Supervised Learning: Additional Topics

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February 2023

This Lecture

- Evaluation metrics for classification
- Decision trees
- Support Vector Machines
- Nearest-neighbour methods
- Supervised learning: summary

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Confusion Matrix

True label	Predicted label			
	Poha	Paratha	Idli	Toast
Poha	95	0	2	3
Paratha	0	98	0	2
Idli	1	15	84	0
Toast	0	0	0	100

Useful device to identify weaknesses of model.

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- F1 score = $\frac{2 \times \text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}}$.

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Sales Data

Colour	**Shape**	**Packed**	Sale
Red	Square	Yes	–
Red	Square	No	–
Red	Triangle	Yes	–
Red	Triangle	No	–
Red	Circle	Yes	+
Red	Circle	No	+
Green	Square	No	–
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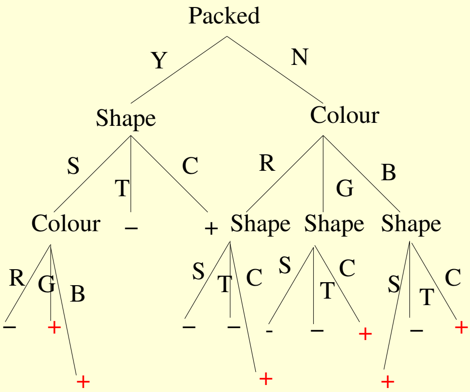
Which products are selling? Which ones are not?

Decision Trees



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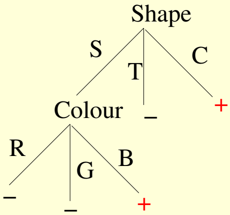
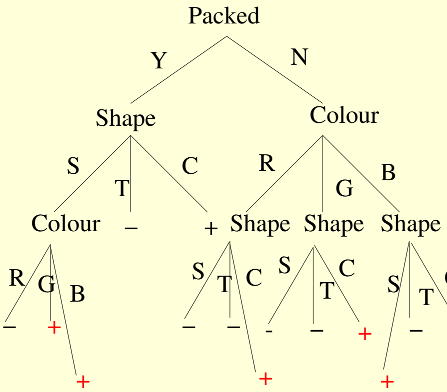


Decision Tree 2

Decision Trees



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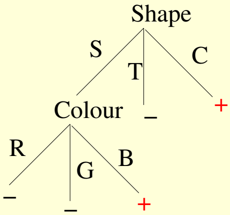

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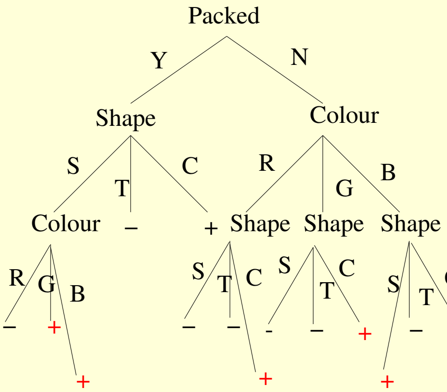

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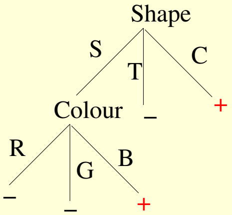

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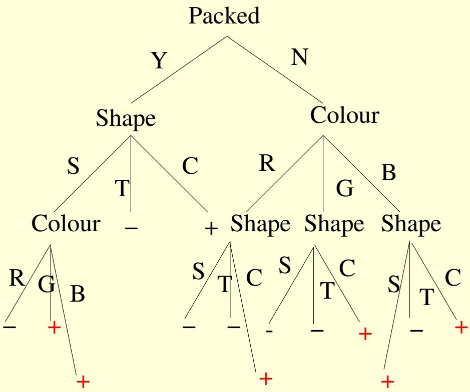

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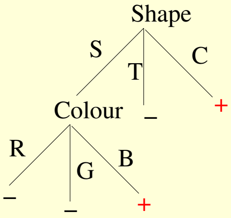

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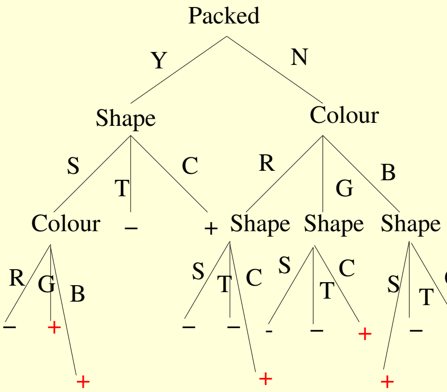

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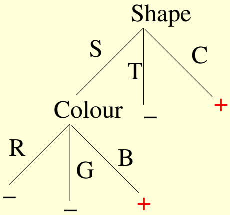

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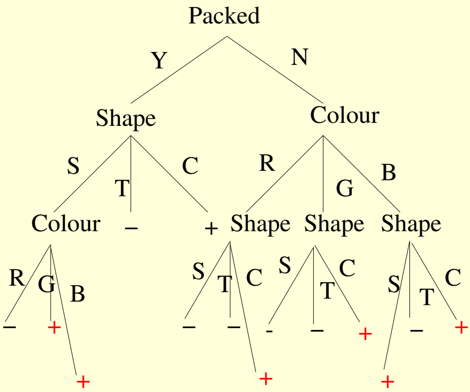

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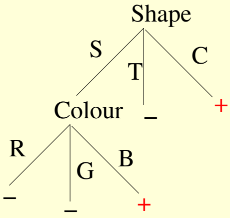

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  A smaller tree is likely to generalise better.
  Occam's Razor: a simpler solution is usually better.

Learning (Compact) Decision Trees

1. If all data have the same class C, create a leaf with prediction C. Return.
2. Identify a feature f that divides data into “homogeneous” (“pure”) sets.
3. Create a node with a split based on f; divide training data based on f and move it into corresponding branches.
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- What are some advantages of decision trees?

  Often human-readable/interpretable.

  Very fast to train (parallelisable).

  Suited to work with categorical features.

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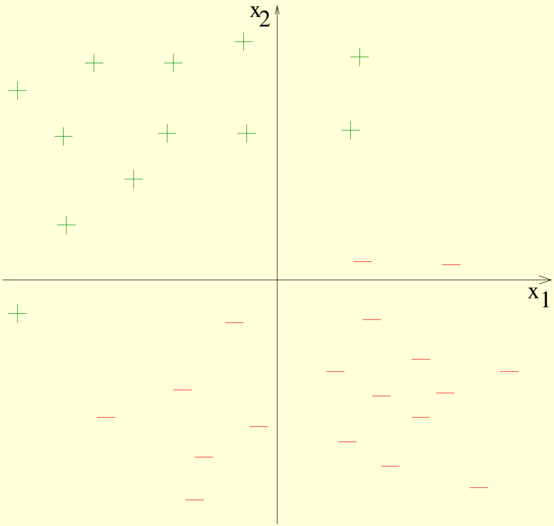
Maximum-margin Hyperplane



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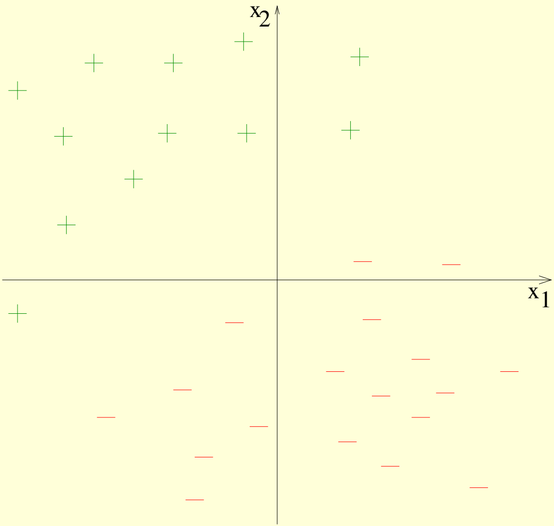

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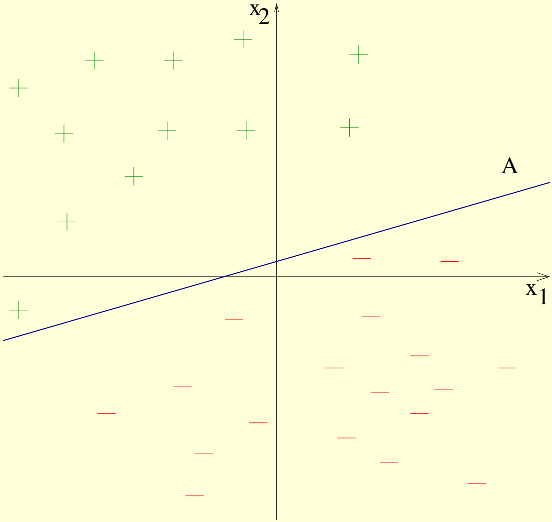

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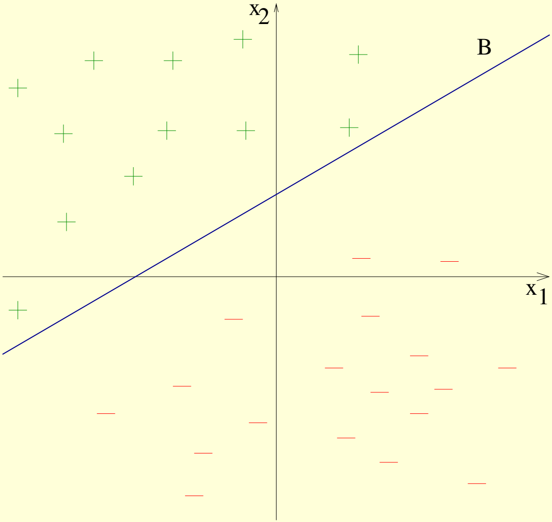

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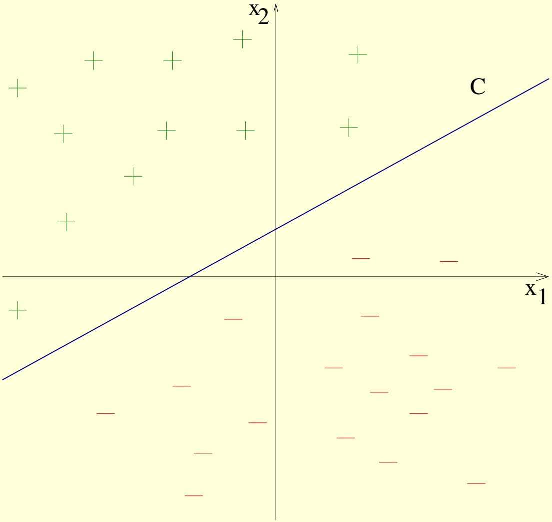

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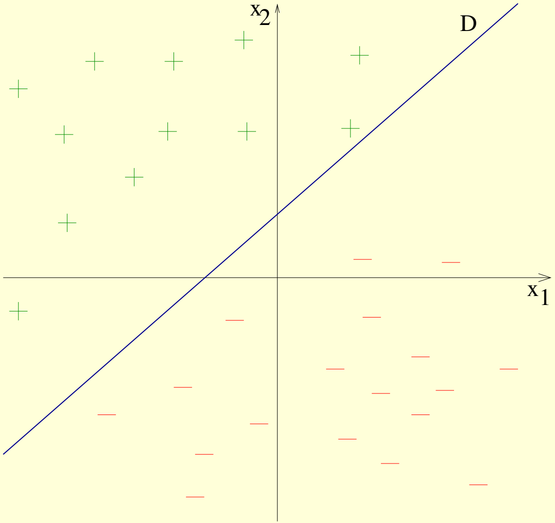

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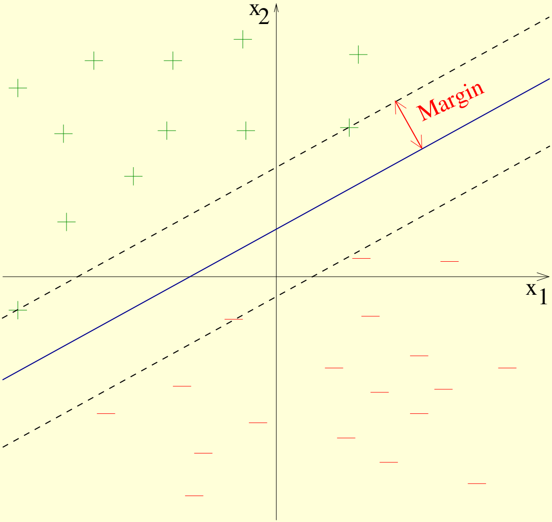

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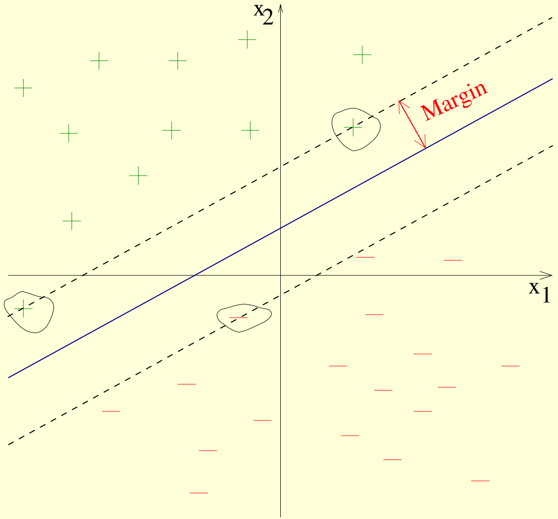

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The circled points are support vectors; the model is a Support Vector Machine.

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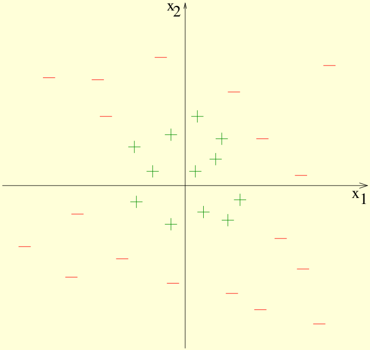
- SVMs were very popular in the 2000's.
- Why all the fuss? The model is still linear, isn't it?

Kernels



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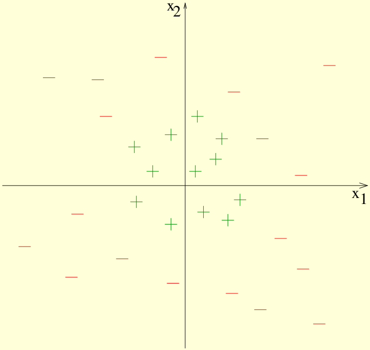

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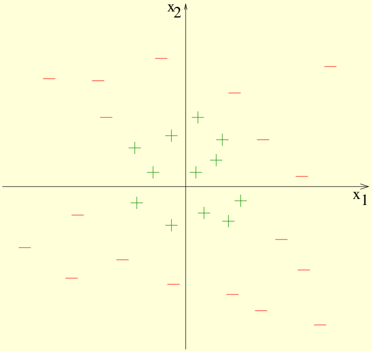

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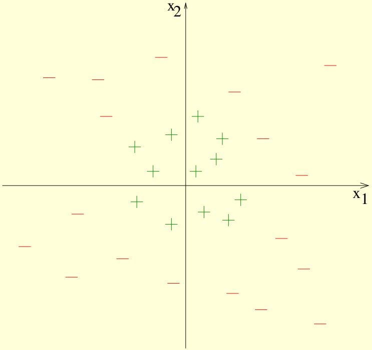

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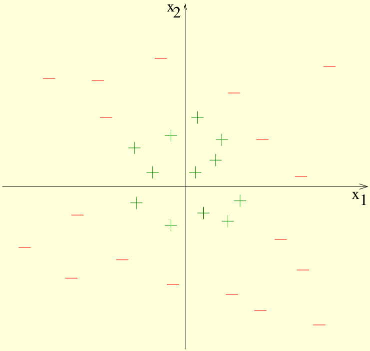

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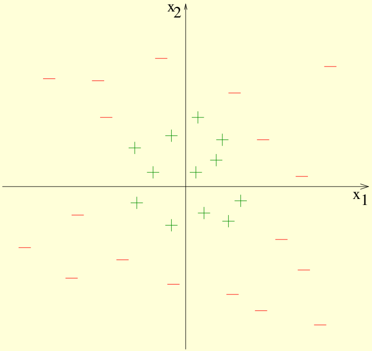

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- While there is no guarantee of linear separability in X', kernelised SVMs have registered many empirical successes.

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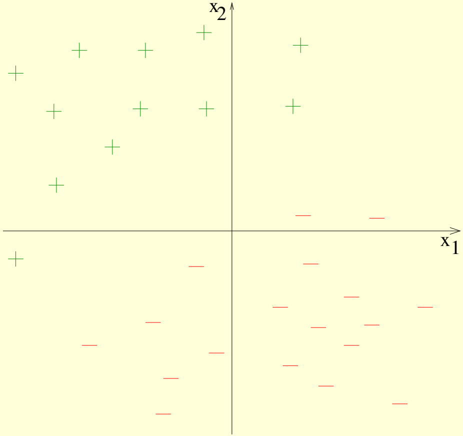

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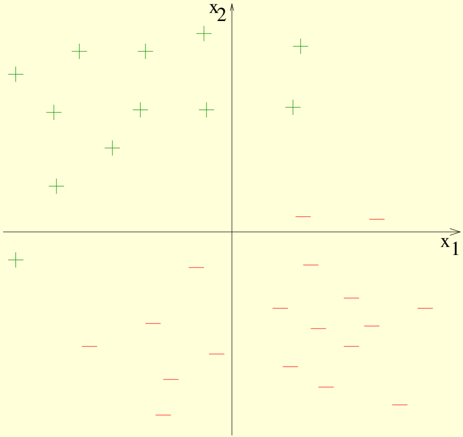

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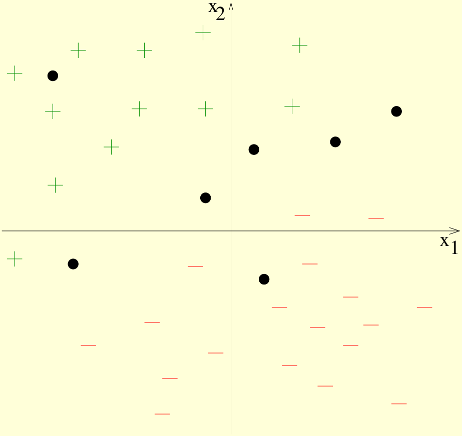

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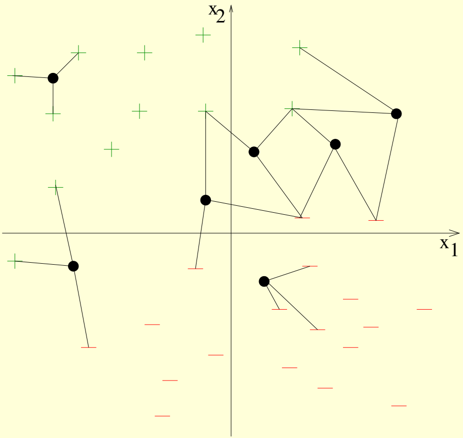

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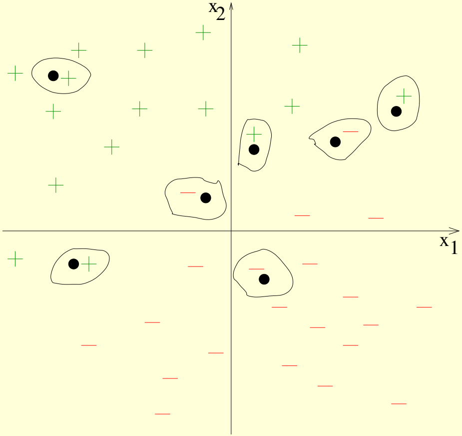

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Nearest Neighbour Methods

- What did we learn here? What is the model?
  The model is the data set itself! There is nothing to "learn".
  However, sometimes only a subset of the training data is stored, so the "model" size is reduced.

- What are the main advantages and disadvantages of this approach?
  Simple, but only tends to work in relatively low dimensions. In high dimensions all neighbours are far away!
  Query time (finding k nearest neighbours) is high (linear in the model size), although finding k "approximate" nearest neighbours can be done quickly.

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  Of course. Predict the average value of the k nearest neighbours. (Sometimes each neighbour is given a weight inversely proportional to its distance.)

This Lecture

- Evaluation metrics for classification
- Decision trees
- Support Vector Machines
- Nearest-neighbour methods
- Supervised learning: summary

Summary of Supervised Learning

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- Topics we did not cover: dimensionality reduction, logistic regression, ensemble methods, error analysis, ...

References

- Wikipedia page on confusion matrix:
  https://en.wikipedia.org/wiki/Confusion_matrix.

- Chapter 1, sections 3.2, 3.3, 7.7, **A Course in Machine Learning**, Hal Daumé III. Available on-line at http://ciml.info/.