CS 747, Autumn 2022: Lecture 20

Shivaram Kalyanakrishnan

Department of Computer Science and Engineering
Indian Institute of Technology Bombay

Autumn 2022

Reinforcement Learning

1. Policy gradient methods
2. Variance reduction
3. Actor-critic methods

Reinforcement Learning

1. Policy gradient methods
2. Variance reduction
3. Actor-critic methods

A Variety of Applications

- **Learning to Trade via Direct Reinforcment**
  Moody and Saffell (2001)

- **Reinforcement learning of motor skills with policy gradients**
  Peters and Schaal (2008).

- **Mastering the game of Go with deep neural networks and tree search**
  Silver et al. (2016)

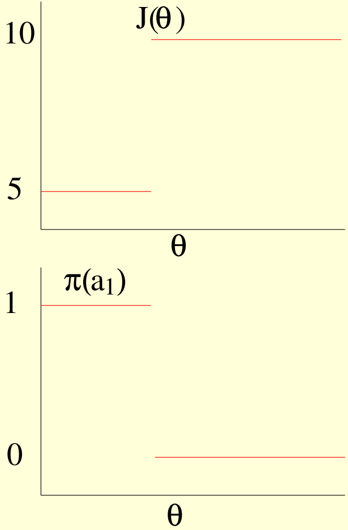
- **Deep Reinforcement Learning for Autonomous Driving: A Survey**
  Ravi Kiran et al. (2021)

Stochastic Policies

- Single state; actions a_1, a_2.
- $R(a_1) = 5$; $R(a_2) = 10$.
- Policy π; parameter θ.

$$\pi(a_1) = \begin{cases} 1 & \text{if } \theta < 0.6, \\ 0 & \text{otherwise.} \end{cases}$$

$$J(\theta) = \pi(a_1) \cdot 5 + \pi(a_2) \cdot 10.$$



Stochastic Policies

- Single state; actions a_1, a_2.
- $R(a_1) = 5$; $R(a_2) = 10$.
- Policy π; parameter θ.

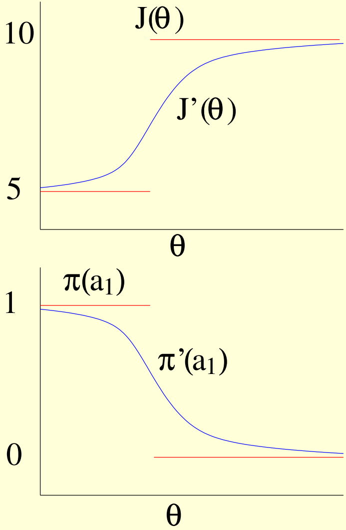
$$\pi(a_1) = \begin{cases} 1 & \text{if } \theta < 0.6, \\ 0 & \text{otherwise.} \end{cases}$$

$$J(\theta) = \pi(a_1) \cdot 5 + \pi(a_2) \cdot 10.$$

- Policy π'; parameter θ.

$$\pi'(a_1) = \frac{1}{1 + e^{\theta - 0.6}}.$$

$$J'(\theta) = \pi'(a_1) \cdot 5 + \pi'(a_2) \cdot 10.$$



Idea

- If π is differentiable w.r.t. θ, so is (scalar) “policy value” J.

Idea

- If π is differentiable w.r.t. θ, so is (scalar) "policy value" J.
- We can "search" for "good" θ by iterating:

$$\theta_{\text{new}} \leftarrow \theta_{\text{old}} + \alpha \nabla_\theta J(\theta_{\text{old}}).$$

Idea

- If π is differentiable w.r.t. θ, so is (scalar) "policy value" J.
- We can "search" for "good" θ by iterating:

$$\theta_{\text{new}} \leftarrow \theta_{\text{old}} + \alpha \nabla_\theta J(\theta_{\text{old}}).$$

- **Example.** If we have features $x(s, a) \in \mathbb{R}^d$ for $s \in S$, $a \in A$, a common template for π is:

$$\pi(s, a) = \frac{e^{\theta \cdot x(s,a)}}{\sum_{b \in A} e^{\theta \cdot x(s,b)}},$$

where $\theta \in \mathbb{R}^d$ is the vector of policy parameters.

Idea

- If π is differentiable w.r.t. θ, so is (scalar) "policy value" J.
- We can "search" for "good" θ by iterating:

$$\theta_{\text{new}} \leftarrow \theta_{\text{old}} + \alpha \nabla_\theta J(\theta_{\text{old}}).$$

- **Example.** If we have features $x(s, a) \in \mathbb{R}^d$ for $s \in S, a \in A$, a common template for π is:

$$\pi(s, a) = \frac{e^{\theta \cdot x(s,a)}}{\sum_{b \in A} e^{\theta \cdot x(s,b)}},$$

where $\theta \in \mathbb{R}^d$ is the vector of policy parameters. In this case, work out that

$$\nabla_\theta \pi(s, a) = \left(x(s, a) - \sum_{b \in B} \pi(s, b) x(s, b) \right) \pi(s, a).$$

Idea

- If π is differentiable w.r.t. θ, so is (scalar) "policy value" J.
- We can "search" for "good" θ by iterating:

$$\theta_{\text{new}} \leftarrow \theta_{\text{old}} + \alpha \nabla_\theta J(\theta_{\text{old}}).$$

- **Example.** If we have features $x(s, a) \in \mathbb{R}^d$ for $s \in S$, $a \in A$, a common template for π is:

$$\pi(s, a) = \frac{e^{\theta \cdot x(s,a)}}{\sum_{b \in A} e^{\theta \cdot x(s,b)}},$$

  where $\theta \in \mathbb{R}^d$ is the vector of policy parameters. In this case, work out that

$$\nabla_\theta \pi(s, a) = \left(x(s, a) - \sum_{b \in B} \pi(s, b) x(s, b) \right) \pi(s, a).$$

- But what's the connection between $\nabla_\theta J$ and $\nabla_\theta \pi(\cdot, \cdot)$?

Policy Gradient Theorem

- For simplicity assume episodic task with $\gamma = 1$.
- Assume there is a fixed start state s^0.
- We leave it implicit that π is fixed by parameter vector θ.
- $J(\theta) = V^{\pi}(s^0)$.
- We shall derive the connection between $\nabla_{\theta} J$ and $\nabla_{\theta} \pi(\cdot, \cdot)$.

Policy Gradient Theorem

$$\text{For } s \in S, \nabla_\theta V^\pi(s) = \nabla_\theta \sum_{a \in A} \pi(s, a) Q^\pi(s, a)$$

Policy Gradient Theorem

$$\text{For } s \in S, \nabla_\theta V^\pi(s) = \nabla_\theta \sum_{a \in A} \pi(s,a) Q^\pi(s,a)$$

$$= \sum_{a \in A} \nabla_\theta \pi(s,a) Q^\pi(s,a)$$

$$+ \sum_{a \in A} \pi(s,a) \nabla_\theta \sum_{s' \in S} T(s,a,s')(R(s,a,s') + V^\pi(s'))$$

Policy Gradient Theorem

For $s \in S$, $\nabla_\theta V^\pi(s) = \nabla_\theta \sum_{a \in A} \pi(s,a) Q^\pi(s,a)$

$$= \sum_{a \in A} \nabla_\theta \pi(s,a) Q^\pi(s,a)$$

$$+ \sum_{a \in A} \pi(s,a) \nabla_\theta \sum_{s' \in S} T(s,a,s')(R(s,a,s') + V^\pi(s'))$$

$$= \sum_{a \in A} \left[\nabla_\theta \pi(s,a) Q^\pi(s,a) + \pi(s,a) \sum_{s' \in S} T(s,a,s') \nabla_\theta V^\pi(s') \right]$$

Policy Gradient Theorem

For $s \in S, \nabla_\theta V^\pi(s) = \nabla_\theta \sum_{a \in A} \pi(s, a) Q^\pi(s, a)$

$$= \sum_{a \in A} \nabla_\theta \pi(s, a) Q^\pi(s, a)$$
$$+ \sum_{a \in A} \pi(s, a) \nabla_\theta \sum_{s' \in S} T(s, a, s')(R(s, a, s') + V^\pi(s'))$$
$$= \sum_{a \in A} \left[\nabla_\theta \pi(s, a) Q^\pi(s, a) + \pi(s, a) \sum_{s' \in S} T(s, a, s') \nabla_\theta V^\pi(s') \right]$$
$$= \cdots = \sum_{x \in S} \sum_{t=0}^{\infty} \mathbb{P}\{s \to x, t, \pi\} \sum_{a \in A} \nabla_\theta \pi(x, a) Q^\pi(x, a),$$

Policy Gradient Theorem

$$\text{For } s \in S, \nabla_\theta V^\pi(s) = \nabla_\theta \sum_{a \in A} \pi(s, a) Q^\pi(s, a)$$

$$= \sum_{a \in A} \nabla_\theta \pi(s, a) Q^\pi(s, a)$$

$$+ \sum_{a \in A} \pi(s, a) \nabla_\theta \sum_{s' \in S} T(s, a, s')(R(s, a, s') + V^\pi(s'))$$

$$= \sum_{a \in A} \left[\nabla_\theta \pi(s, a) Q^\pi(s, a) + \pi(s, a) \sum_{s' \in S} T(s, a, s') \nabla_\theta V^\pi(s') \right]$$

$$= \cdots = \sum_{x \in S} \sum_{t=0}^{\infty} \mathbb{P}\{s \to x, t, \pi\} \sum_{a \in A} \nabla_\theta \pi(x, a) Q^\pi(x, a),$$

where $\mathbb{P}\{s \to x, t, \pi\}$ is the probability of reaching x from s in t steps following π.

Policy Gradient Theorem

- Recall that $J(\theta) = V^{\pi}(s^0)$.

$$\nabla_{\theta} J(\theta) = \sum_{s \in S} \sum_{t=0}^{\infty} \mathbb{P}\{s^0 \rightarrow s, t, \pi\} \sum_{a \in A} \nabla_{\theta} \pi(s, a) Q^{\pi}(s, a).$$

Policy Gradient Theorem

- Recall that $J(\theta) = V^{\pi}(s^0)$.

$$\nabla_\theta J(\theta) = \sum_{s \in S} \sum_{t=0}^{\infty} \mathbb{P}\{s^0 \to s, t, \pi\} \sum_{a \in A} \nabla_\theta \pi(s, a) Q^{\pi}(s, a).$$

- But how to do gradient ascent? We don't know $\mathbb{P}\{s^0 \to s, t, \pi\}$, $Q^{\pi}(s, a)$!

Policy Gradient Theorem

- Recall that $J(\theta) = V^{\pi}(s^0)$.

$$\nabla_{\theta} J(\theta) = \sum_{s \in S} \sum_{t=0}^{\infty} \mathbb{P}\{s^0 \to s, t, \pi\} \sum_{a \in A} \nabla_{\theta} \pi(s, a) Q^{\pi}(s, a).$$

- But how to do gradient ascent? We don't know $\mathbb{P}\{s^0 \to s, t, \pi\}$, $Q^{\pi}(s, a)$!
- We perform stochastic gradient ascent.
- We use the following fact. For any discrete, real-valued random variable X with pmf $p : X \to [0, 1]$,

$$\sum_{x \in X} p(x)x = \mathbb{E}[X].$$

Towards Gradient Ascent

- Generate episode $s^0, a^0, r^0, s^1, a^1, r^1, s^2, \ldots, s^T = s_\top$ by acting according to π, parameterised by θ.

Towards Gradient Ascent

- Generate episode $s^0, a^0, r^0, s^1, a^1, r^1, s^2, \ldots, s^T = s_\top$ by acting according to π, parameterised by θ. Now consider:

$$\nabla_\theta J(\theta) = \sum_{s \in S} \sum_{t=0}^{\infty} \mathbb{P}\{s^0 \to s, t, \pi\} \sum_{a \in A} \nabla_\theta \pi(s, a) Q^\pi(s, a)$$

Towards Gradient Ascent

- Generate episode $s^0, a^0, r^0, s^1, a^1, r^1, s^2, \ldots, s^T = s_\top$ by acting according to π, parameterised by θ. Now consider:

$$\nabla_\theta J(\theta) = \sum_{s \in S} \sum_{t=0}^{\infty} \mathbb{P}\{s^0 \to s, t, \pi\} \sum_{a \in A} \nabla_\theta \pi(s, a) Q^\pi(s, a)$$

$$= \sum_{t=0}^{\infty} \sum_{s \in S} \mathbb{P}\{s^0 \to s, t, \pi\} \sum_{a \in A} \nabla_\theta \pi(s, a) Q^\pi(s, a)$$

Towards Gradient Ascent

- Generate episode $s^0, a^0, r^0, s^1, a^1, r^1, s^2, \ldots, s^T = s_\top$ by acting according to π, parameterised by θ. Now consider:

$$\nabla_\theta J(\theta) = \sum_{s \in S} \sum_{t=0}^{\infty} \mathbb{P}\{s^0 \to s, t, \pi\} \sum_{a \in A} \nabla_\theta \pi(s, a) Q^\pi(s, a)$$

$$= \sum_{t=0}^{\infty} \sum_{s \in S} \mathbb{P}\{s^0 \to s, t, \pi\} \sum_{a \in A} \nabla_\theta \pi(s, a) Q^\pi(s, a)$$

$$= \sum_{t=0}^{\infty} \mathbb{E}_\pi \left[\sum_{a \in A} \nabla_\theta \pi(s^t, a) Q^\pi(s^t, a) \right]$$

Towards Gradient Ascent

- Generate episode $s^0, a^0, r^0, s^1, a^1, r^1, s^2, \ldots, s^T = s_\top$ by acting according to π, parameterised by θ. Now consider:

$$\nabla_\theta J(\theta) = \sum_{s \in S} \sum_{t=0}^{\infty} \mathbb{P}\{s^0 \rightarrow s, t, \pi\} \sum_{a \in A} \nabla_\theta \pi(s, a) Q^\pi(s, a)$$

$$= \sum_{t=0}^{\infty} \sum_{s \in S} \mathbb{P}\{s^0 \rightarrow s, t, \pi\} \sum_{a \in A} \nabla_\theta \pi(s, a) Q^\pi(s, a)$$

$$= \sum_{t=0}^{\infty} \mathbb{E}_\pi \left[\sum_{a \in A} \nabla_\theta \pi(s^t, a) Q^\pi(s^t, a) \right]$$

$$= \mathbb{E}_\pi \left[\sum_{t=0}^{T-1} \sum_{a \in A} \nabla_\theta \pi(s^t, a) Q^\pi(s^t, a) \right].$$

Towards Gradient Ascent

- Generate episode $s^0, a^0, r^0, s^1, a^1, r^1, s^2, \ldots, s^T = s_\top$ by acting according to π, parameterised by θ. Now consider:

$$\nabla_\theta J(\theta) = \mathbb{E}_\pi \left[\sum_{t=0}^{T-1} \sum_{a \in A} \nabla_\theta \pi(s^t, a) Q^\pi(s^t, a) \right]$$

Towards Gradient Ascent

- Generate episode $s^0, a^0, r^0, s^1, a^1, r^1, s^2, \ldots, s^T = s_\top$ by acting according to π, parameterised by θ. Now consider:

$$\nabla_\theta J(\theta) = \mathbb{E}_\pi \left[\sum_{t=0}^{T-1} \sum_{a \in A} \nabla_\theta \pi(s^t, a) Q^\pi(s^t, a) \right]$$

$$= \mathbb{E}_\pi \left[\sum_{t=0}^{T-1} \sum_{a \in A} \pi(s^t, a) \frac{\nabla_\theta \pi(s^t, a)}{\pi(s^t, a)} Q^\pi(s^t, a) \right]$$

Towards Gradient Ascent

- Generate episode $s^0, a^0, r^0, s^1, a^1, r^1, s^2, \ldots, s^T = s_\top$ by acting according to π, parameterised by θ. Now consider:

$$\begin{aligned}
\nabla_\theta J(\theta) &= \mathbb{E}_\pi \left[\sum_{t=0}^{T-1} \sum_{a \in A} \nabla_\theta \pi(s^t, a) Q^\pi(s^t, a) \right] \\
&= \mathbb{E}_\pi \left[\sum_{t=0}^{T-1} \sum_{a \in A} \pi(s^t, a) \frac{\nabla_\theta \pi(s^t, a)}{\pi(s^t, a)} Q^\pi(s^t, a) \right] \\
&= \mathbb{E}_\pi \left[\sum_{t=0}^{T-1} \frac{\nabla_\theta \pi(s^t, a^t)}{\pi(s^t, a^t)} Q^\pi(s^t, a^t) \right]
\end{aligned}$$

Towards Gradient Ascent

- Generate episode $s^0, a^0, r^0, s^1, a^1, r^1, s^2, \ldots, s^T = s_\top$ by acting according to π, parameterised by θ. Now consider:

$$\begin{aligned}
\nabla_\theta J(\theta) &= \mathbb{E}_\pi \left[\sum_{t=0}^{T-1} \sum_{a \in A} \nabla_\theta \pi(s^t, a) Q^\pi(s^t, a) \right] \\
&= \mathbb{E}_\pi \left[\sum_{t=0}^{T-1} \sum_{a \in A} \pi(s^t, a) \frac{\nabla_\theta \pi(s^t, a)}{\pi(s^t, a)} Q^\pi(s^t, a) \right] \\
&= \mathbb{E}_\pi \left[\sum_{t=0}^{T-1} \frac{\nabla_\theta \pi(s^t, a^t)}{\pi(s^t, a^t)} Q^\pi(s^t, a^t) \right] = \mathbb{E}_\pi \left[\sum_{t=0}^{T-1} \frac{\nabla_\theta \pi(s^t, a^t)}{\pi(s^t, a^t)} G_{t:T} \right]
\end{aligned}$$

Towards Gradient Ascent

- Generate episode $s^0, a^0, r^0, s^1, a^1, r^1, s^2, \ldots, s^T = s_\top$ by acting according to π, parameterised by θ. Now consider:

$$\begin{aligned}
\nabla_\theta J(\theta) &= \mathbb{E}_\pi\left[\sum_{t=0}^{T-1}\sum_{a\in A}\nabla_\theta \pi(s^t, a) Q^\pi(s^t, a)\right] \\
&= \mathbb{E}_\pi\left[\sum_{t=0}^{T-1}\sum_{a\in A}\pi(s^t, a)\frac{\nabla_\theta \pi(s^t, a)}{\pi(s^t, a)} Q^\pi(s^t, a)\right] \\
&= \mathbb{E}_\pi\left[\sum_{t=0}^{T-1}\frac{\nabla_\theta \pi(s^t, a^t)}{\pi(s^t, a^t)} Q^\pi(s^t, a^t)\right] = \mathbb{E}_\pi\left[\sum_{t=0}^{T-1}\frac{\nabla_\theta \pi(s^t, a^t)}{\pi(s^t, a^t)} G_{t:T}\right] \\
&= \mathbb{E}_\pi\left[\sum_{t=0}^{T-1}\left(\nabla_\theta \ln \pi(s^t, a^t)\right) G_{t:T}\right].
\end{aligned}$$

REINFORCE Algorithm

- Reference: Williams (1992).
- For clarity we show explicit dependence of π on parameter vector $\theta \in \mathbb{R}^d$.
- Assume θ is initialised arbitrarily.

Repeat for ever:

$\theta_{\text{new}} \leftarrow \theta$.

Generate episode $s^0, a^0, r^0, s^1, \dots, s^T = s_\top$, following π_θ.

For $t = 0, 1, \dots, T-1$:

$G \leftarrow \sum_{k=t}^{T-1} r^k$. //This is $G_{t:T}$.

$\theta_{\text{new}} \leftarrow \theta_{\text{new}} + \alpha G \nabla_\theta \ln \pi_\theta(s^t, a^t)$.

$\theta \leftarrow \theta_{\text{new}}$.

REINFORCE Algorithm

- Reference: Williams (1992).
- For clarity we show explicit dependence of π on parameter vector $\theta \in \mathbb{R}^d$.
- Assume θ is initialised arbitrarily.

Repeat for ever:

  $\theta_{\text{new}} \leftarrow \theta$.

  Generate episode $s^0, a^0, r^0, s^1, \ldots, s^T = s_\top$, following π_θ.

  For $t = 0, 1, \ldots, T-1$:

    $G \leftarrow \sum_{k=t}^{T-1} r^k$. //This is $G_{t:T}$.

    $\theta_{\text{new}} \leftarrow \theta_{\text{new}} + \alpha G \nabla_\theta \ln \pi_\theta(s^t, a^t)$.

    //REward Increment = Nonnegative Factor $\times$

    //Offset Reinforcement $\times$ Characteristic Eligibility.

  $\theta \leftarrow \theta_{\text{new}}$.

Reinforcement Learning

1. Policy gradient methods
2. Variance reduction
3. Actor-critic methods

Baseline Subtraction

- Policy Gradient Theorem

$$\nabla_\theta J(\theta) = \sum_{s \in S} \sum_{t=0}^{\infty} \mathbb{P}\{s^0 \to s, t, \pi\} \sum_{a \in A} \nabla_\theta \pi(s, a) Q^\pi(s, a).$$

Baseline Subtraction

- Policy Gradient Theorem

$$\nabla_\theta J(\theta) = \sum_{s \in S} \sum_{t=0}^{\infty} \mathbb{P}\{s^0 \rightarrow s, t, \pi\} \sum_{a \in A} \nabla_\theta \pi(s, a) Q^\pi(s, a).$$

- Let $B : S \rightarrow \mathbb{R}$ be an *arbitrary* function of state.

Baseline Subtraction

- Policy Gradient Theorem

$$\nabla_\theta J(\theta) = \sum_{s \in S} \sum_{t=0}^{\infty} \mathbb{P}\{s^0 \to s, t, \pi\} \sum_{a \in A} \nabla_\theta \pi(s, a) Q^\pi(s, a).$$

- Let $B : S \to \mathbb{R}$ be an *arbitrary* function of state. We claim

$$\nabla_\theta J(\theta) = \sum_{s \in S} \sum_{t=0}^{\infty} \mathbb{P}\{s^0 \to s, t, \pi\} \sum_{a \in A} \nabla_\theta \pi(s, a) (Q^\pi(s, a) - B(s)).$$

Baseline Subtraction

- Policy Gradient Theorem

$$\nabla_\theta J(\theta) = \sum_{s \in S} \sum_{t=0}^{\infty} \mathbb{P}\{s^0 \to s, t, \pi\} \sum_{a \in A} \nabla_\theta \pi(s, a) Q^\pi(s, a).$$

- Let $B : S \to \mathbb{R}$ be an *arbitrary* function of state. We claim

$$\nabla_\theta J(\theta) = \sum_{s \in S} \sum_{t=0}^{\infty} \mathbb{P}\{s^0 \to s, t, \pi\} \sum_{a \in A} \nabla_\theta \pi(s, a)(Q^\pi(s, a) - B(s)).$$

- How come?

Baseline Subtraction

- Policy Gradient Theorem

$$\nabla_\theta J(\theta) = \sum_{s \in S} \sum_{t=0}^{\infty} \mathbb{P}\{s^0 \to s, t, \pi\} \sum_{a \in A} \nabla_\theta \pi(s, a) Q^\pi(s, a).$$

- Let $B : S \to \mathbb{R}$ be an *arbitrary* function of state. We claim

$$\nabla_\theta J(\theta) = \sum_{s \in S} \sum_{t=0}^{\infty} \mathbb{P}\{s^0 \to s, t, \pi\} \sum_{a \in A} \nabla_\theta \pi(s, a)(Q^\pi(s, a) - B(s)).$$

- How come? Observe that

$$\sum_{s \in S} \sum_{t=0}^{\infty} \mathbb{P}\{s^0 \to s, t, \pi\} \sum_{a \in A} \nabla_\theta \pi(s, a) B(s)$$

$$= \sum_{s \in S} \sum_{t=0}^{\infty} \mathbb{P}\{s^0 \to s, t, \pi\} B(s) \nabla_\theta \sum_{a \in A} \pi(s, a) = 0.$$

Baseline Subtraction

- The policy gradient estimate can have high variance.

s	$Q^\pi(s, a_1)$	$Q^\pi(s, a_2)$	$Q^\pi(s, a_3)$	$V^\pi(s)$
s_1	105	79	100	90
s_2	10	6	13	12
s_3	-50	-60	-50	-55

Baseline Subtraction

- The policy gradient estimate can have high variance.

s	$Q^{\pi}(s, a_1)$	$Q^{\pi}(s, a_2)$	$Q^{\pi}(s, a_3)$	$V^{\pi}(s)$
s_1	105	79	100	90
s_2	10	6	13	12
s_3	-50	-60	-50	-55

- Common to subtract out $V^{\pi}(s)$—approximated independently as $\hat{V}(s)$.
- REINFORCE with baseline: revise pseudocode to

$$\theta_{\text{new}} \leftarrow \theta_{\text{new}} + \alpha \sum_{t=0}^{T-1} (G_{t:T} - \hat{V}(s^t)) \nabla_{\theta} \ln \pi_{\theta}(s^t, a^t).$$

Reinforcement Learning

1. Policy gradient methods
2. Variance reduction
3. Actor-critic methods

Actor-critic Methods

- Even for fixed (s^t, a^t), can have high variance in $G_{t:T}$.

Actor-critic Methods

- Even for fixed (s^t, a^t), can have high variance in $G_{t:T}$.
- One approach is to do gradient ascent after averaging the gradient from a few episodes.

Actor-critic Methods

- Even for fixed (s^t, a^t), can have high variance in $G_{t:T}$.
- One approach is to do gradient ascent after averaging the gradient from a few episodes.
- Another approach is to bootstrap: to use $r^t + \hat{V}(s^{t+1})$ in place of $G_{t:T}$, where $\hat{V}(s^{t+1})$ is estimated independently.

Actor-critic Methods

- Even for fixed (s^t, a^t), can have high variance in $G_{t:T}$.
- One approach is to do gradient ascent after averaging the gradient from a few episodes.
- Another approach is to bootstrap: to use $r^t + \hat{V}(s^{t+1})$ in place of $G_{t:T}$, where $\hat{V}(s^{t+1})$ is estimated independently.
- Called the Actor-Critic architecture.
  - Actor updates θ and hence π_θ.
  - Critic evaluates π_θ (say using TD(0)) and provides input for the gradient ascent update.

$$\theta_{\text{new}} \leftarrow \theta_{\text{new}} + \alpha \sum_{t=0}^{T-1} (r^t + \hat{V}(s^{t+1}) - \hat{V}(s^t)) \nabla_\theta \ln \pi_\theta(s^t, a^t).$$

Actor-critic Methods

- Even for fixed (s^t, a^t), can have high variance in $G_{t:T}$.
- One approach is to do gradient ascent after averaging the gradient from a few episodes.
- Another approach is to bootstrap: to use $r^t + \hat{V}(s^{t+1})$ in place of $G_{t:T}$, where $\hat{V}(s^{t+1})$ is estimated independently.
- Called the Actor-Critic architecture.

 - Actor updates θ and hence π_θ.
 - Critic evaluates π_θ (say using TD(0)) and provides input for the gradient ascent update.

$$\theta_{\text{new}} \leftarrow \theta_{\text{new}} + \alpha \sum_{t=0}^{T-1} (r^t + \hat{V}(s^{t+1}) - \hat{V}(s^t)) \nabla_\theta \ln \pi_\theta(s^t, a^t).$$

- Not always provably convergent, but widely used in practice.

Reinforcement Learning

1. Policy gradient methods
2. Variance reduction
3. Actor-critic methods

Reinforcement Learning

1. Policy gradient methods
2. Variance reduction
3. Actor-critic methods

Next class: Batch reinforcement learning