CS 748, Spring 2021: Week 2, Lecture 1

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Department of Computer Science and Engineering
Indian Institute of Technology Bombay

Spring 2021

Navigation System

How to go from IIT Bombay to Marine Drive?

[1]

[1] https://www.flickr.com/photos/nat507/16088993607. CC image courtesy of Nathan Hughes Hamilton on Flickr licensed under CC BY 2.0.

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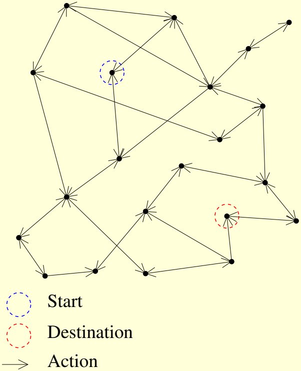

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Some Popular Puzzles

How to solve?

								1
							2	3
		4			5			
			1					
				3		6		
		7				5	8	
				6	7			
	1				4			
5	2							

Sudoku [1]

[1] https://upload.wikimedia.org/wikipedia/commons/e/eb/Sudoku_Puzzle_%28a_symmetrical_puzzle_with_17_clues%29.png. CC image courtesy of LithiumFlash on WikiCommons licensed under CC-BY-SA-4.0.

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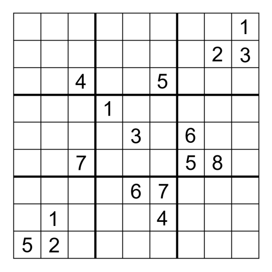

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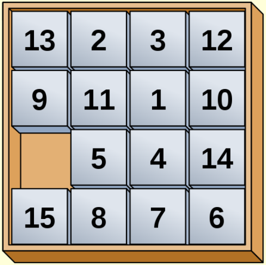

15-puzzle [2]

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[2] https://commons.wikimedia.org/wiki/File:15-puzzle-solvable.svg. CC image courtesy of Stannic on WikiMedia Commons licensed under CC-BY-SA-3.0

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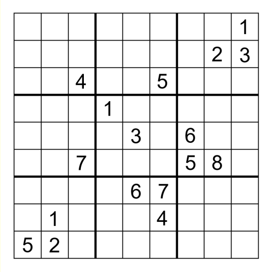

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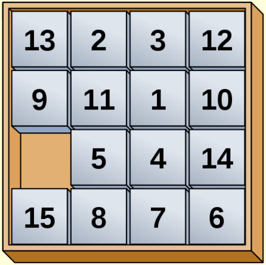

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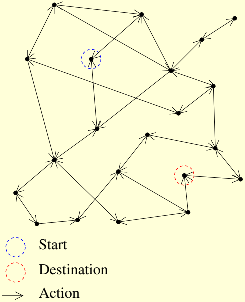

Same abstraction?

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Search

- Classical search
  - Problem instances
  - Generic search template
  - Uninformed search
  - Informed search (a.k.a. heuristic search)
  - Minimax search
- Decision-time planning in MDPs
  - Problem
  - Rollout policies
  - Monte Carlo tree search
- Discussion

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- Set of states, including designated start state.
- Set of actions available from each state.
- NextState(s, a) for each state s and action a.
- Cost(s, a) for each state s and action a (assumed ≥ 0).
- IsGoal(s) for each state s.

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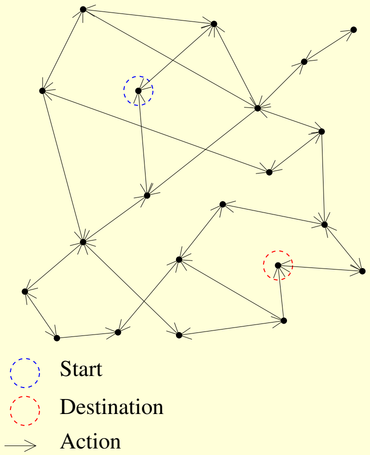
- Number of available actions in each state is called the branching factor b.
- Length of optimal path to reach goal state is called the depth d of the search instance.

Problem Formulation: Navigation System



States?

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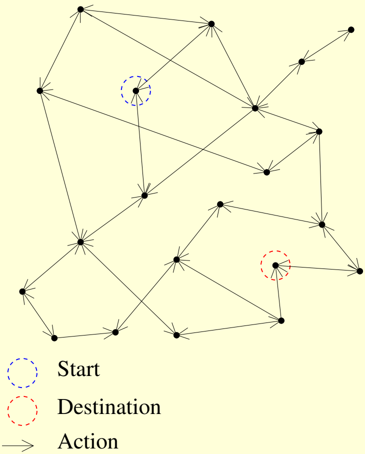

States?

Start state?

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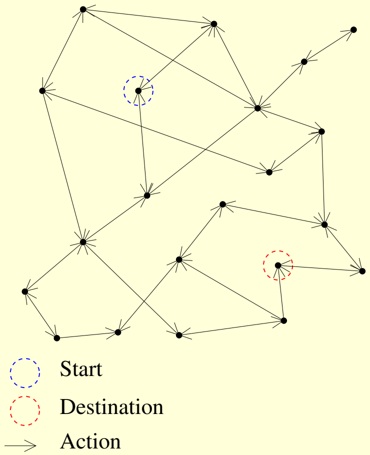

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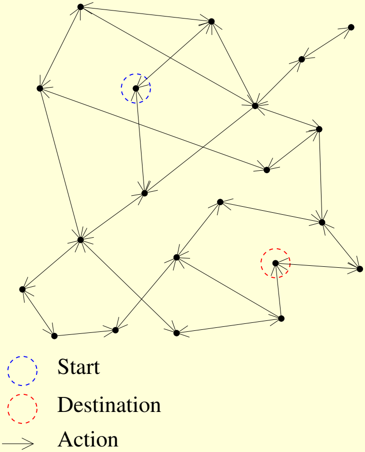


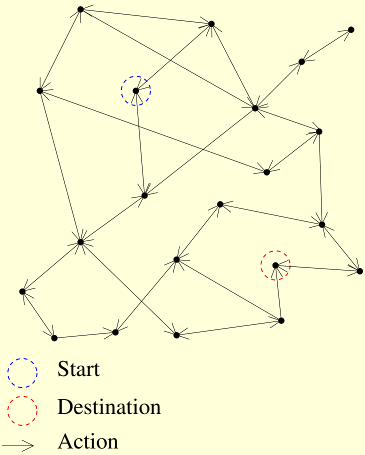
States?

Start state?

Actions?

NextState()?

Problem Formulation: Navigation System



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NextState()?

Cost()?

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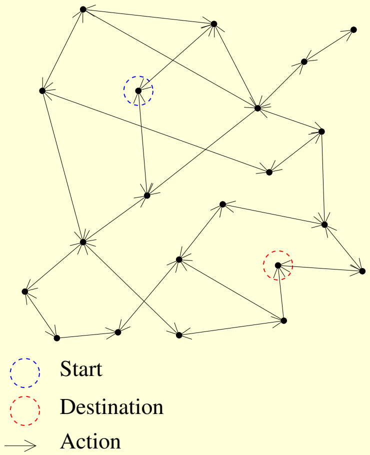

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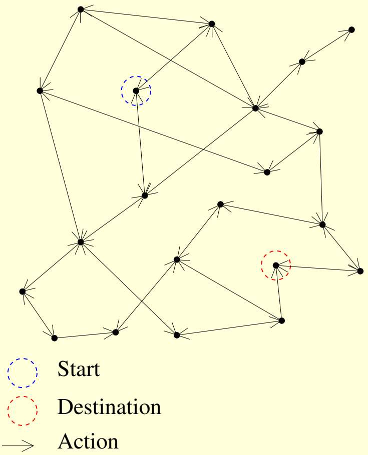

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A solver needs to find the least-cost path.

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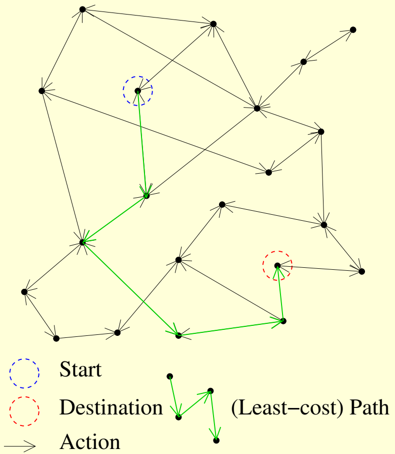

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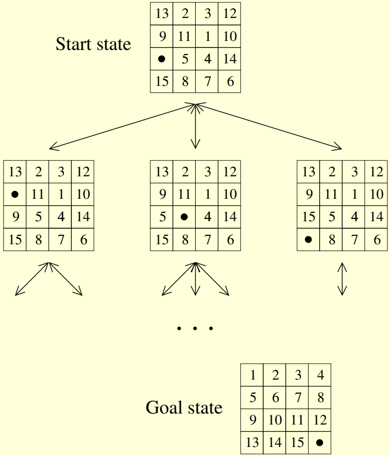
Problem Formulation: 15 Puzzle



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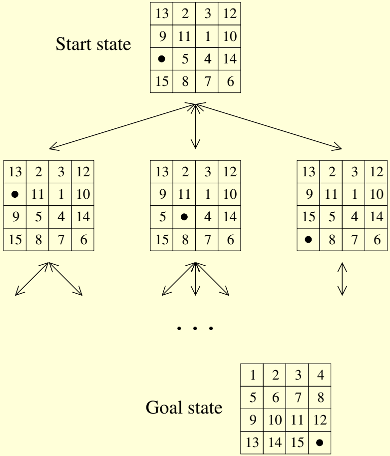
Problem Formulation: 15 Puzzle



States?

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A solver needs to find the shortest path to goal state.

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Generic Search Template: Pseudocode

- Primary data element is a Node, which a tuple of the form

$$(state, pathFromStartState, pathCost).$$

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$$(state, pathFromStartState, pathCost).$$

- At every stage of the search,
- some states have been explored
- some states remain unexplored, and
- The *Frontier* is a set of nodes due for imminent expansion.

Generic Search Template: Pseudocode

$Frontier \leftarrow \{Node(startState, (startState), 0)\}$.
Repeat for ever:
    Select a node n from $Frontier$.
    //Expand n.
    If $isGoal(n.state)$:
        Return n.
    For each action a available from $n.state$:
        $s \leftarrow NextState(n.state, a)$.
        $c \leftarrow Cost(n.state, a)$.
        $n' \leftarrow Node(s, n.path + (a, s), n.pathCost + c)$.
        Merge n' with $Frontier$.//Typically insertion; might also allow deletions.

Generic Search Template: Pseudocode

$Frontier \leftarrow \{Node(startState, (startState), 0)\}$.
Repeat for ever:
    Select a node n from $Frontier$.//Which one?
    //Expand n.
    If $isGoal(n.state)$:
        Return n.
    For each action a available from $n.state$:
        $s \leftarrow NextState(n.state, a)$.
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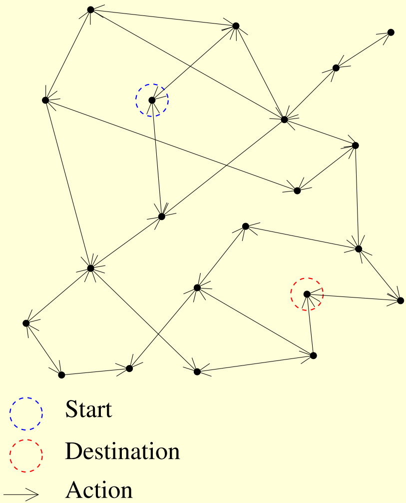
Generic Search Template: Illustration



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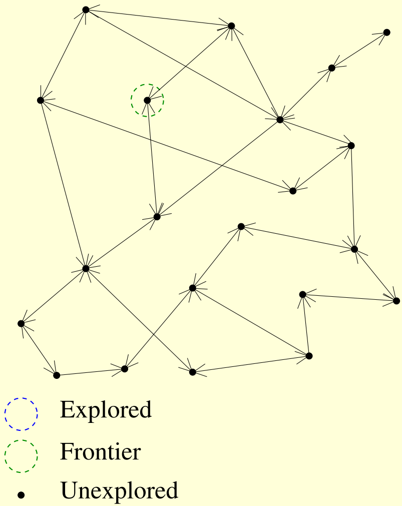

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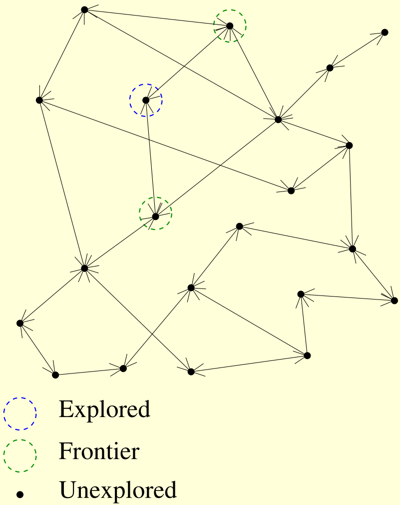

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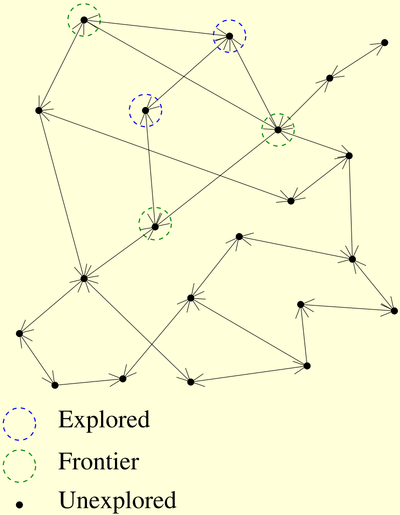

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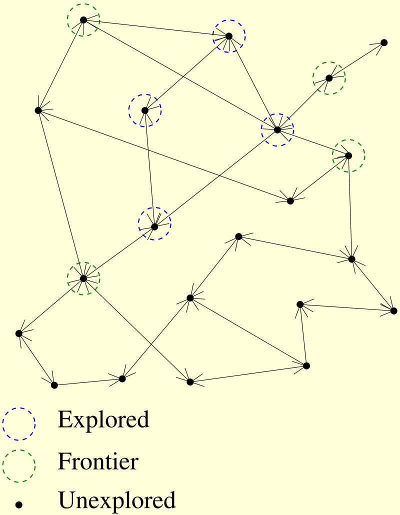

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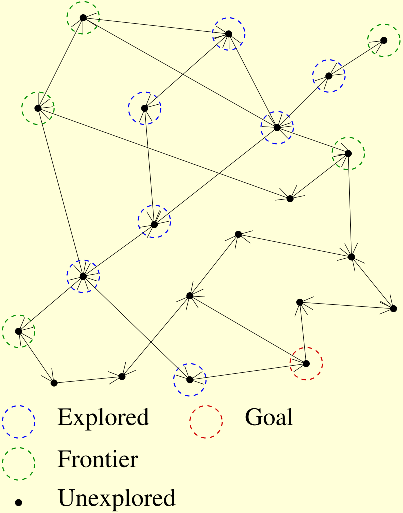

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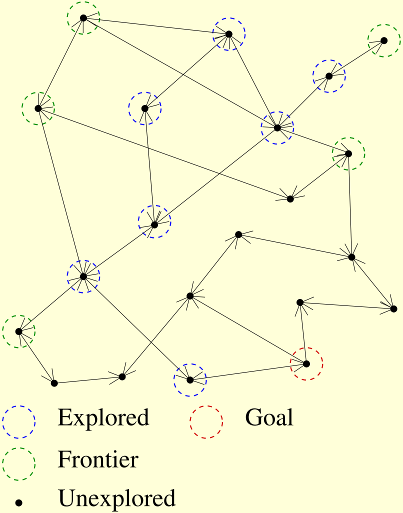

How did we decide which frontier nodes to expand?

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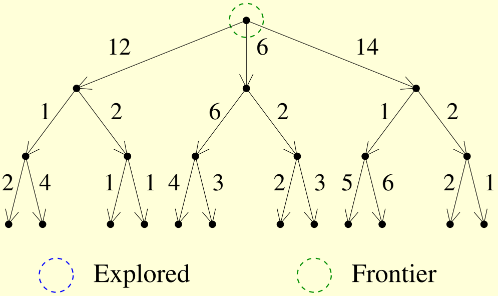
Depth-first Search (DFS)

Expand frontier node with longest path from start state.



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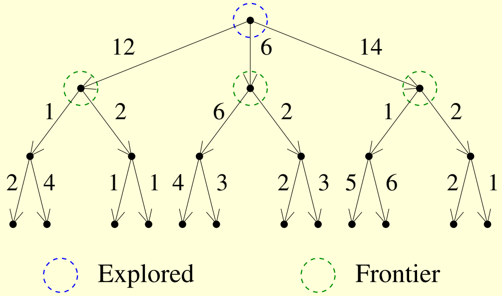

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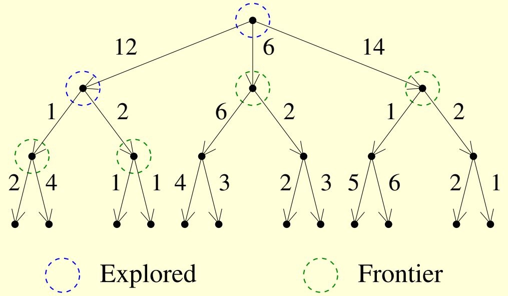

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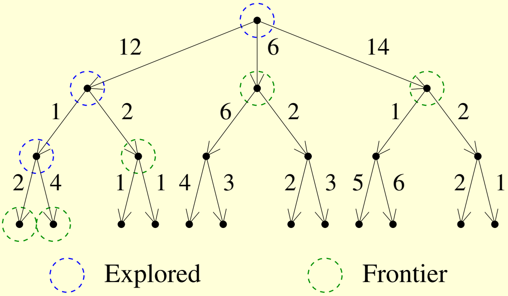

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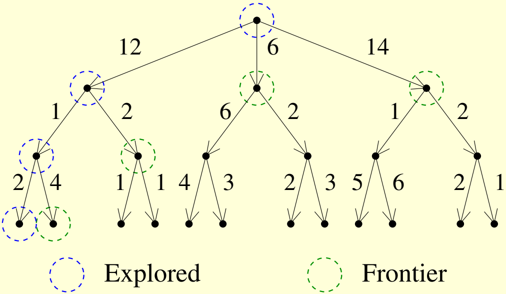

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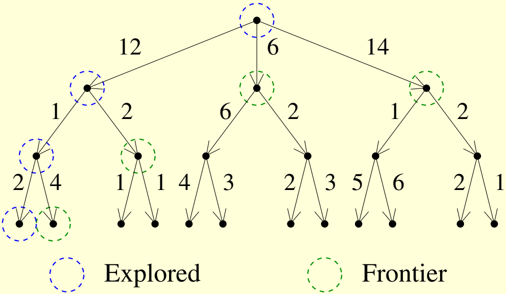

- Frontier treated like a stack (LIFO).

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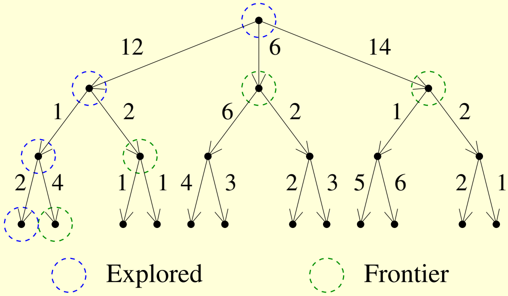

- Frontier treated like a stack (LIFO).
- No need to explicitly maintain frontier (construct on-line).

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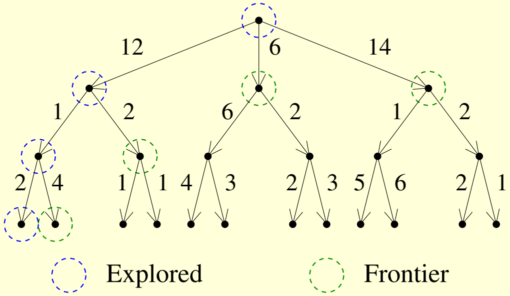

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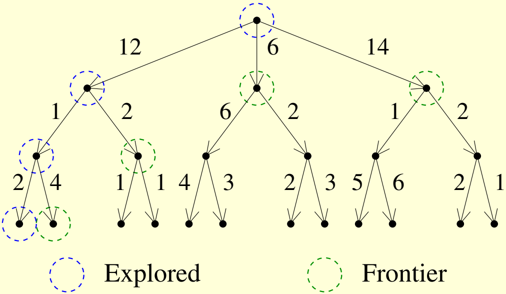

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- No need to explicitly maintain frontier (construct on-line).
- Guaranteed to terminate on finite search instances.
- Memory requirement linear in depth d.

Breadth-first Search (BFS)

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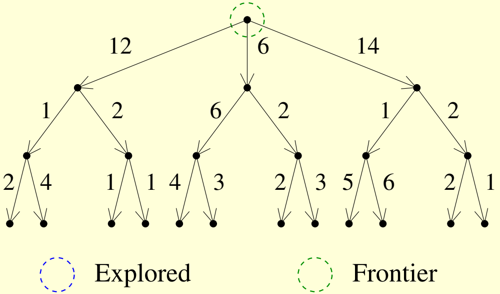

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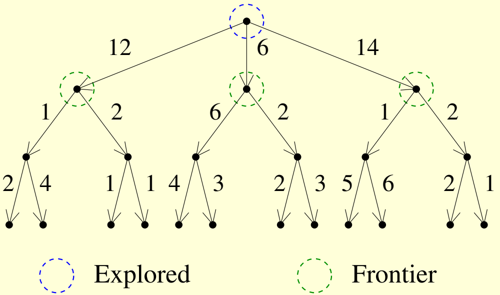

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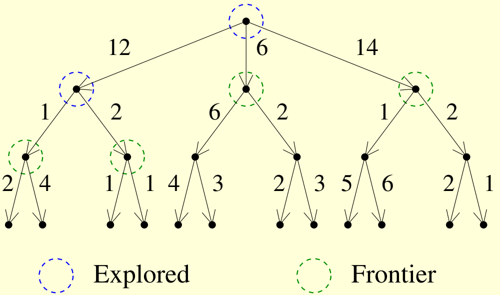

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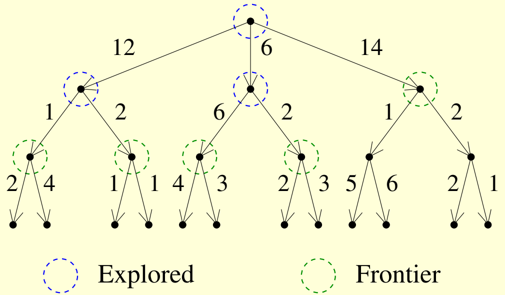

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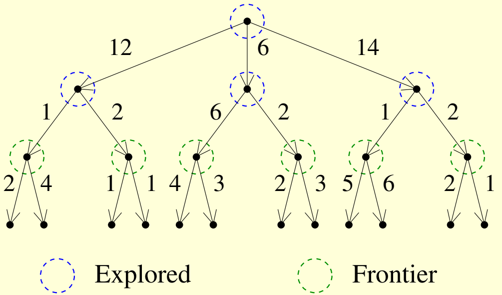

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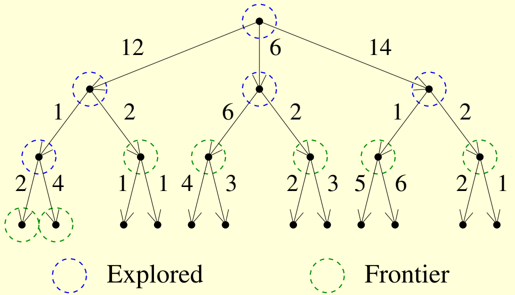

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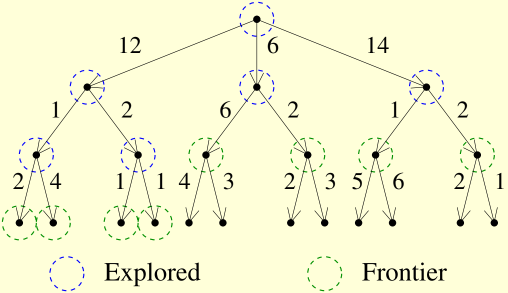

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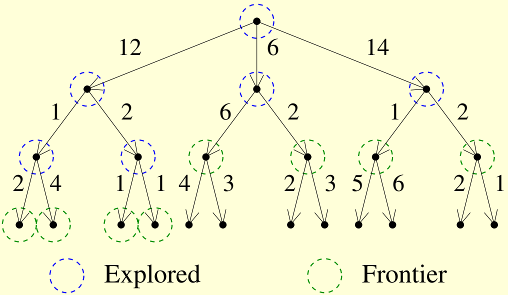

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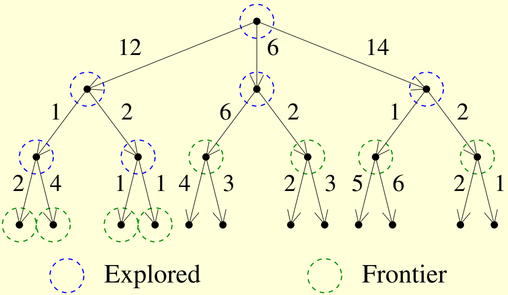

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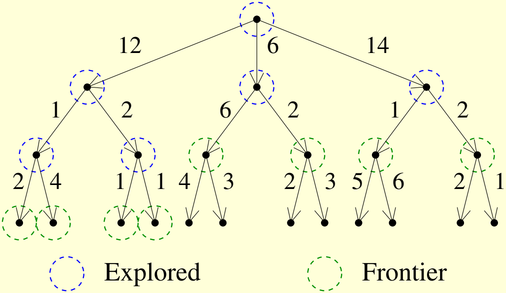

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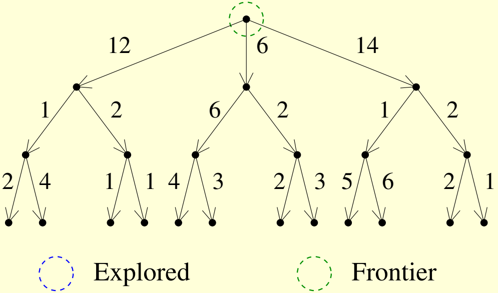
- Memory requirement $O(b^d)$.

Lowest-cost-first Search (LCFS)

Expand frontier node with lowest path-cost from start state.



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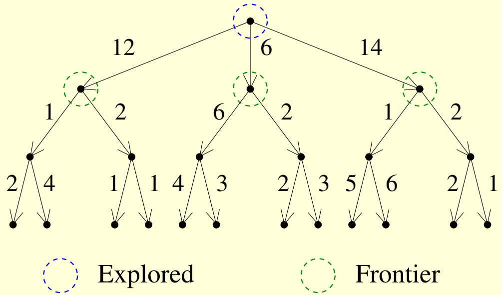

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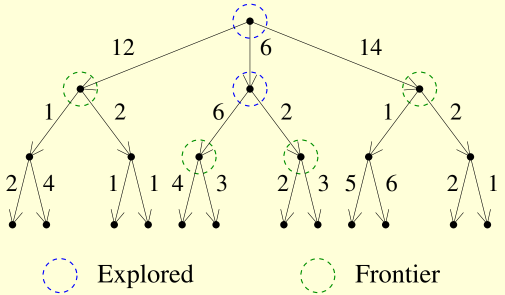

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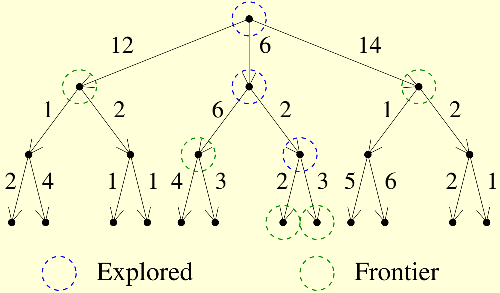

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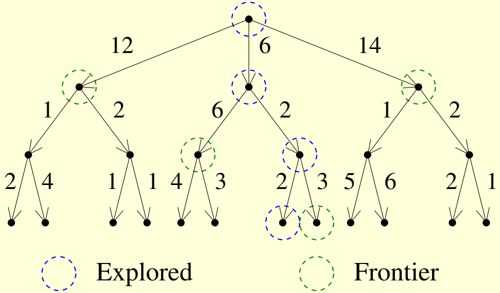

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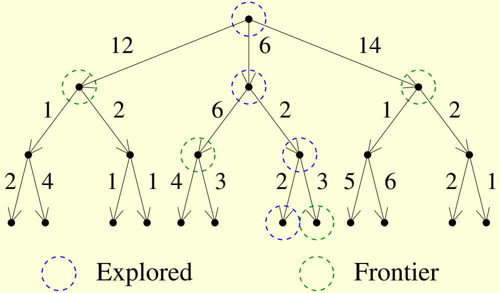

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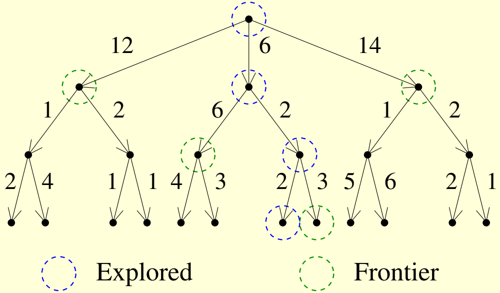

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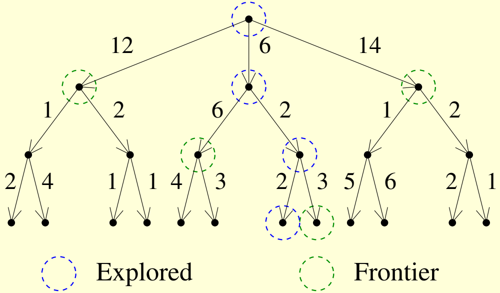

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- Memory requirement depends heavily on instance.

Search

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  - Generic search template
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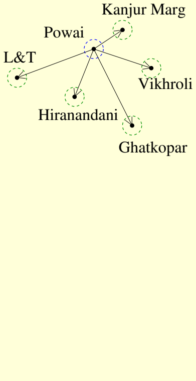

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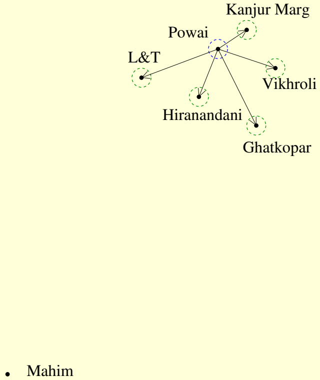

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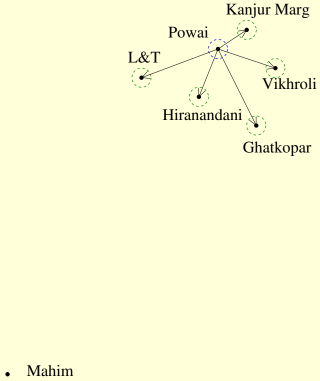

- First you expand the Powai node.
- Which node will you expand next?
- L&T and Hiranandani are geographically closer to Mahim: should that count?

Heuristic Functions and A* Search Algorithm

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- A* search originally conceived for robotic path planning.

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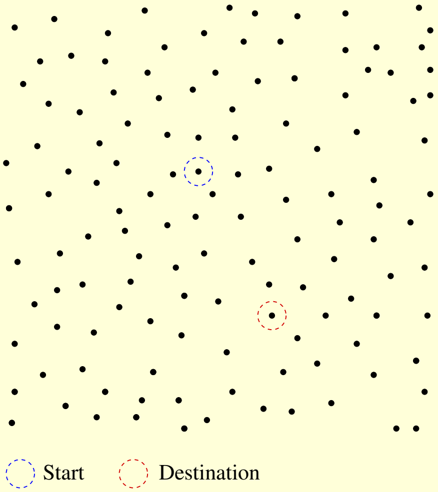
- For a given task, which is the best heuristic function to use?

Effect of Heuristic



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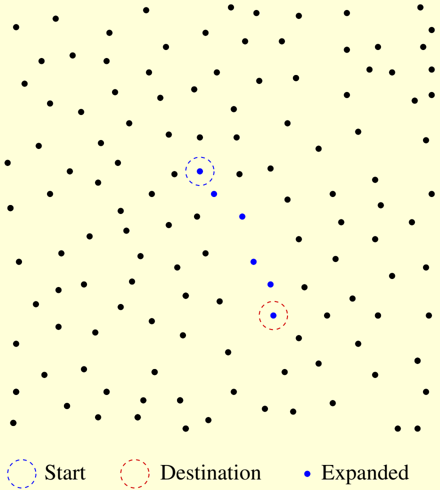

$h(n) = c^{\star}(n)$. Will only expand nodes along optimal path!
Unfortunately $c^{\star}(n)$ is not known!

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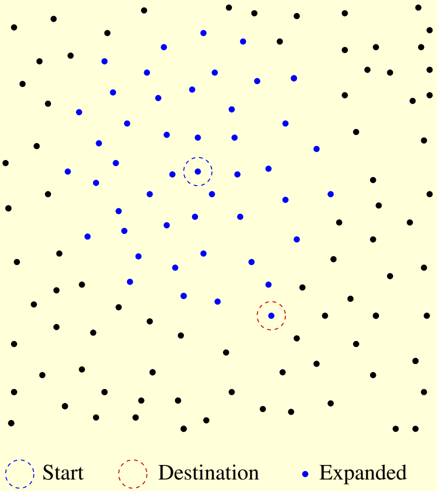
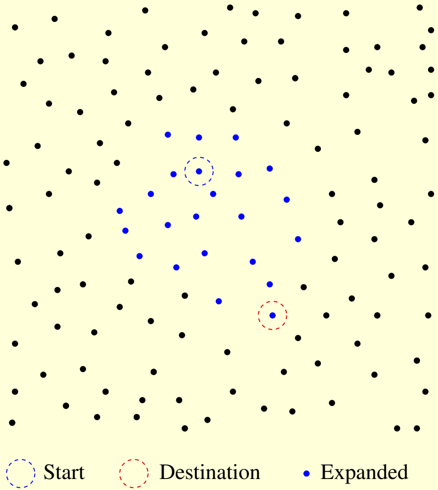


$h(n) = 0$. Identical to LCFS.

Effect of Heuristic



Intermediate/typical $h(n)$ expands fewer nodes than LCFS.

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Start state

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9	11	1	10
•	5	4	14
15	8	7	6

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1	2	3	4
5	6	7	8
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- Can we make do with inadmissible heuristics? Yes—example coming up in next section. But try to avoid.

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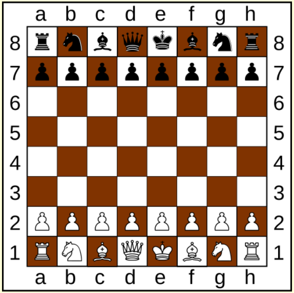
Search and Games



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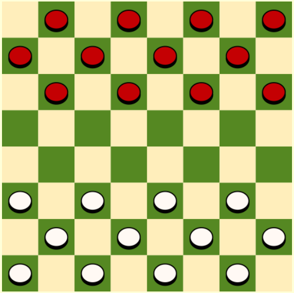

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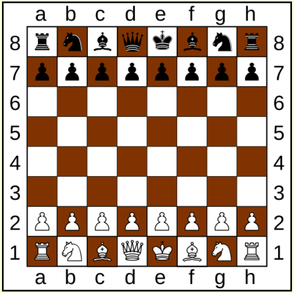

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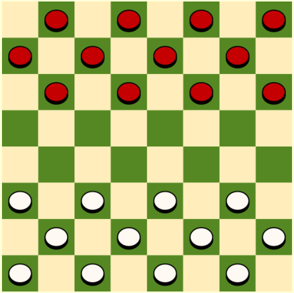

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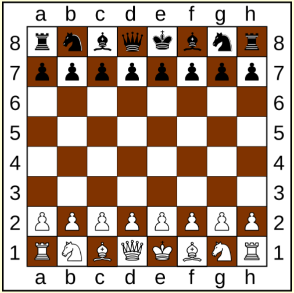

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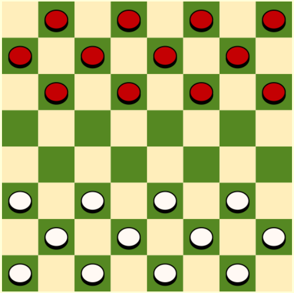

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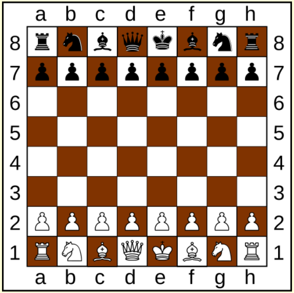

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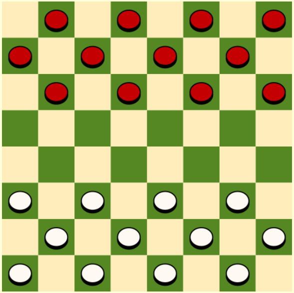

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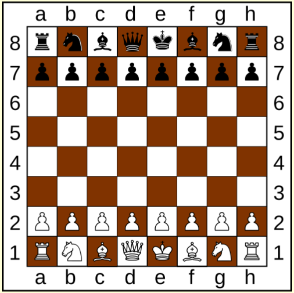

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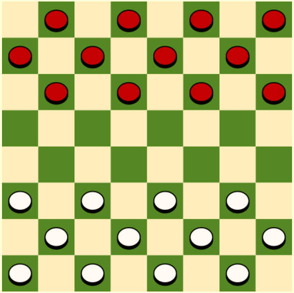

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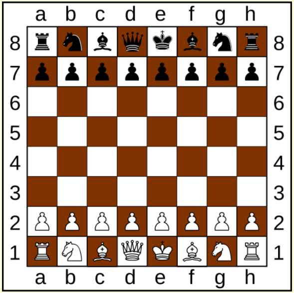

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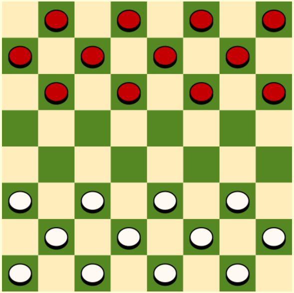

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- Instances of turn-based two player zero-sum games.

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Game Tree

- Assume turn-taking zero sum game played by Max and Min.
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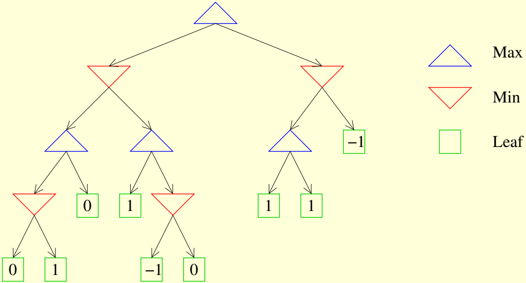

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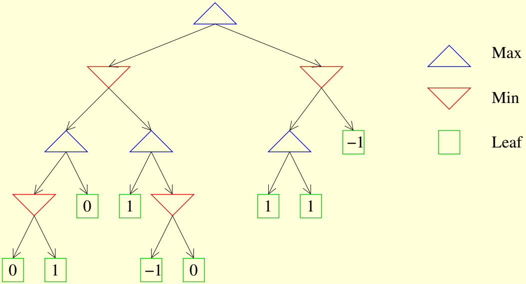

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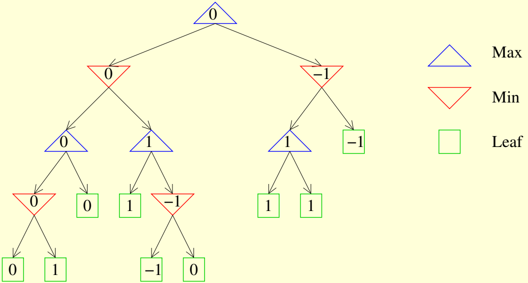

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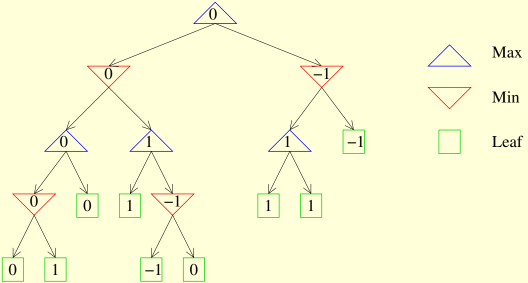
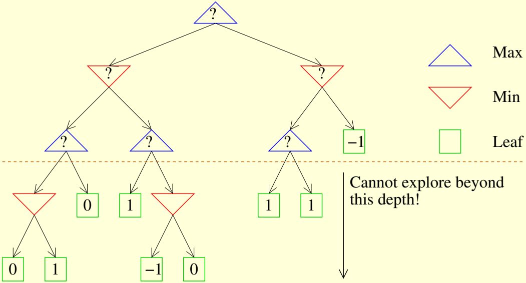


- In 2007, a massive, long-running computation concluded that the value of the root node for Checkers is 0 (draw).

Evaluation Function

- The Checkers tree has $\approx 10^{40}$ nodes; Chess has $\approx 10^{120}$.
  Infeasible to solve!



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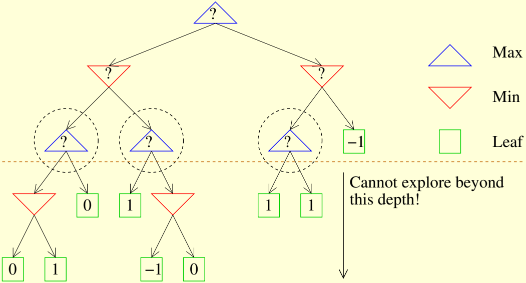

- At some depth d from current node, estimate node value using features.
- For example, in Chess, set evaluation as

$$w_1 \times \text{Material diff.} + w_2 \times \text{King safety} + w_3 \times \text{pawn strength} + \ldots.$$

- Weights $w_1, w_2, w_3, \ldots$ are tuned or learned.

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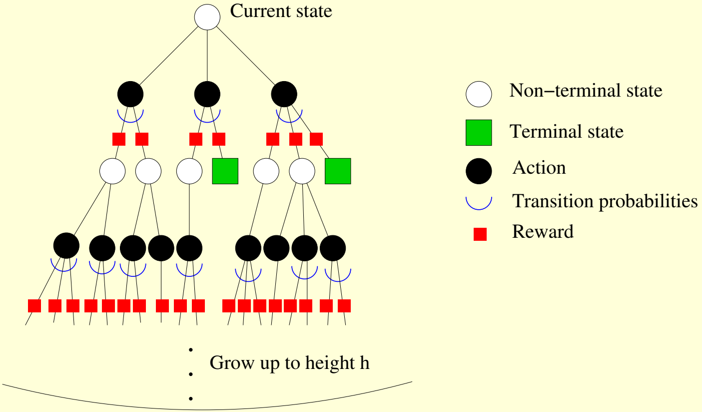
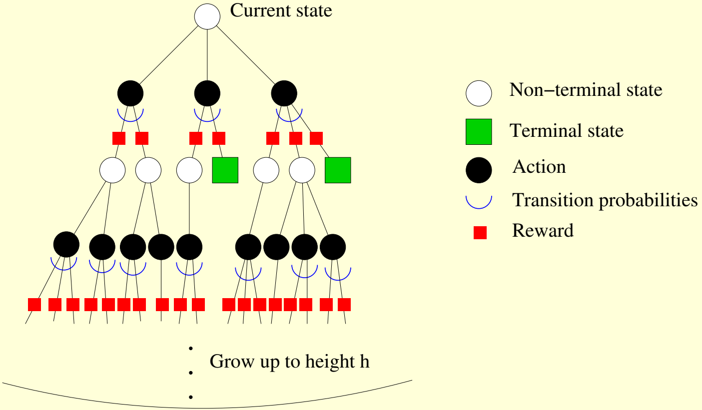
- How to rigorously do so?

Tree Search on MDPs



- Expectimax calculation. Set $Q^h \leftarrow \mathbf{0}$ //Leaves.
  For $d = h-1, h-2, \ldots, 0$://Bottom-up calculation.
  $V^d(s) \leftarrow \max_{a \in A} Q^{d+1}(s, a)$;
  $Q^d(s, a) \leftarrow \sum_{s' \in S} T(s, a, s')\{R(s, a, s') + \gamma V^d(s')\}$.

Tree Search on MDPs



- Need $h = \theta(\frac{1}{1-\gamma})$ for sufficient accuracy.
- With branching factor b, tree size is $\theta(b^h)$. Expensive!
- Often M is only a sampling model (not distribution model).
- Can we avoid expanding (clearly) inferior branches?

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Rollout Policies

- Suppose we have a (look-up) policy π.
- Let policy π' satisfy $\pi'(s) = \max_{a \in A} Q^{\pi}(s, a)$ for $s \in S$.
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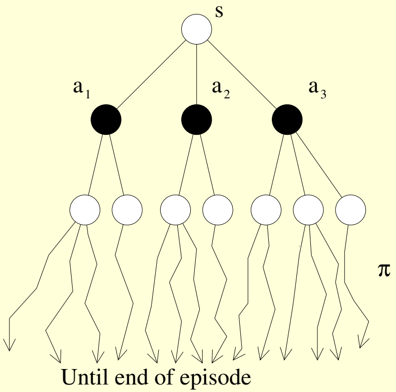

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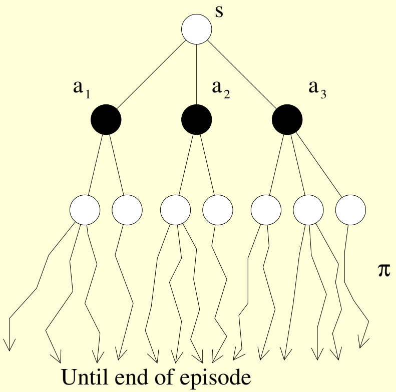

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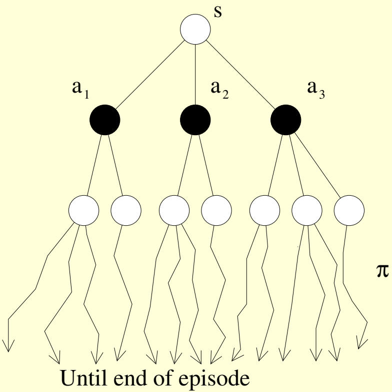

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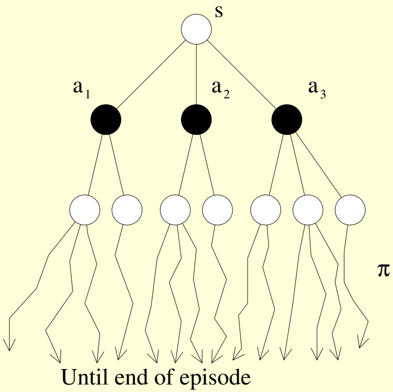

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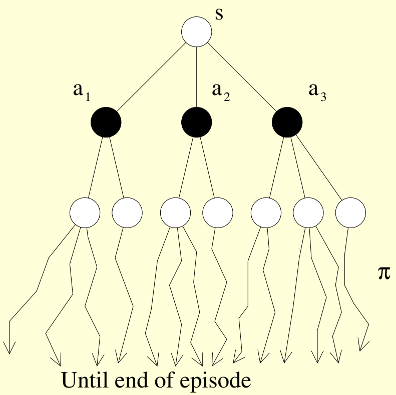

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- Take action $\pi'(s) = \text{argmax}_{a \in A} \hat{Q}^{\pi}(s, a)$.
- Repeat same process from next state s'.

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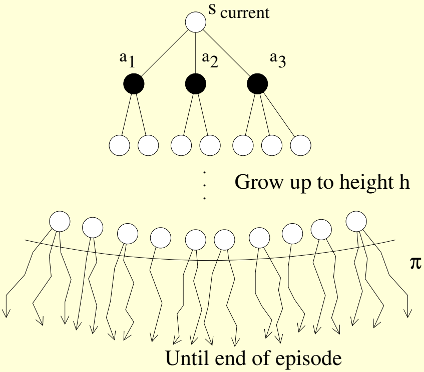
Monte Carlo Tree Search (UCT Algorithm)

- Build out a tree up to height h (say 5–10) from current state s_{current}. "Data" for the tree are samples returned by M.
- For (s, a) pairs reachable from s_{current} in $\leq h$ steps, maintain
  - $Q(s, a)$: average of returns of rollouts passing through (s, a).
  - $ucb(s, a) = Q(s, a) + C_p \sqrt{\frac{\ln(t)}{\text{visits}(s,a)}}$.



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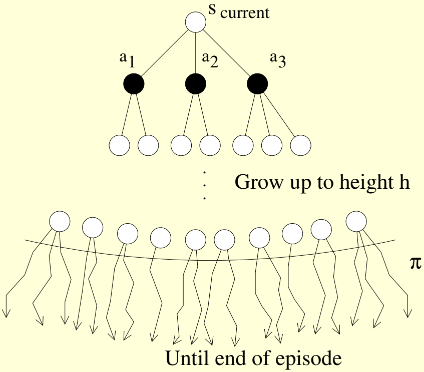

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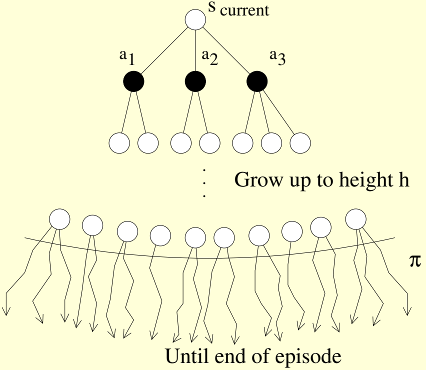

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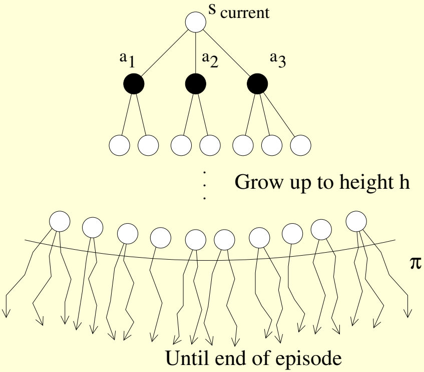

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Take $\text{argmax}_{a \in A}\, ucb(s, a)$.

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- UCT focuses attention on rewarding regions of state space.
- Rollouts can easily be parallelised.
- Extremely successful algorithm in practice.

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Search in AI/ML

- Heuristic search ("problem-solving") is among the earliest topics studied in AI.
- Applications: Theorem-proving, constraint satisfaction problems/integer programming, robotic path planning, logistics, Video games (movement of characters).
- $A^\star$ search (and variants such as $IDA^\star$) used widely in practice.
- Search is different from learning, although these two attributes of intelligence often come together.
- Main technical challenge: large (exponentially growing) number of states in most practical tasks.

Search in On-line Decision Making

- Key requirement: simulator (model).
- More computationally expensive than lookup of π or Q.
- MCTS with rollout policies an effective approach to handle stochasticity as well as large state spaces.
- Learning (say an evaluation function) can also help solution quality of search in practice.
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- We'll cover more model-based methods, as well as AlphaGo, in upcoming lectures.