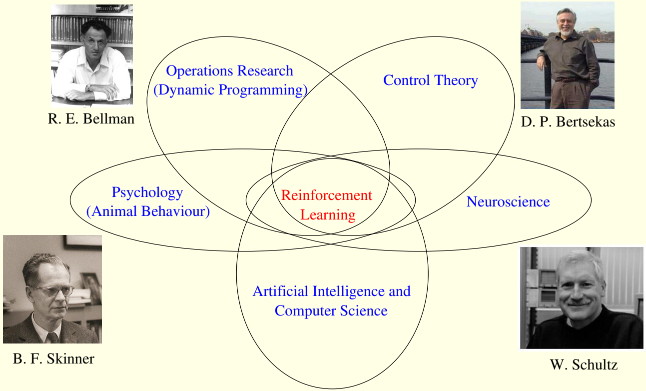

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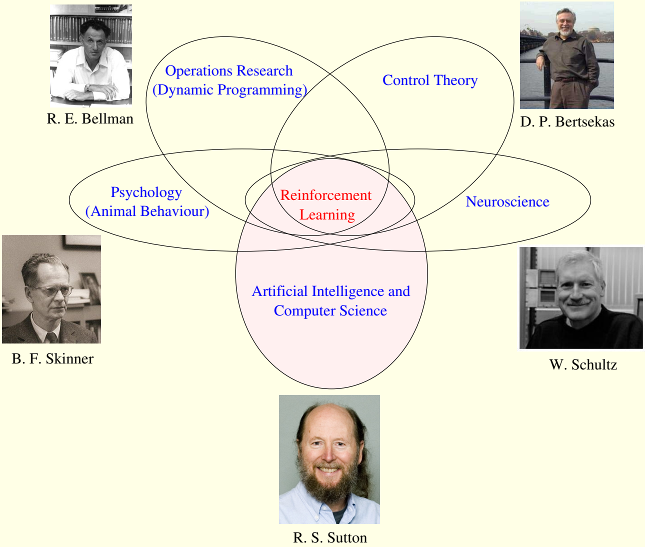

Resources

Reinforcement Learning: A Survey.
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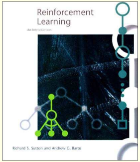

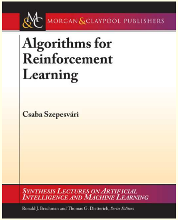

Reinforcement Learning: An Introduction
Richard S. Sutton and Andrew G. Barto. MIT Press, 1998. (2017-8 draft also now on-line).

Algorithms for Reinforcement Learning
Csaba Szepesvári. Morgan & Claypool, 2010.

Today's Class

1. Markov Decision Problems
2. Planning and learning
3. Deep Reinforcement Learning
4. Summary

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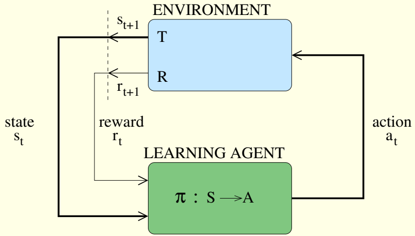

S: set of states.
A: set of actions.
T: transition function. $\forall s \in S, \forall a \in A$, $T(s, a)$ is a distribution over S.
R: reward function. $\forall s, s' \in S, \forall a \in A$, $R(s, a, s')$ is a finite real number.
γ: discount factor. $0 \leq \gamma < 1$.

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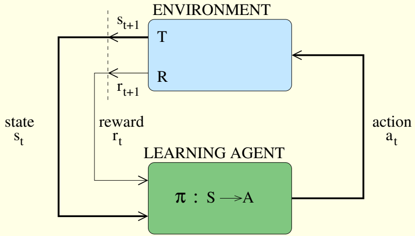

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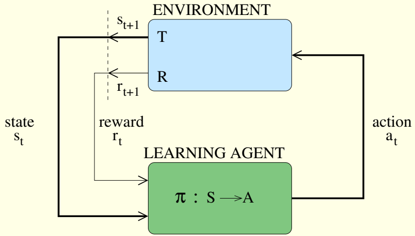

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Value, or expected long-term reward, of state s under policy π:

$$V^\pi(s) = \mathbb{E}[r^0 + \gamma r^1 + \gamma^2 r^2 + \ldots \text{ to } \infty | s^0 = s, a^i = \pi(s^i)].$$

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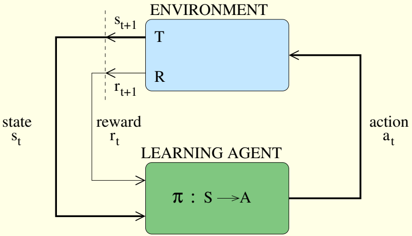

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Objective: "Find π such that $V^\pi(s)$ is maximal $\forall s \in S$."

Examples

What are the agent and environment? What are S, A, T, and R?

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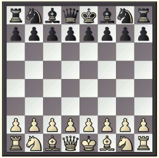

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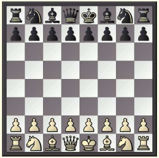

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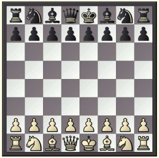

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Bellman's Equations

Recall that

$$V^{\pi}(s) = \mathbb{E}[r^0 + \gamma r^1 + \gamma^2 r^2 + \ldots | s^0 = s, a^i = \pi(s^i)].$$

Bellman's Equations ($\forall s \in S$):

$$\boxed{V^{\pi}(s) = \sum_{s' \in S} T(s, \pi(s), s') \left[R(s, \pi(s), s') + \gamma V^{\pi}(s')\right].}$$

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Thus, given S, A, T, R, γ, and a fixed policy π, we can solve Bellman's equations efficiently to obtain, $\forall s \in S, \forall a \in A$, $V^{\pi}(s)$ and $Q^{\pi}(s, a)$.

Bellman's Optimality Equations

Let Π be the set of all policies. What is its cardinality?

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It can be shown that there exists a policy $\pi^* \in \Pi$ such that

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Bellman's Optimality Equations ($\forall s \in S$):

$$\boxed{V^*(s) = \max_{a \in A} \sum_{s' \in S} T(s, a, s') \left[R(s, a, s') + \gamma V^*(s')\right].}$$

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V^* can be computed up to arbitrary precision using **Value Iteration**.

Planning and Learning

Planning problem:

Given S, A, T, R, γ, how can we find an optimal policy π^*? We need to be computationally efficient.

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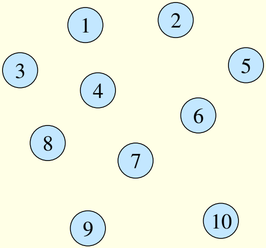
Given S, A, T, R, γ, how can we find an optimal policy π^*? We need to be computationally efficient.

Learning problem:

Given S, A, γ, and the facility to follow a trajectory by sampling from T and R, how can we find an optimal policy π^*? We need to be sample-efficient.

The Learning Problem



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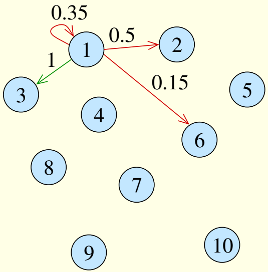

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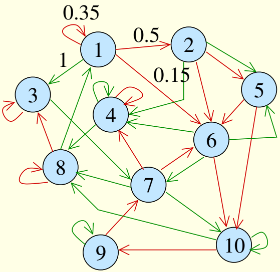

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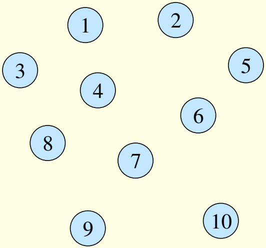

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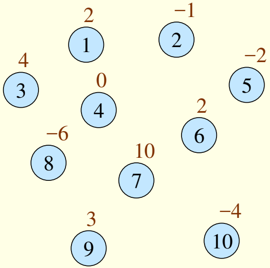

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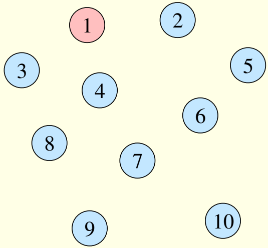

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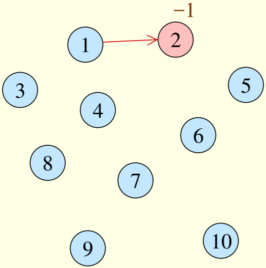

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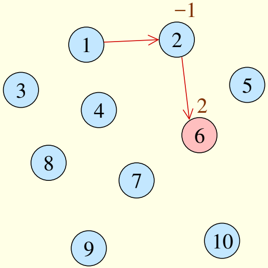

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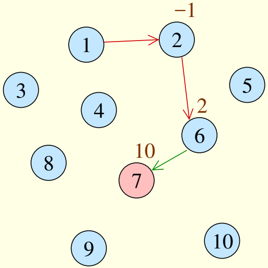

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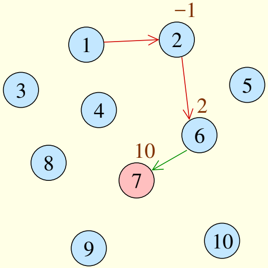

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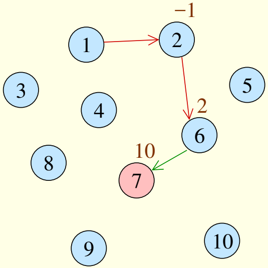

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[Note that there exists an (unknown) *optimal policy*.]

Q-Learning

- Keep a running estimate of the expected long-term reward obtained by taking each action from each state s, and acting *optimally* thereafter.

Q	red	green
1	-0.2	10
2	4.5	13
3	6	-8
4	0	0.2
5	-4.2	-4.2
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- Make sure to explore each action enough times.

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- Make sure to explore each action enough times.

Q-learning will converge and induce an optimal policy!

Q-Learning Algorithm

■ Let Q be our "guess" of Q^*: for every state s and action a, initialise $Q(s, a)$ arbitrarily. We will start in some state s^0.

■For $t = 0, 1, 2, \ldots$

- ■Take an action a^t, chosen uniformly at random with probability ϵ, and to be $\text{argmax}_a Q(s^t, a)$ with probability $1 - \epsilon$.
- ■The environment will generate next state s^{t+1} and reward r^{t+1}.
- ■Update: $Q(s^t, a^t) \leftarrow Q(s^t, a^t) + \alpha_t(r^{t+1} + \gamma \max_{a \in A} Q(s^{t+1}, a) - Q(s^t, a^t))$.

  [ϵ: parameter for "ϵ-greedy" exploration] [α_t: learning rate]

  [$r^{t+1} + \gamma \max_{a \in A} Q(s^{t+1}, a) - Q(s^t, a^t)$: temporal difference prediction error]

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For $\epsilon \in (0, 1]$ and $\alpha_t = \frac{1}{t}$, it can be proven that as $t \to \infty$, $Q \to Q^*$.

Q-Learning

Christopher J. C. H. Watkins and Peter Dayan. Machine Learning, 1992.

Practice In Spite of the Theory!

Task	State Aliasing	State Space	Policy Representation (Number of features)
Backgammon (T1992)	Absent	Discrete	Neural network (198)
Job-shop scheduling (ZD1995)	Absent	Discrete	Neural network (20)
Tetris (BT1906)	Absent	Discrete	Linear (22)
Elevator dispatching (CB1996)	Present	Continuous	Neural network (46)
Acrobot control (S1996)	Absent	Continuous	Tile coding (4)
Dynamic channel allocation (SB1997)	Absent	Discrete	Linear (100's)
Active guidance of finless rocket (GM2003)	Present	Continuous	Neural network (14)
Fast quadrupedal locomotion (KS2004)	Present	Continuous	Parameterized policy (12)
Robot sensing strategy (KF2004)	Present	Continuous	Linear (36)
Helicopter control (NKJS2004)	Present	Continuous	Neural network (10)
Dynamic bipedal locomotion (TZS2004)	Present	Continuous	Feedback control policy (2)
Adaptive job routing/scheduling (WS2004)	Present	Discrete	Tabular (4)
Robot soccer keepaway (SSK2005)	Present	Continuous	Tile coding (13)
Robot obstacle negotiation (LSYSN2006)	Present	Continuous	Linear (10)
Optimized trade execution (NFK2007)	Present	Discrete	Tabular (2-5)
Blimp control (RPHB2007)	Present	Continuous	Gaussian Process (2)
9 $\times$ 9 Go (SSM2007)	Absent	Discrete	Linear ($\approx$1.5 million)
Ms. Pac-Man (SL2007)	Absent	Discrete	Rule list (10)
Autonomic resource allocation (TJDB2007)	Present	Continuous	Neural network (2)
General game playing (FB2008)	Absent	Discrete	Tabular (part of state space)
Soccer opponent "hassling" (GRT2009)	Present	Continuous	Neural network (9)
Adaptive epilepsy treatment (GVAP2008)	Present	Continuous	Extremely rand. trees (114)
Computer memory scheduling (IMMC2008)	Absent	Discrete	Tile coding (6)
Motor skills (PS2008)	Present	Continuous	Motor primitive coeff. (100's)
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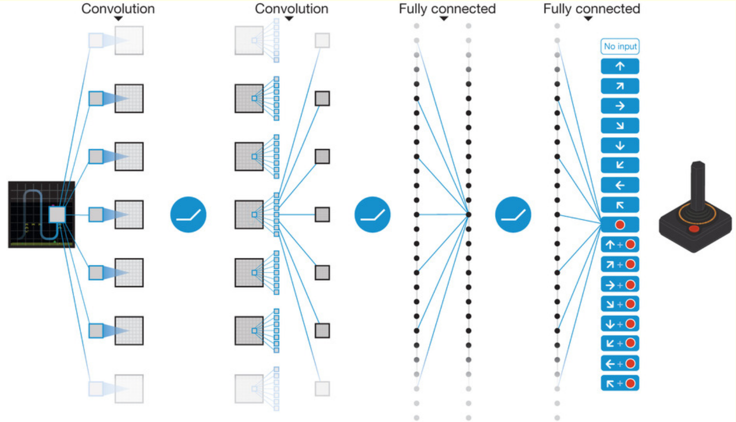
Perfect representations (fully observable, enumerable states) are impractical.

Today's Class

1. Markov Decision Problems
2. Planning and learning
3. Deep Reinforcement Learning
4. Summary

Typical Neural Network-based Representation of Q



1. `http://www.nature.com/nature/journal/v518/n7540/carousel/nature14236-f1.jpg`

ATARI 2600 Games (MKSRVBGRFOPBSAKKWLH2015)

[Breakout video[1]]

1. http://www.nature.com/nature/journal/v518/n7540/extref/nature14236-sv2.mov

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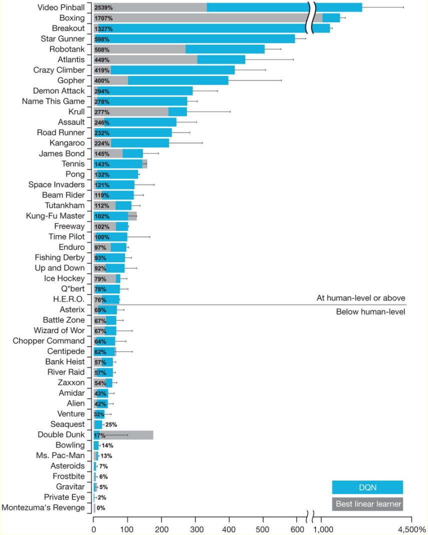

1. http://www.nature.com/nature/journal/v518/n7540/extref/nature14236-sv2.mov

AlphaGo (SHMGSDSAPLDGNKSLLKGH2016)

March 2016: DeepMind's program beats Go champion Lee Sedol 4-1.

1. http://www.kurzweilai.net/images/AlphaGo-vs.-Sedol.jpg

AlphaGo (SHMGSDSAPLDGNKSLLKGH2016)

1. http://static1.uk.businessinsider.com/image/56e0373052bcd05b008b5217-810-602/screen%20shot%202016-03-09%20at%2014.png

Learning Algorithm: Batch Q-learning

1. Represent action value function Q as a neural network.

2. Gather data (on the simulator) by taking ϵ-greedy actions w.r.t. Q:
   $(s_1, a_1, r_1, s_2, a_2, r_2, s_3, a_3, r_3, \ldots s_D, a_D, r_D, s_{D+1})$.

3. Train the network such that $Q(s_t, a_t) \approx r_t + \max_a Q(s_{t+1}, a)$.
   Go to 2.

Learning Algorithm: Batch Q-learning

1. Represent action value function Q as a neural network.
   AlphaGo: Use both a policy network and an action value network.

2. Gather data (on the simulator) by taking ϵ-greedy actions w.r.t. Q:
   $(s_1, a_1, r_1, s_2, a_2, r_2, s_3, a_3, r_3, \ldots s_D, a_D, r_D, s_{D+1})$.
   AlphaGo: Use Monte Carlo Tree Search for action selection

3. Train the network such that $Q(s_t, a_t) \approx r_t + \max_a Q(s_{t+1}, a)$.
   Go to 2.

AlphaGo: Trained using self-play.

Today's Class

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Summary

- Learning by trial and error to perform sequential decision making.
- Do not program behaviour! Rather, specify goals.
- Rich history, at confluence of several fields of study, firm foundation.

- Given an MDP (S, A, T, R, γ), we have to find a policy $\pi : S \rightarrow A$ that yields high expected long-term reward from states.
- An optimal value function V^* exists, and it induces an optimal policy π^* (several optimal policies might exist).
- Under planning, we are given S, A, T, R, and γ. We may compute V^* and π^* using a dynamic programming algorithm such as policy iteration.

- In the learning context, we are given S, A, and γ: we may sample T and R in a sequential manner. We can still converge to V^* and π^* by applying a temporal difference learning method such as Q-learning.
- Limited in practice by quality of the representation used.
- Deep neural networks address the representation problem in some domains, and have yielded impressive results.