Search 1

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Indian Institute of Technology Bombay

February 2023

Navigation System

How to go from IIT Bombay to Marine Drive?

[1]

[1] https://www.flickr.com/photos/nat507/16088993607. CC image courtesy of Nathan Hughes Hamilton on Flickr licensed under CC BY 2.0.

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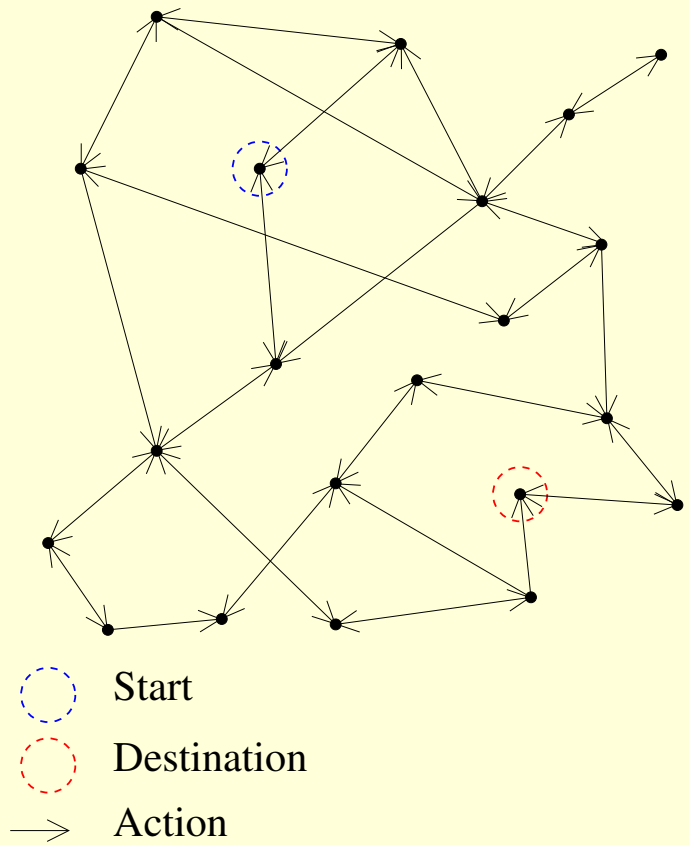

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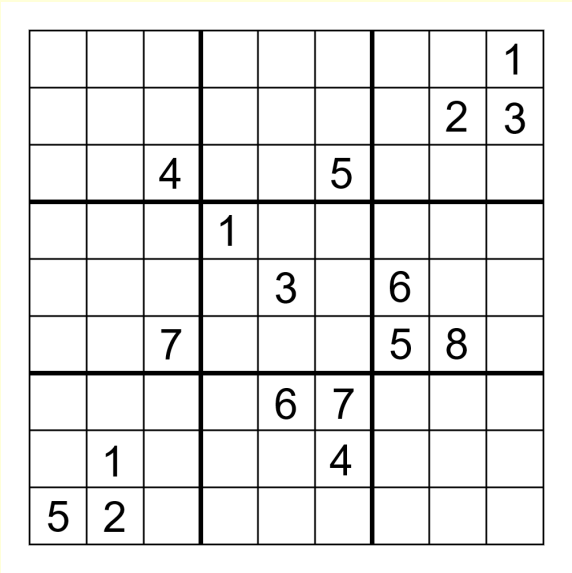
Some Popular Puzzles

How to solve?



								1
							2	3
		4			5			
			1					
				3		6		
		7				5	8	
				6	7			
	1				4			
5	2							

Sudoku [1]

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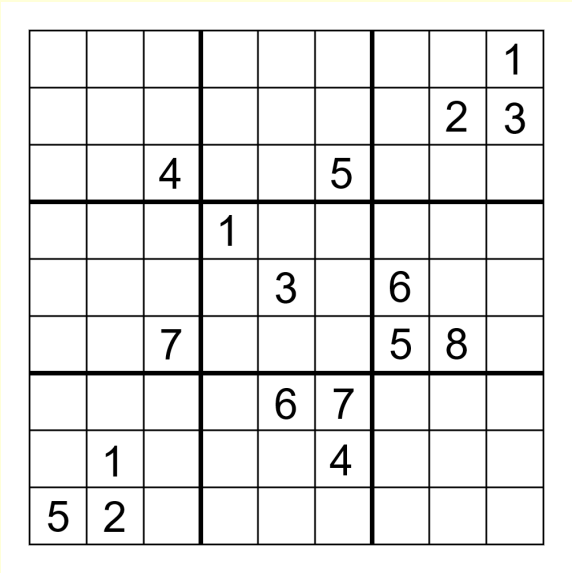

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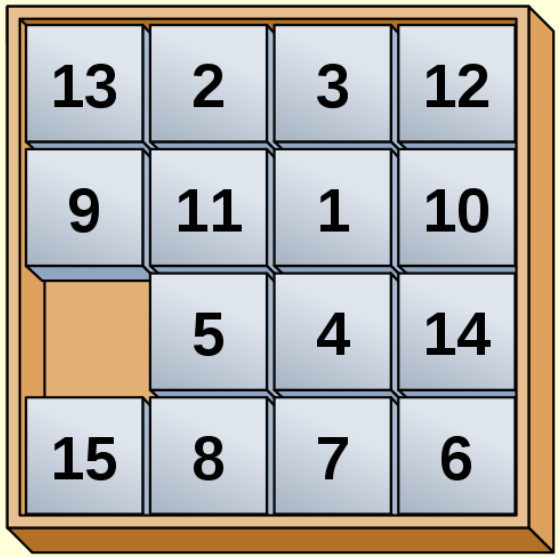

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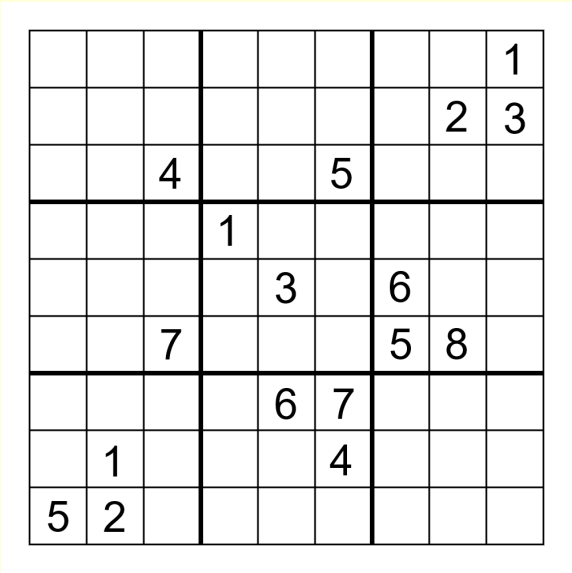

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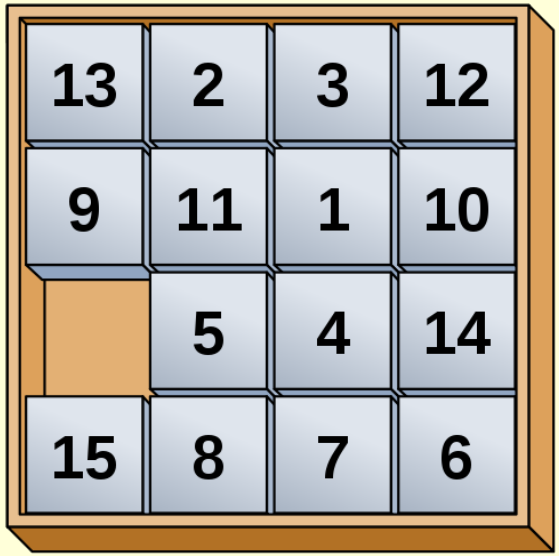

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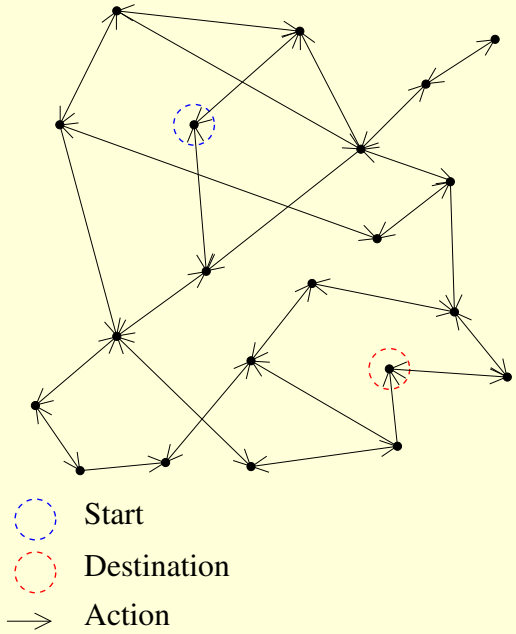

Same abstraction?

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This Lecture

- Search problem instances
- Uninformed search
  - Depth-first search
  - Breadth-first search
  - Lowest-cost-first search

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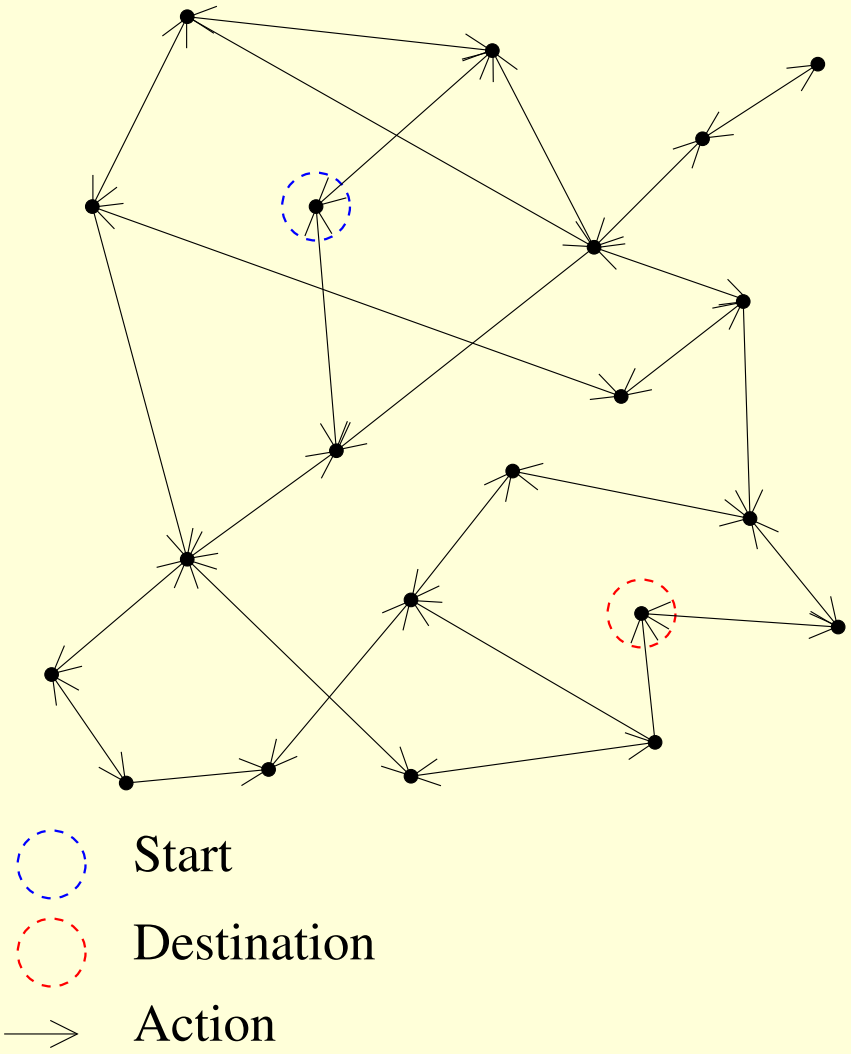
Number of available actions in each state is called the branching factor b.

Minimum number of actions (or length of optimal path) to reach goal state is called the depth d of the search instance.

Problem Formulation: Navigation System



States?

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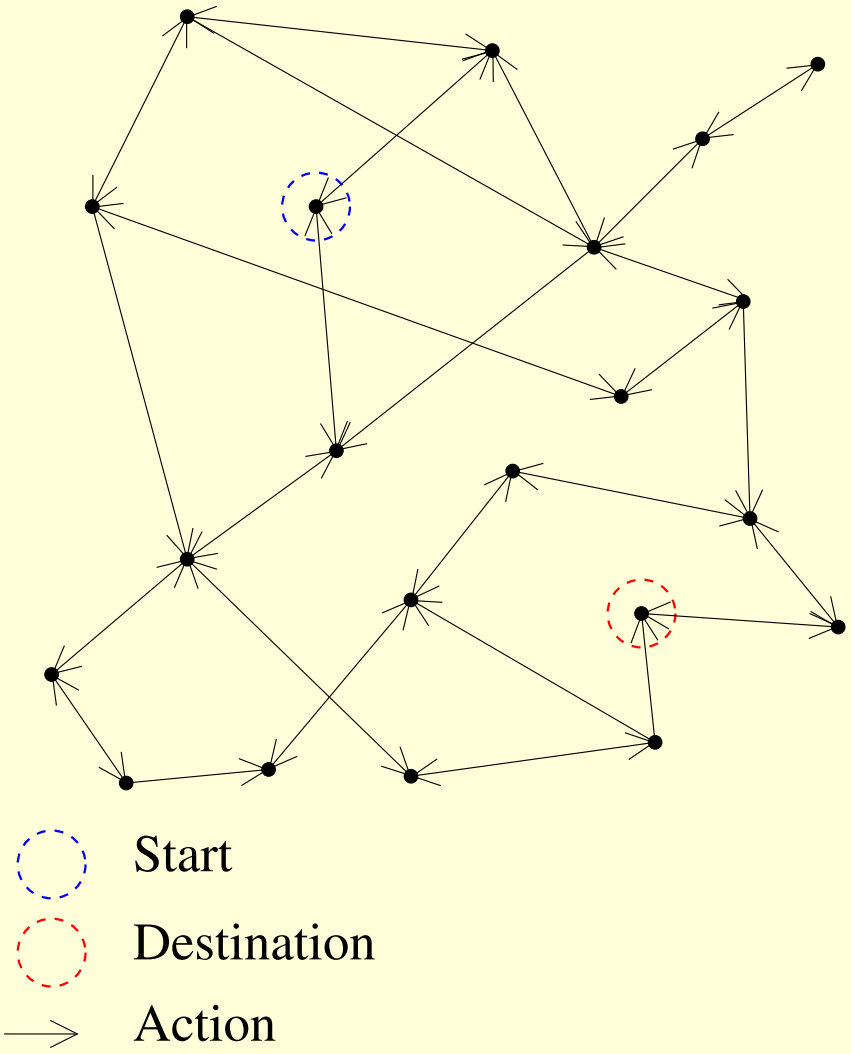

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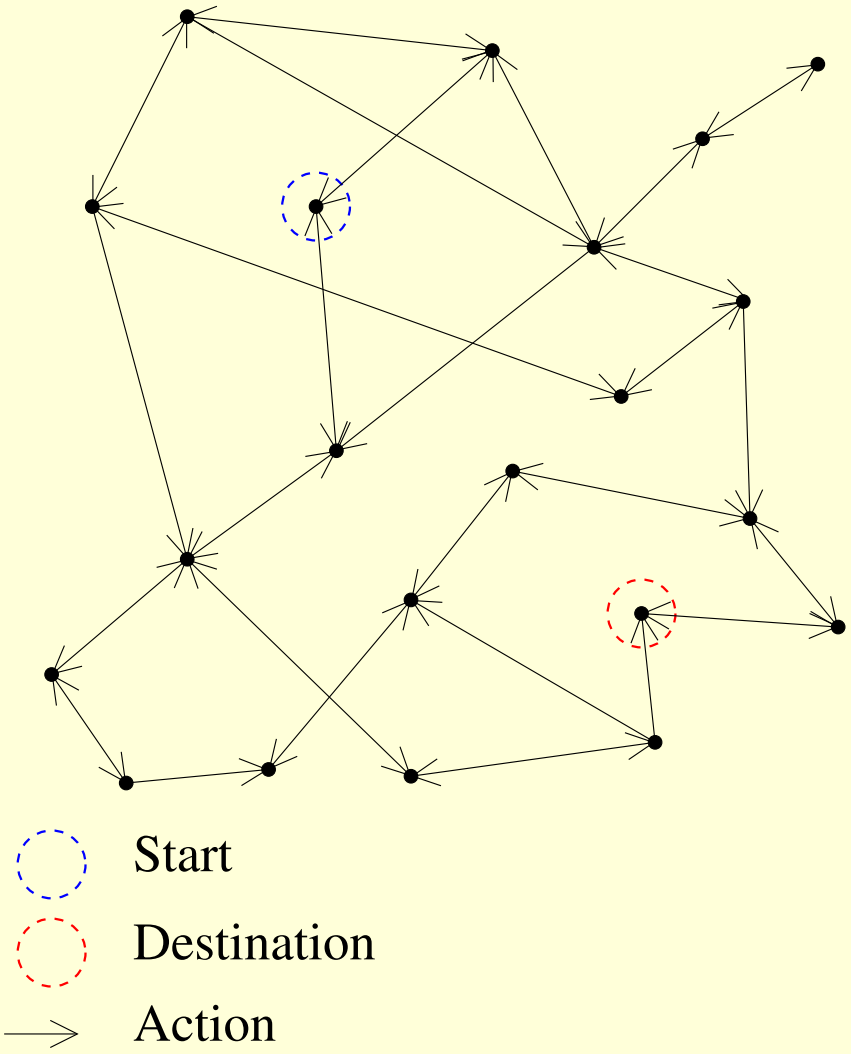

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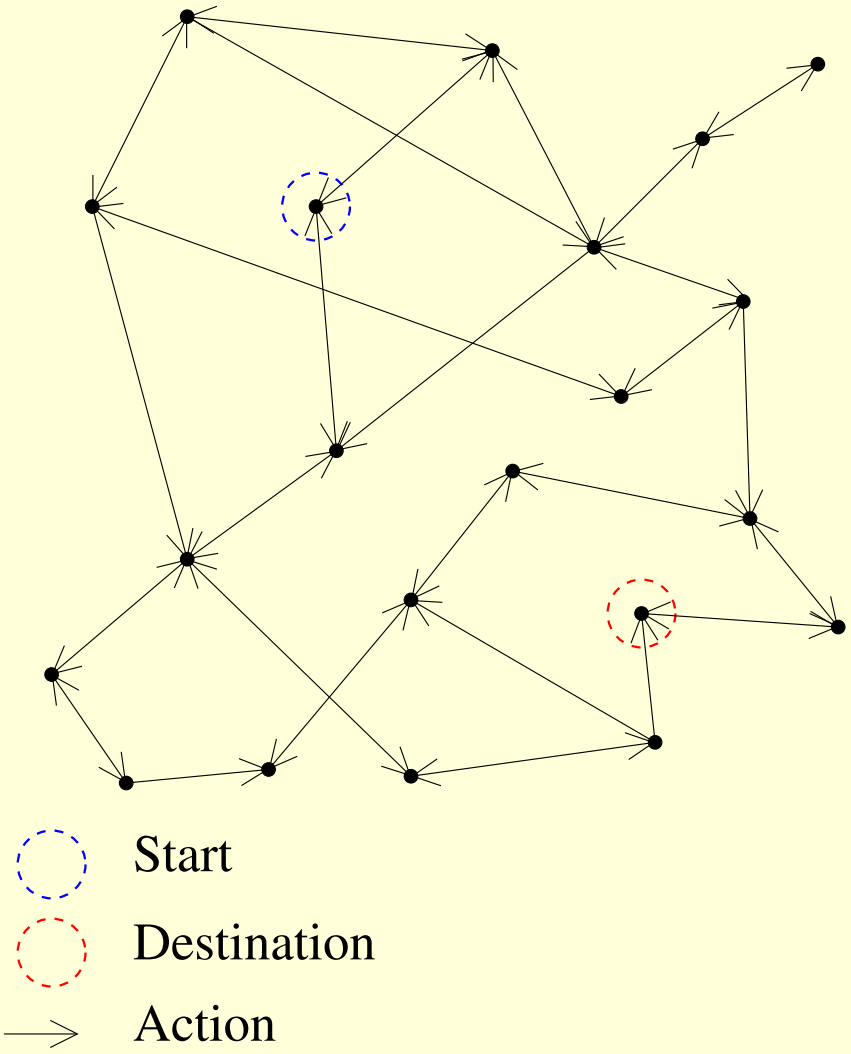

States?, start state?, actions? , NextState()?

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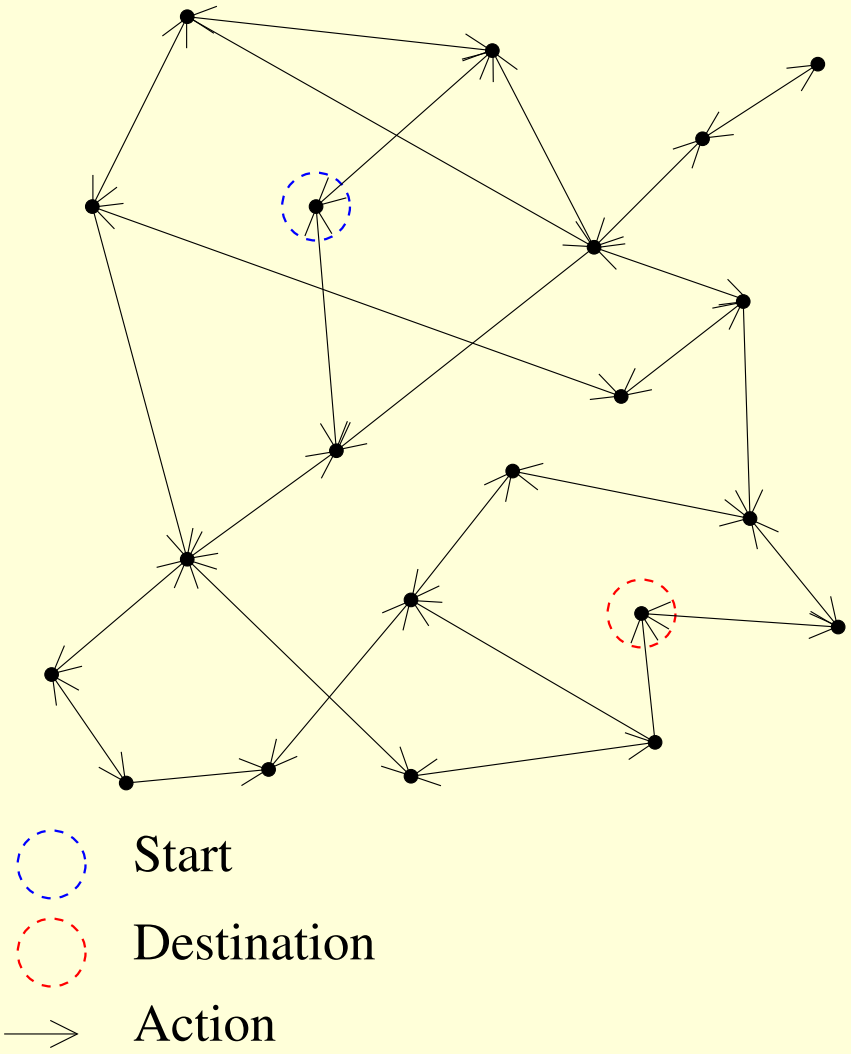

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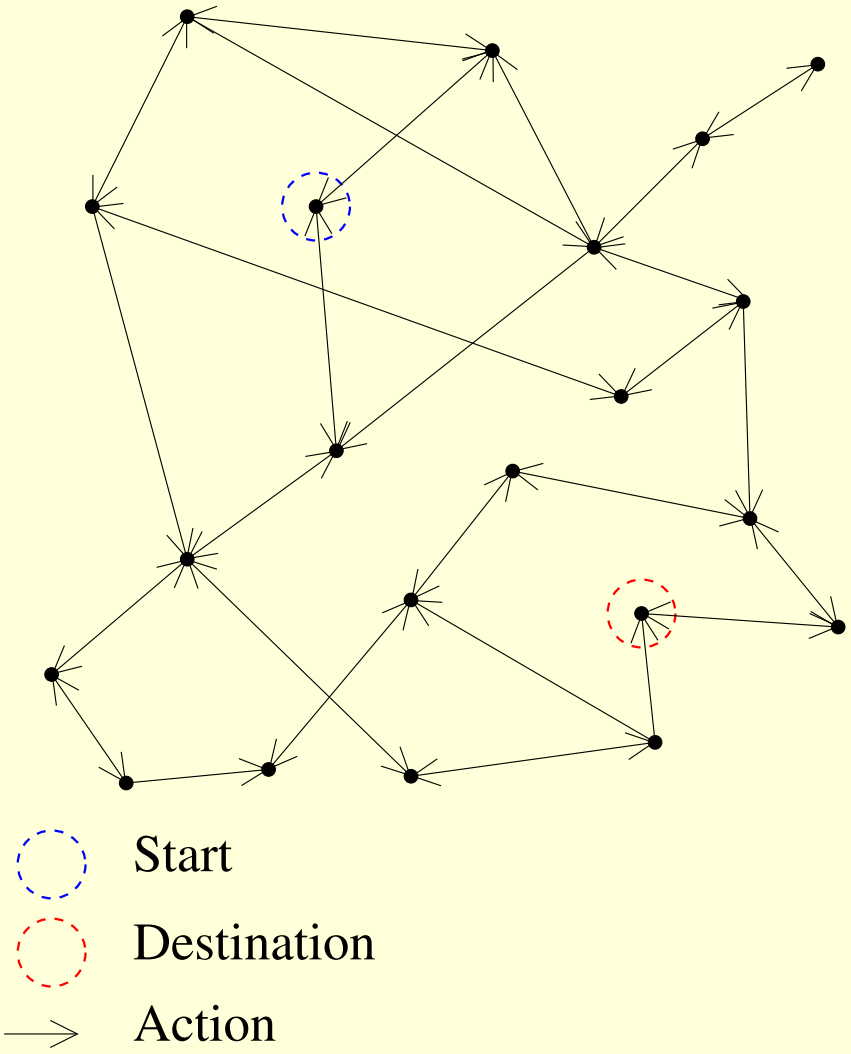

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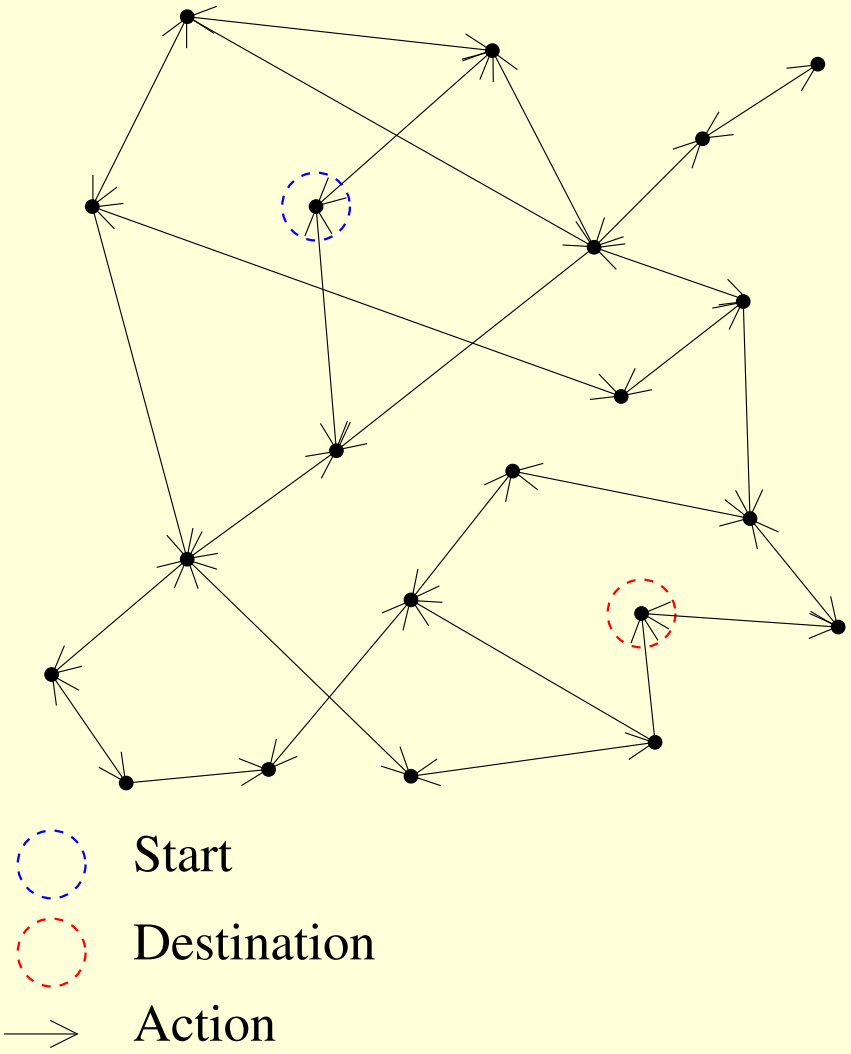

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A solver needs to find the least-cost path.

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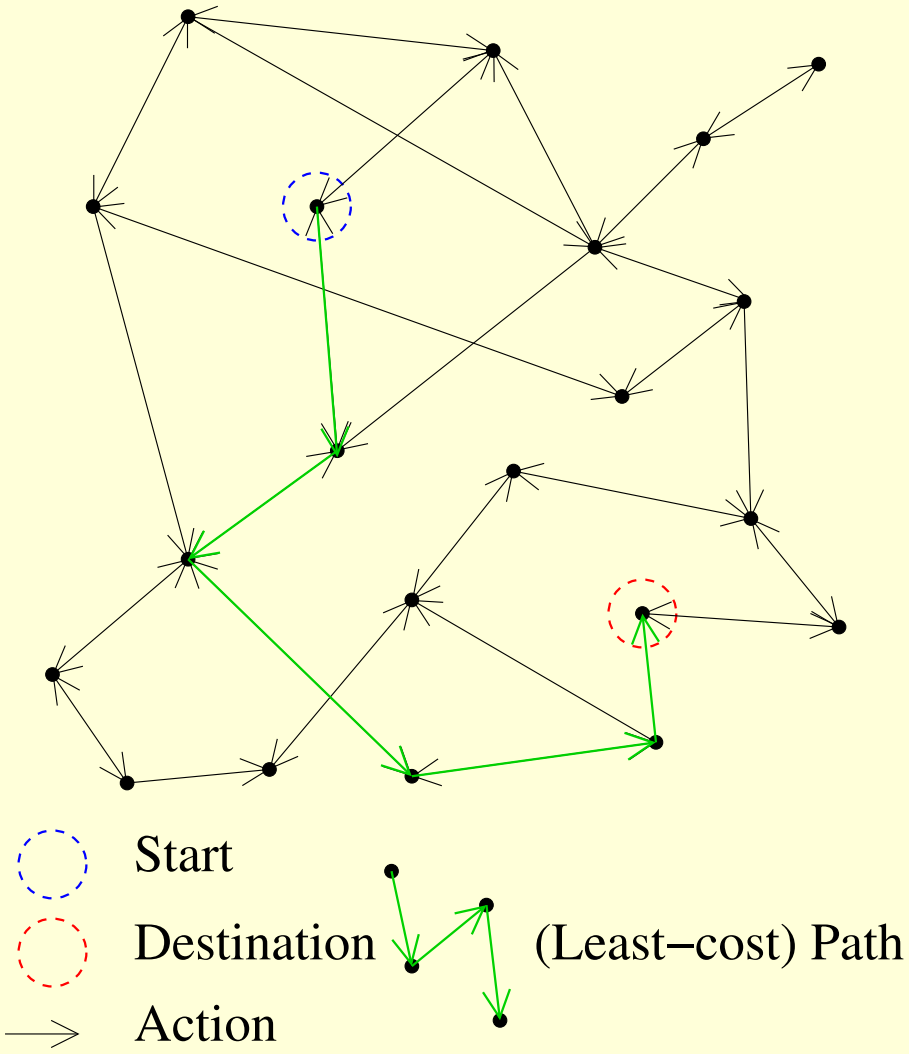

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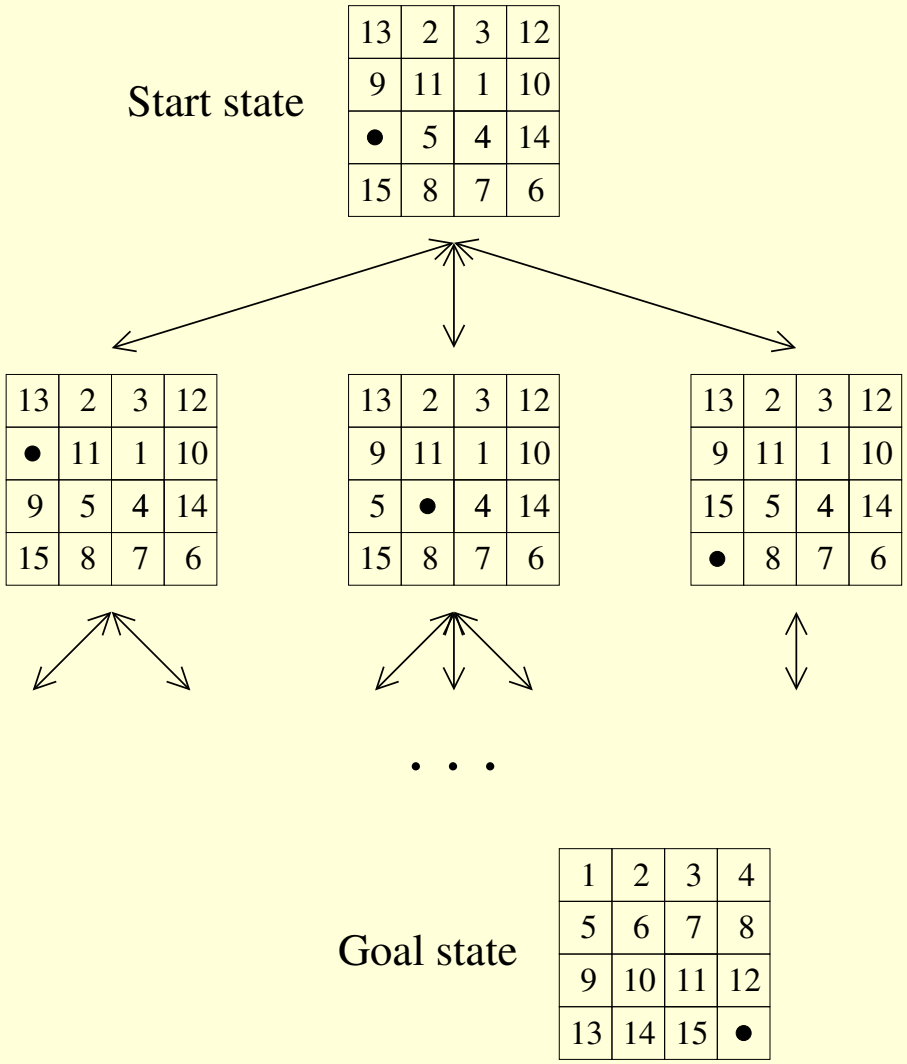

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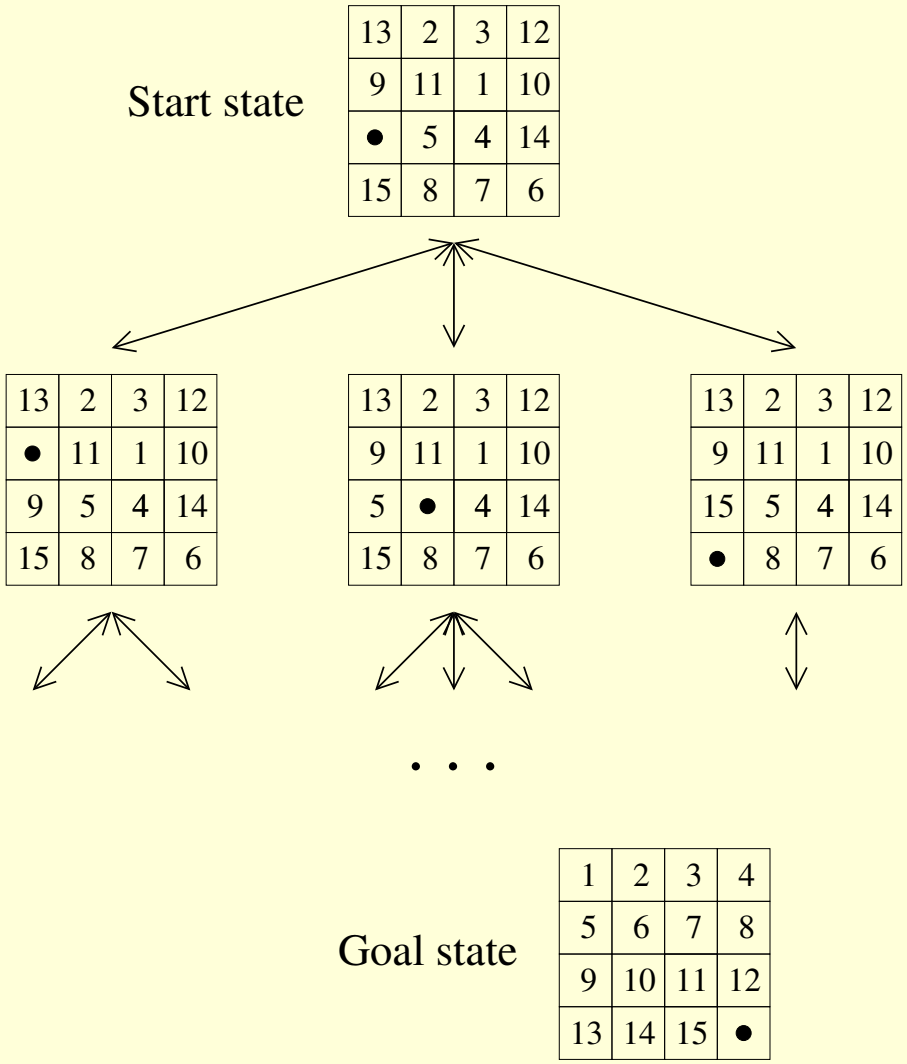

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- What is the main technical challenge in search?

  The large (exponentially growing) number of states in most practical tasks.

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Frontier $\leftarrow \{Node(startState, (startState), 0)\}$.
Repeat for ever:
    Select a node n from *Frontier*.
    //Expand n.
    If $isGoal(n.state)$:
        Return n.
    For each action a available from $n.state$:
        $s \leftarrow NextState(n.state, a)$.
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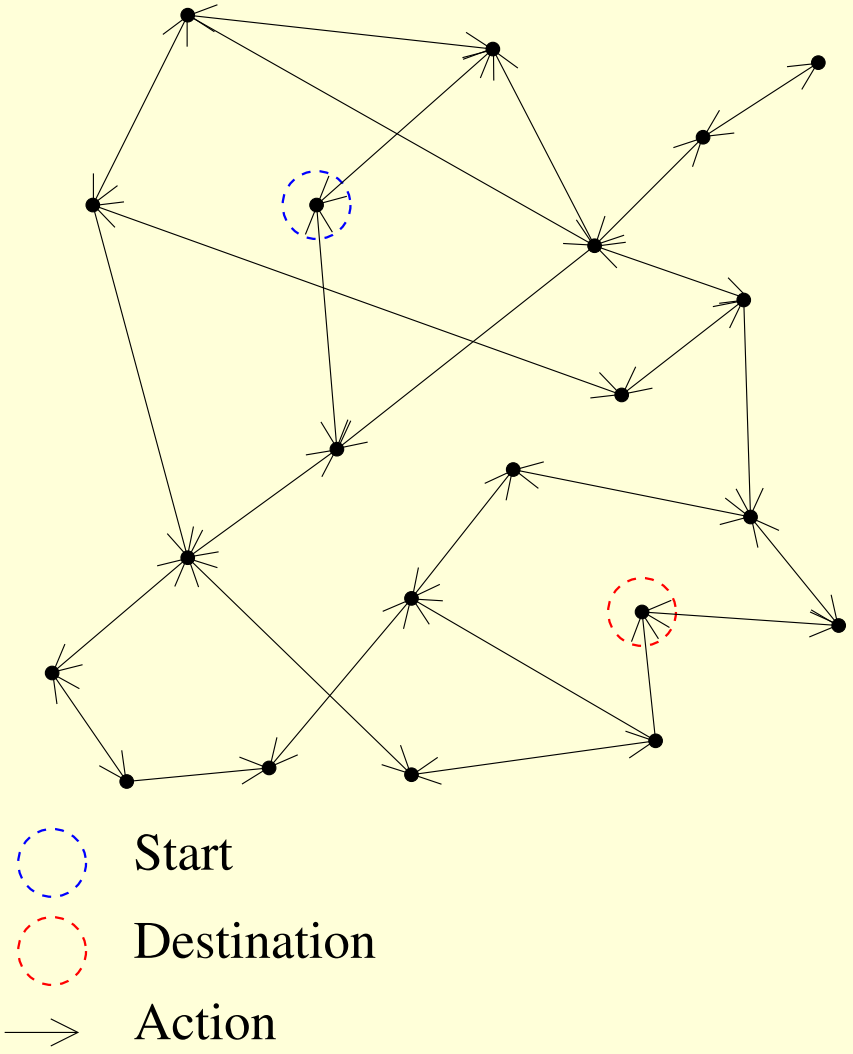

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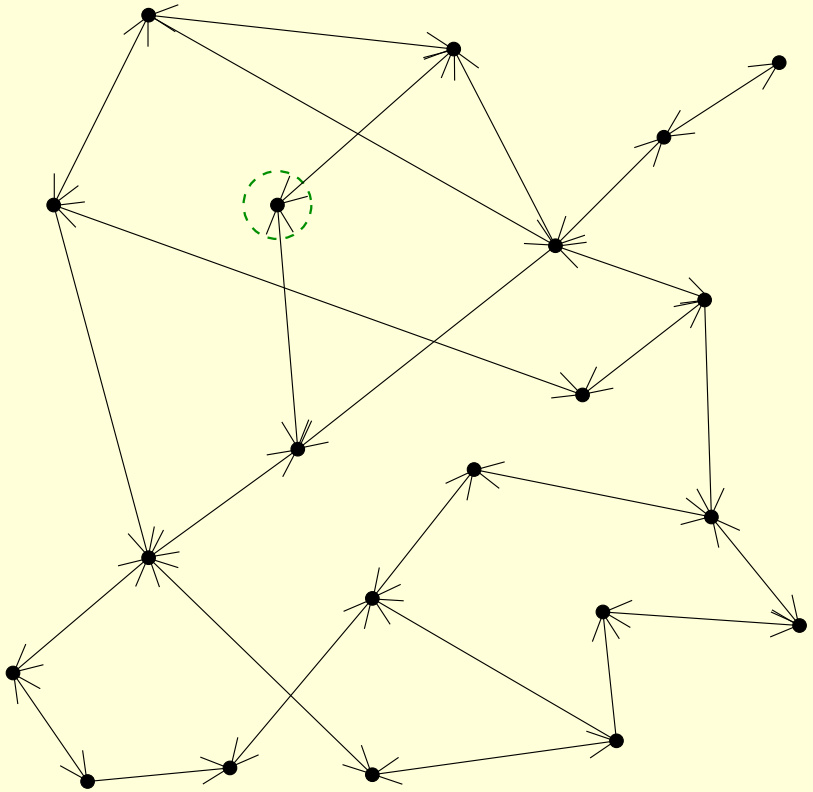

Explored

Frontier

Unexplored

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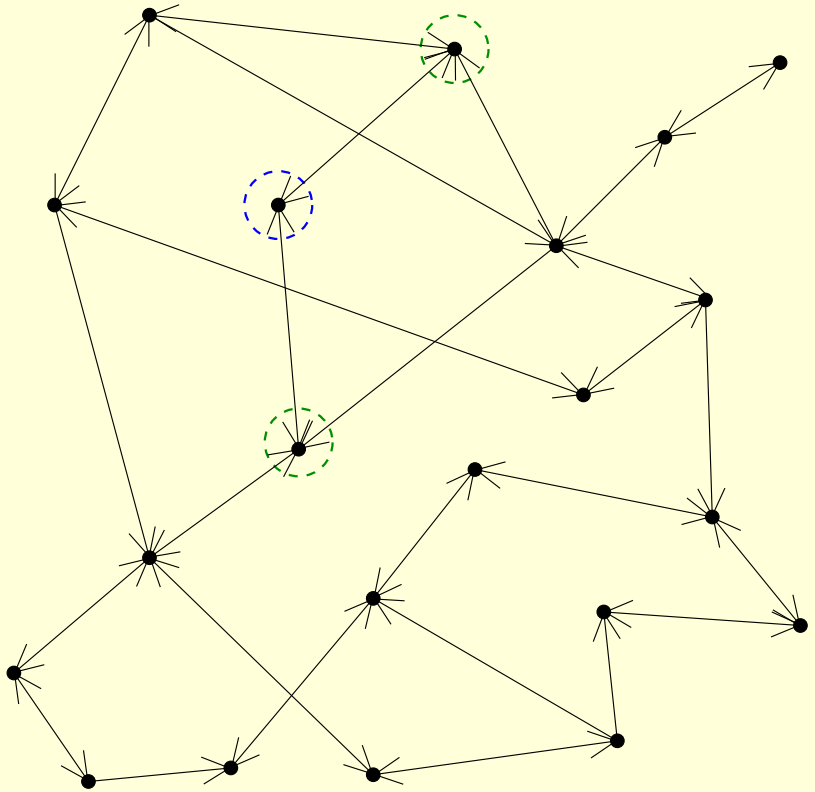

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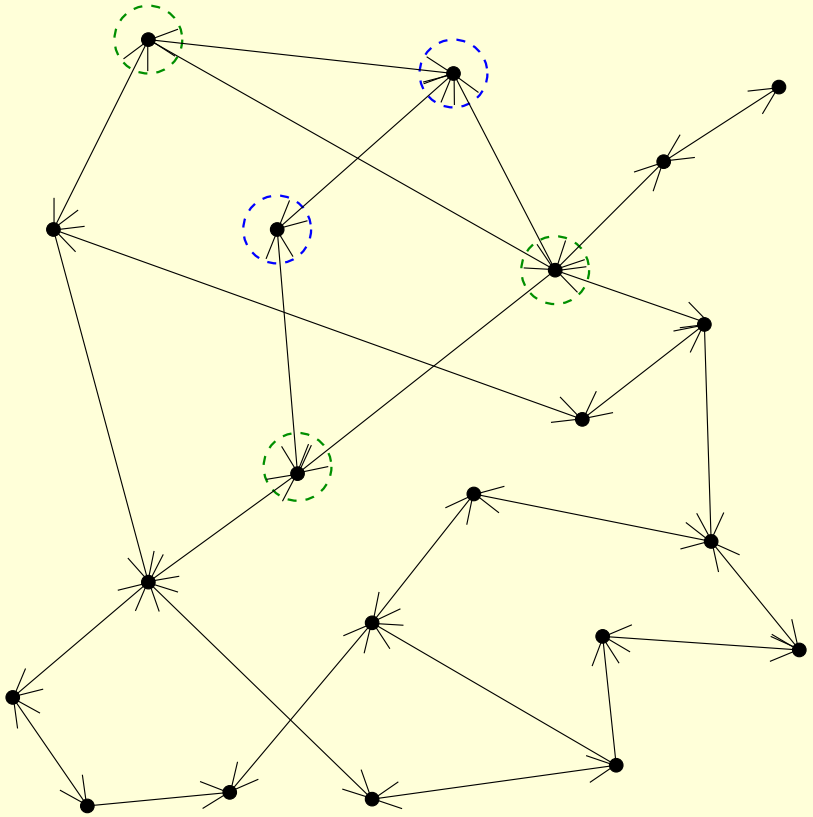

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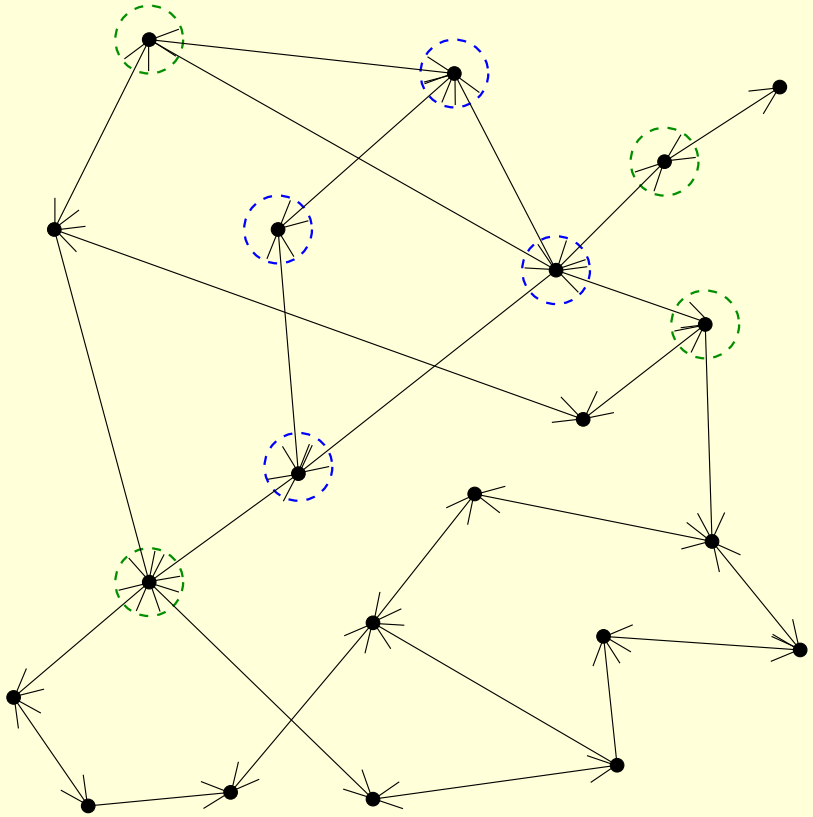

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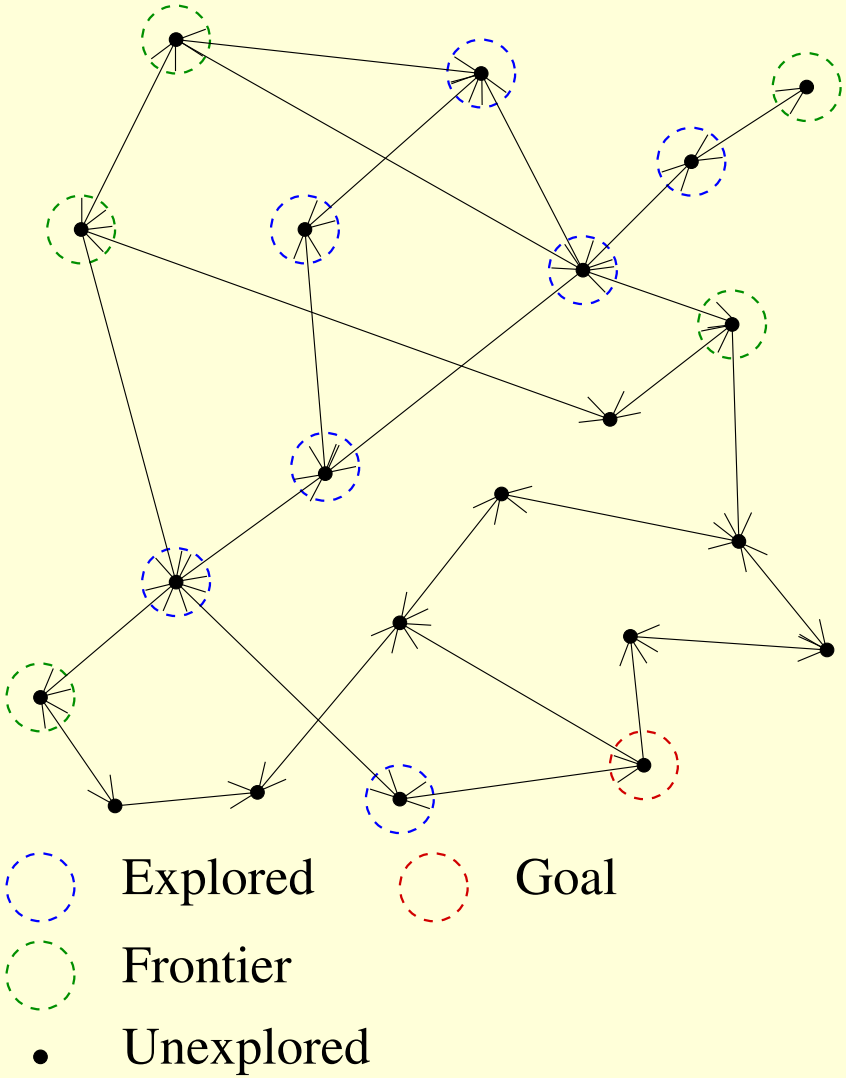

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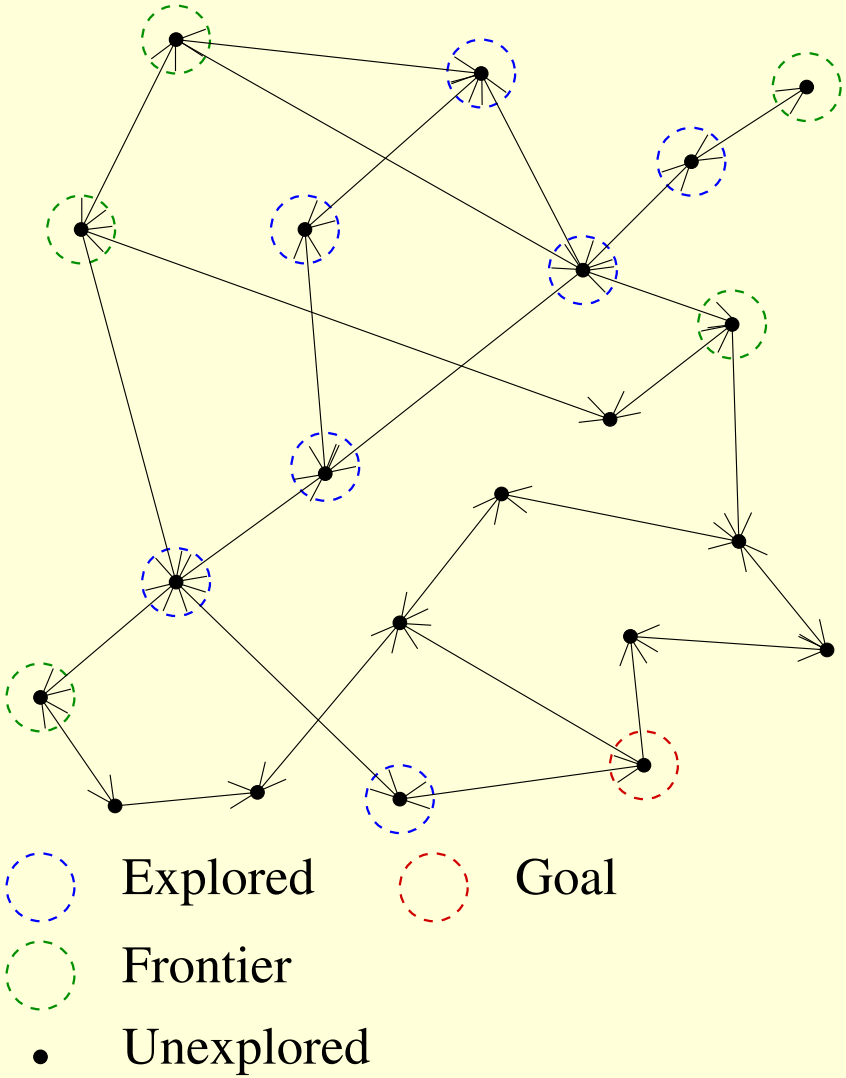
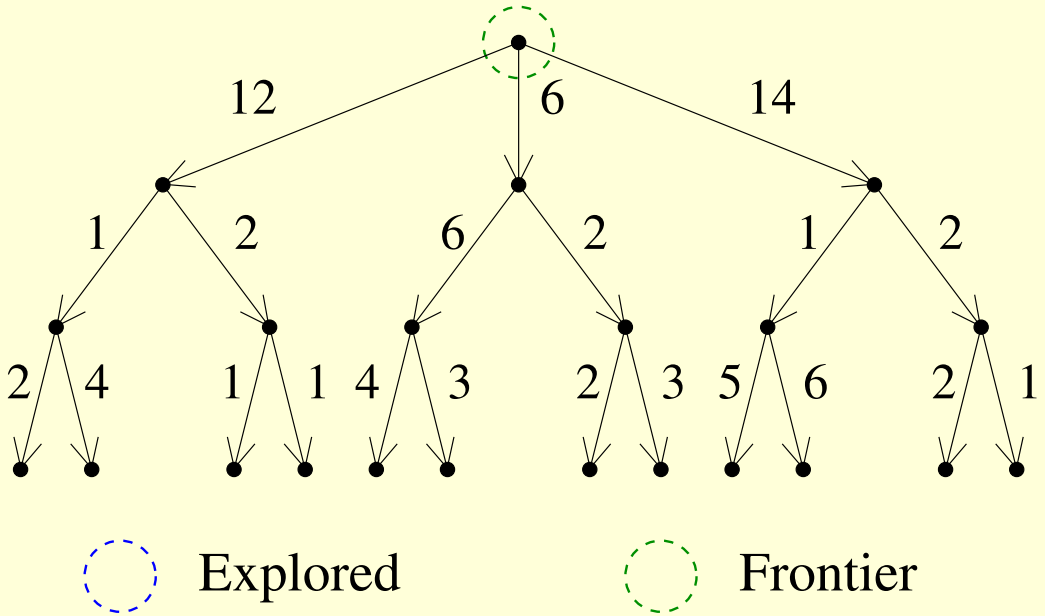


How did we decide which frontier nodes to expand?

Depth-first Search (DFS)

Expand frontier node with longest path from start state.



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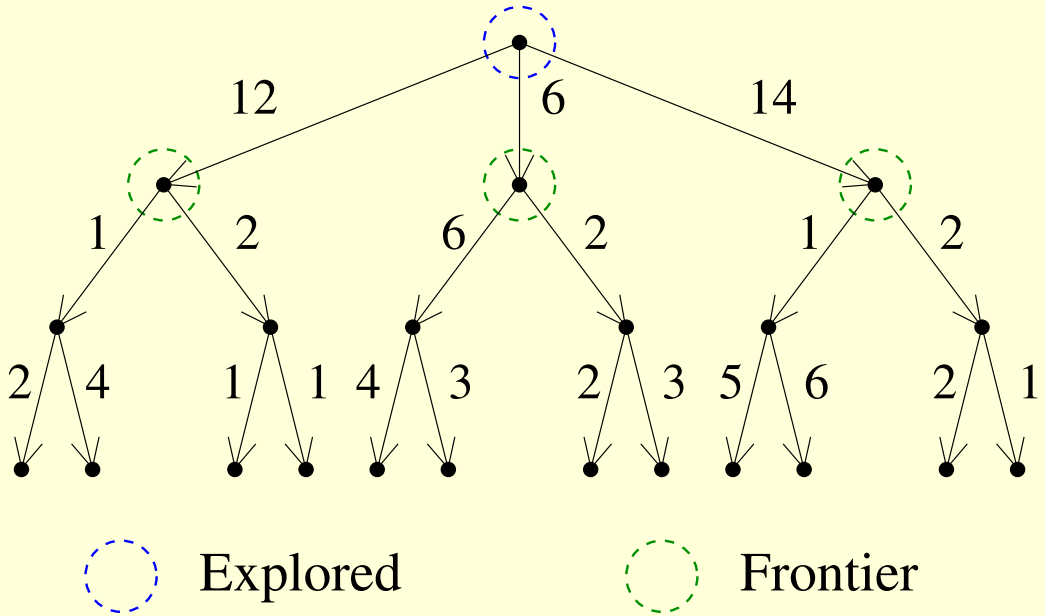

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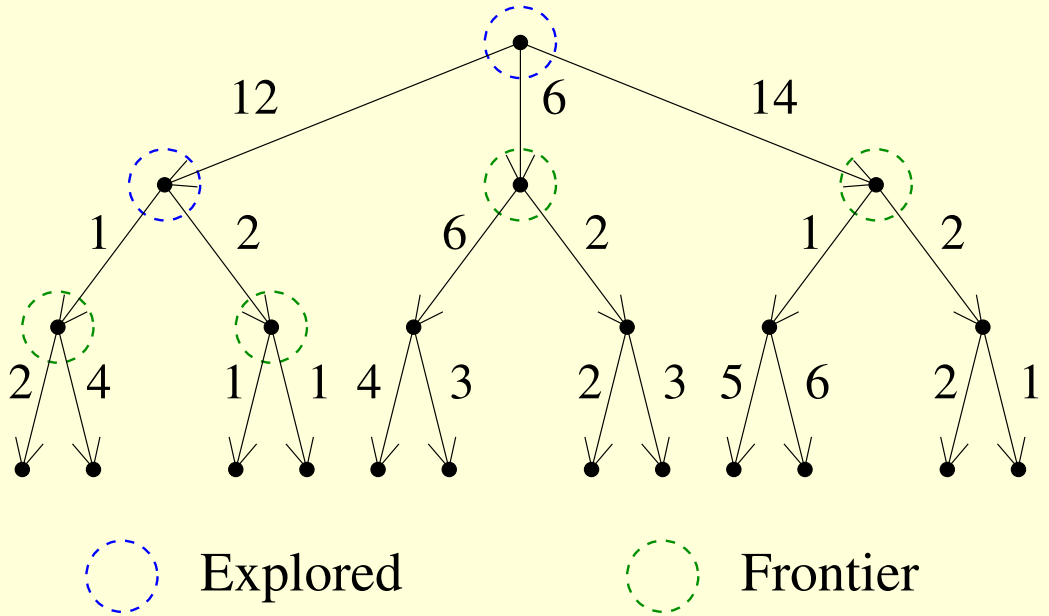

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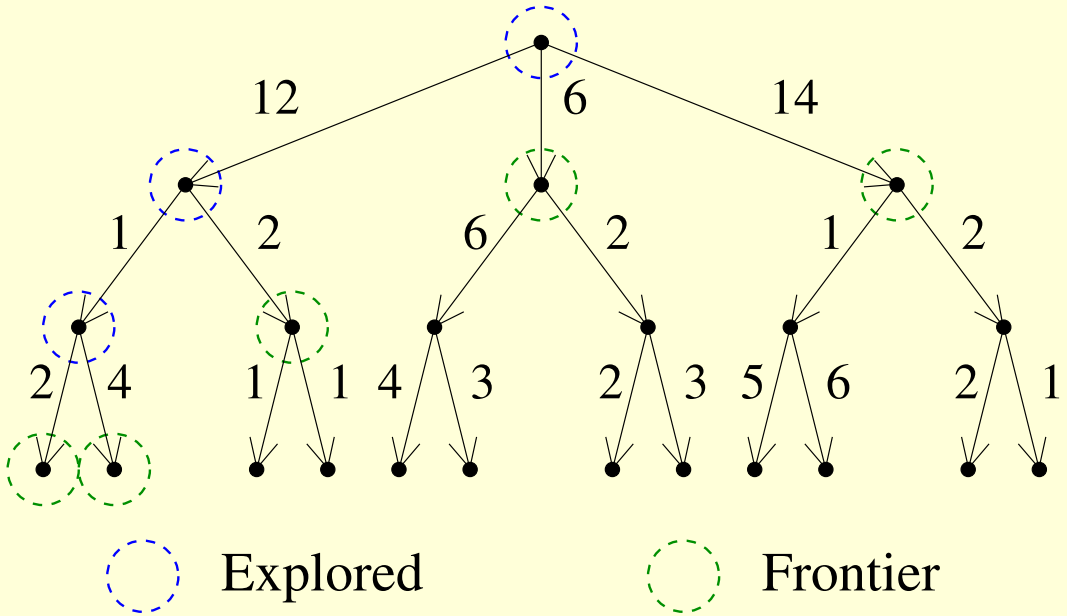

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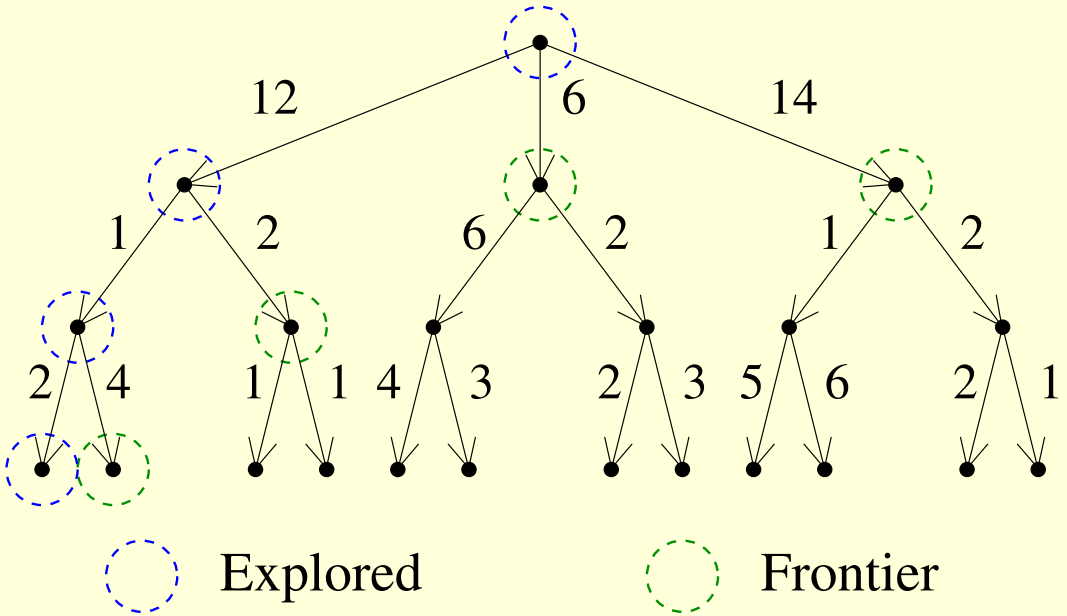

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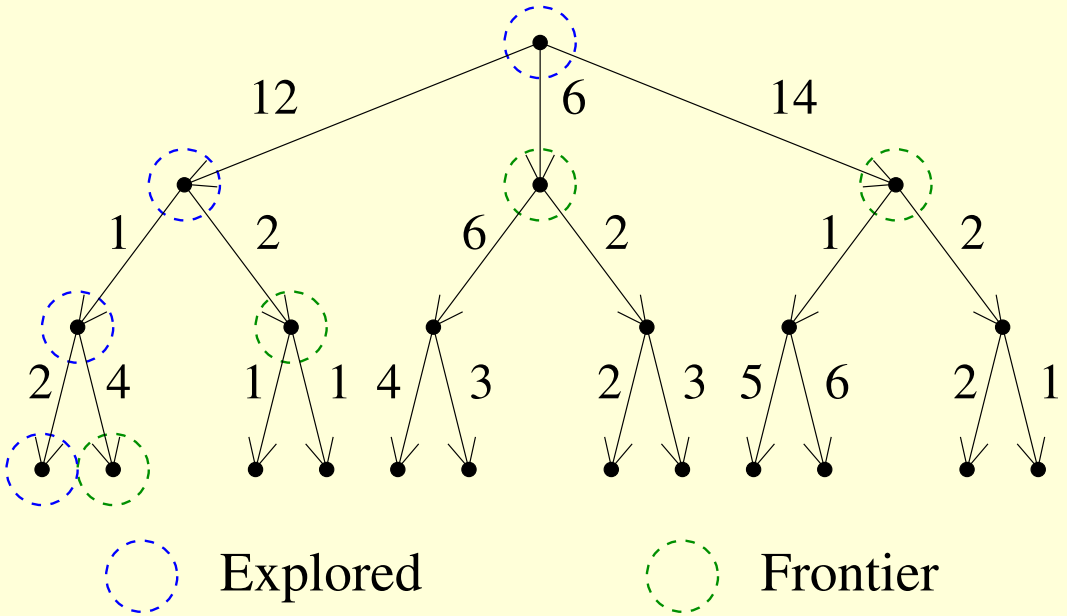

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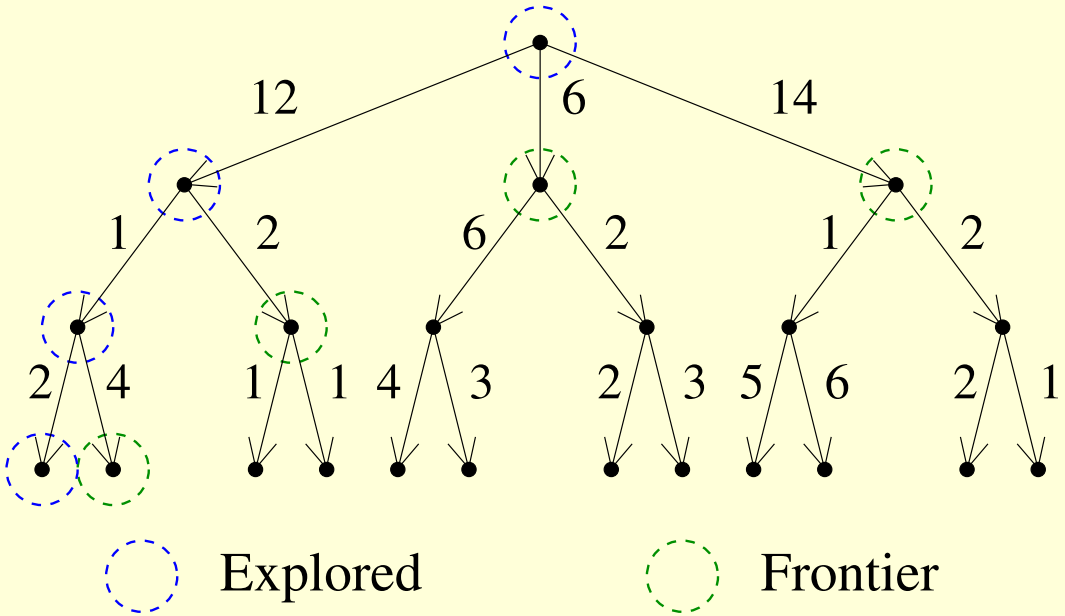

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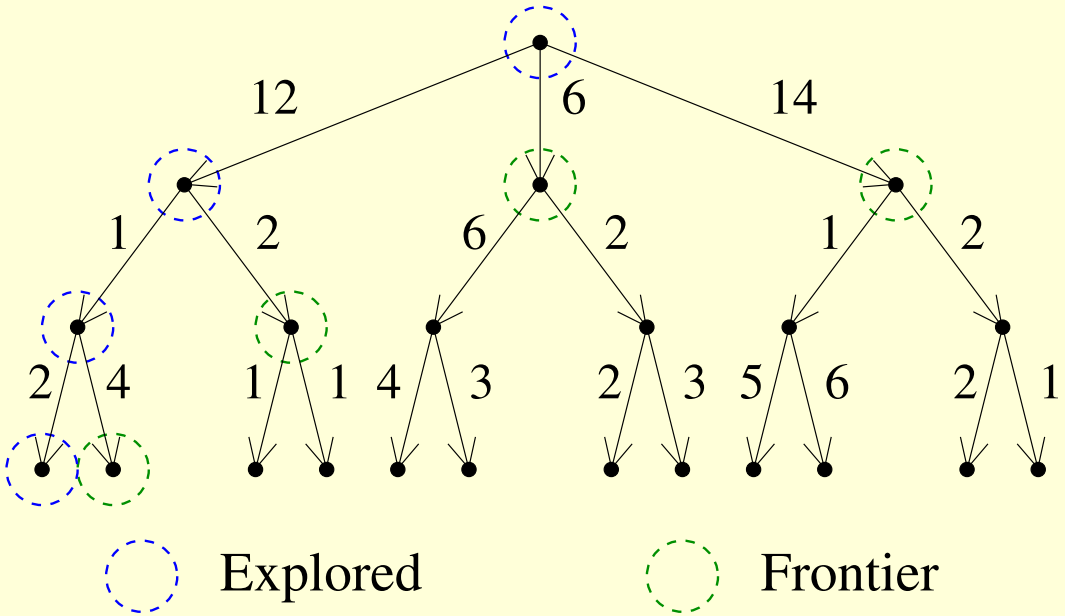

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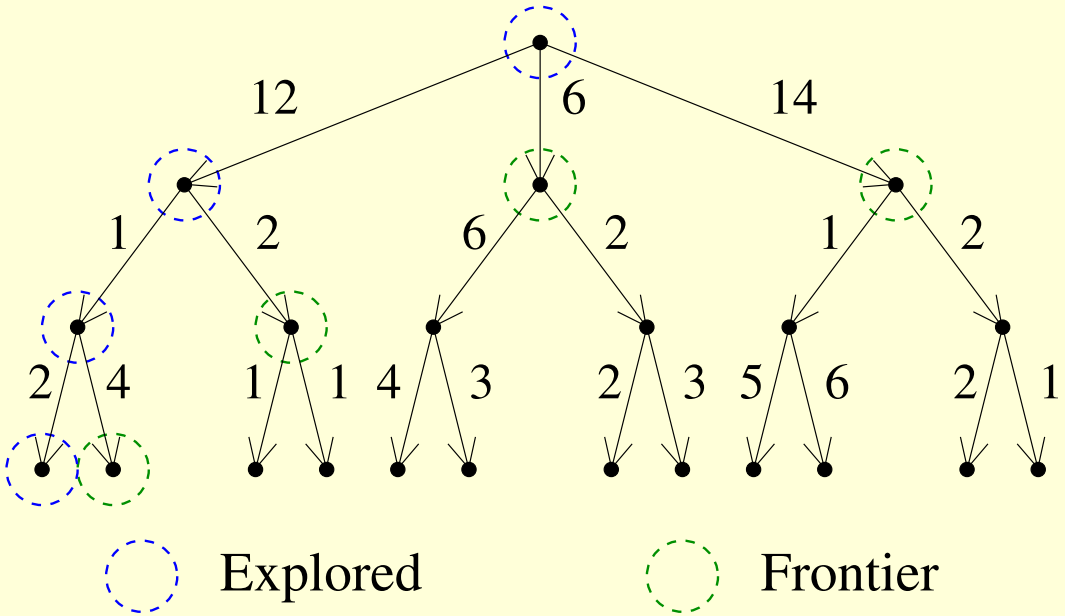

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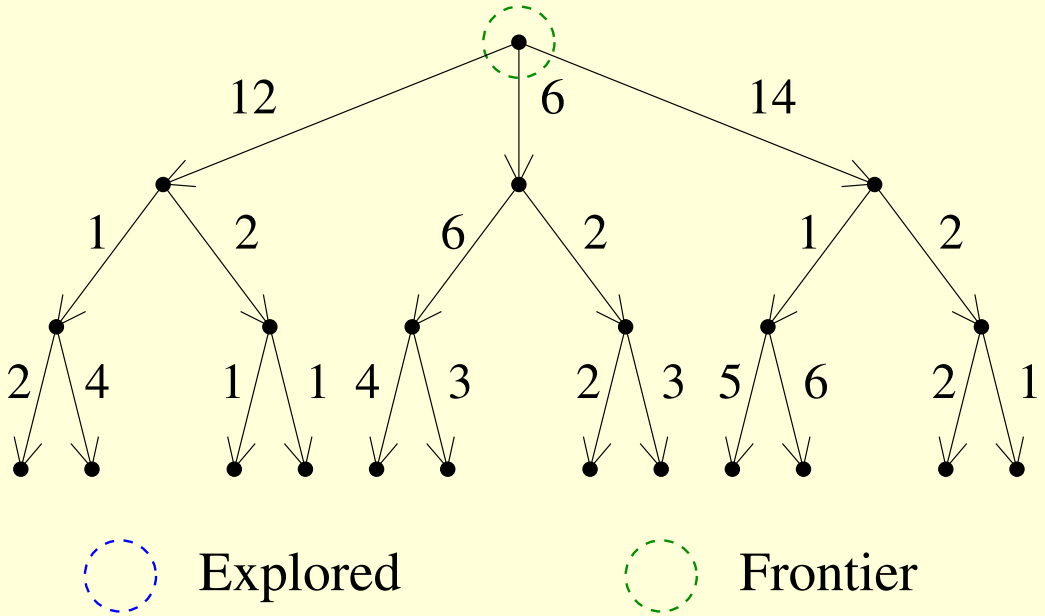

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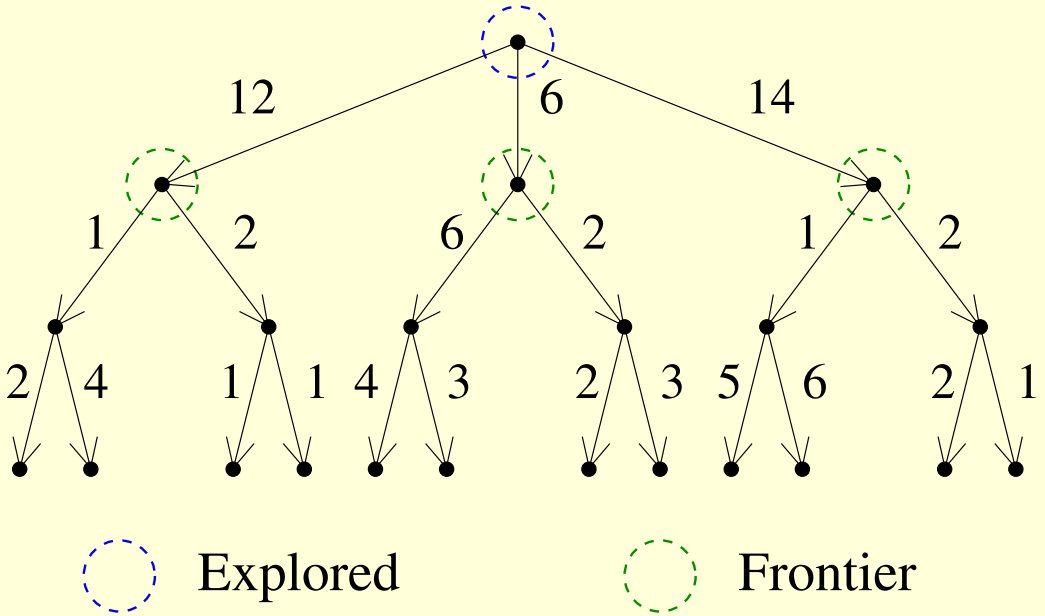

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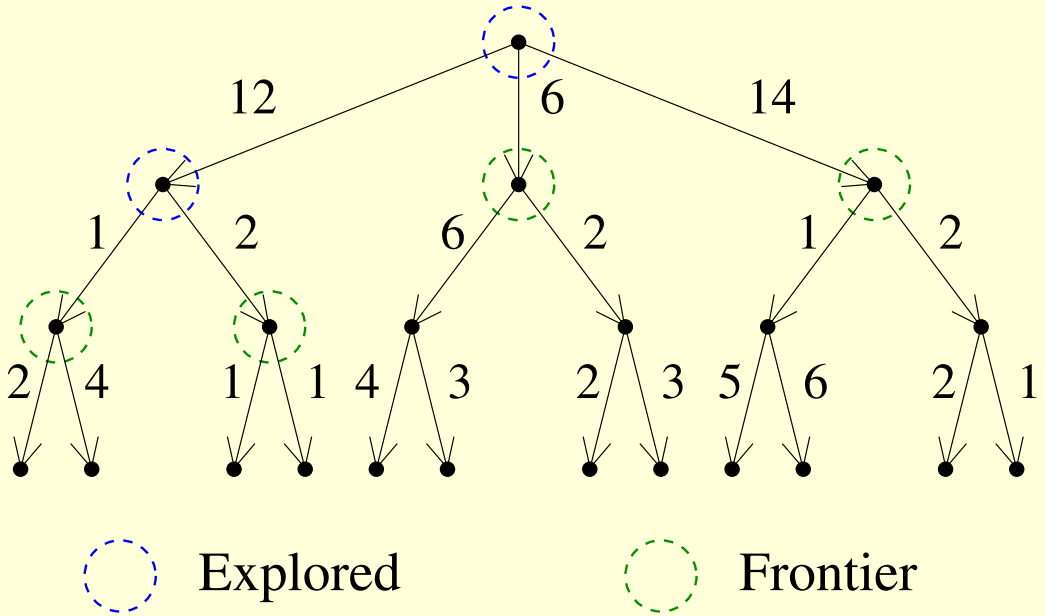

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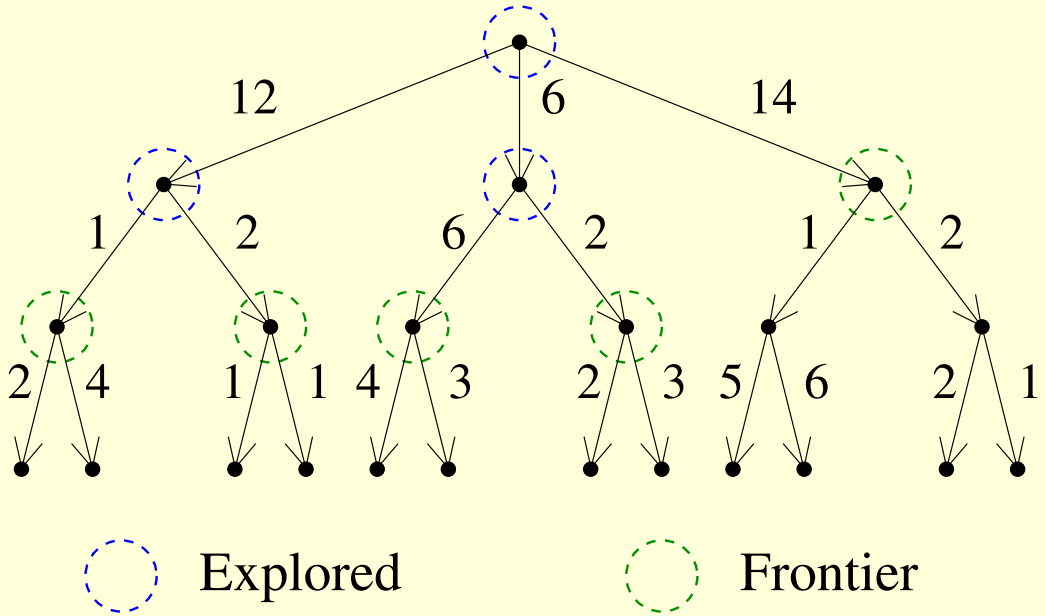

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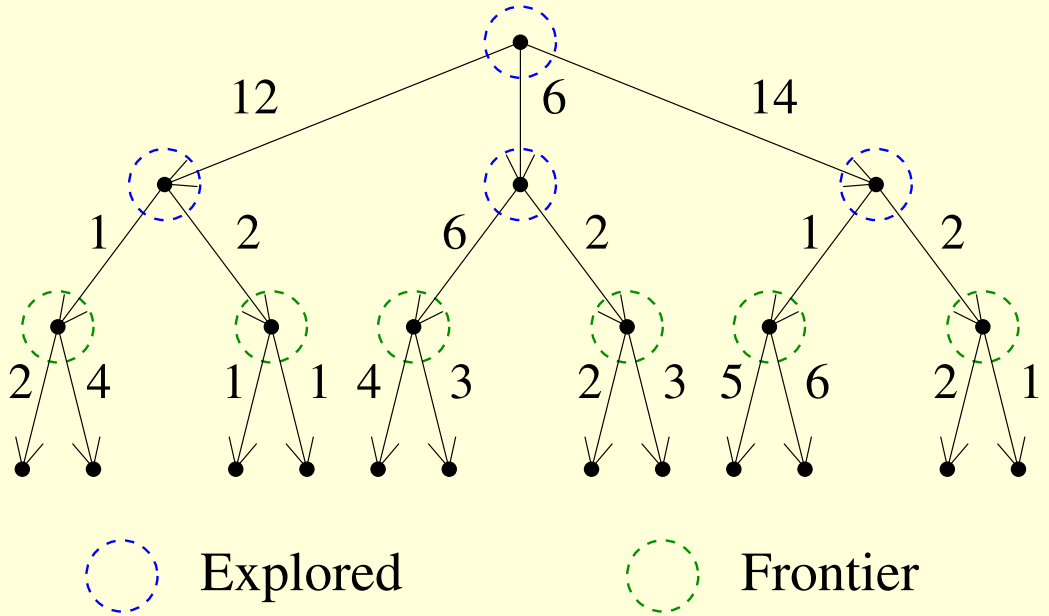
Expand frontier node with shortest path from start state.



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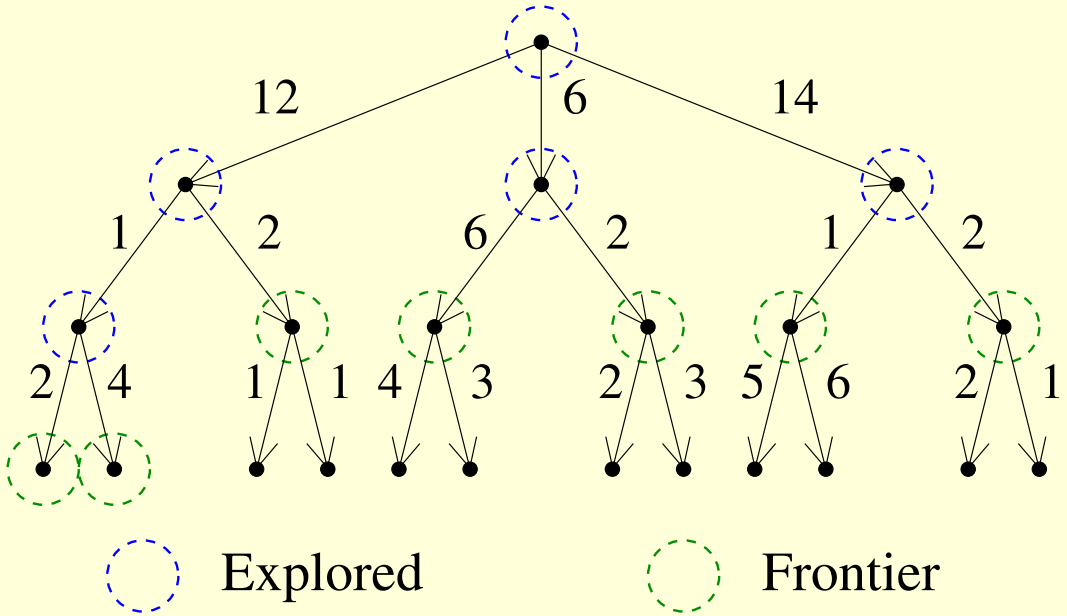

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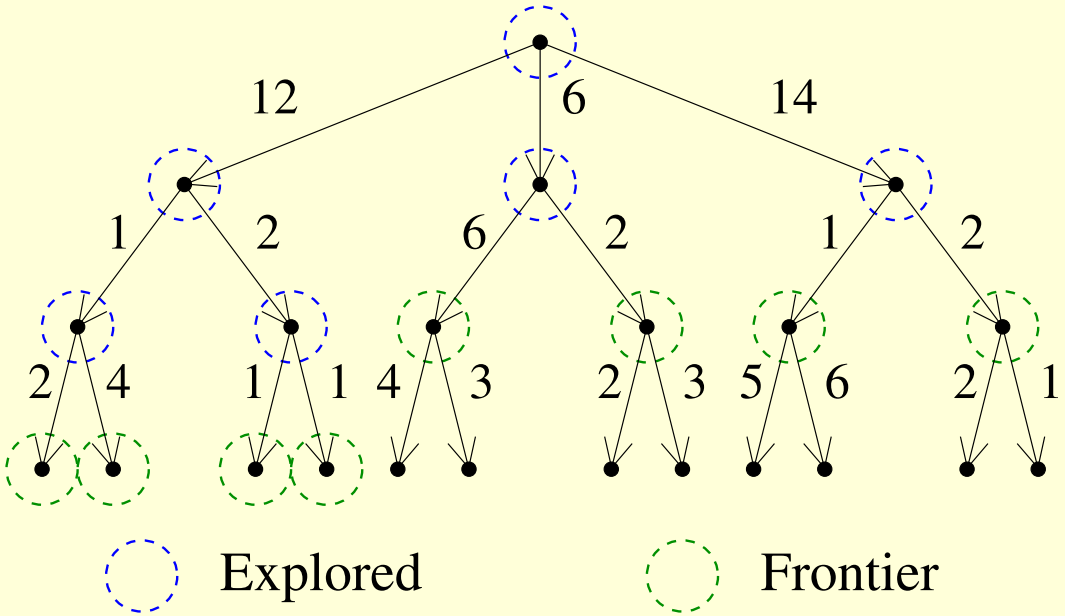

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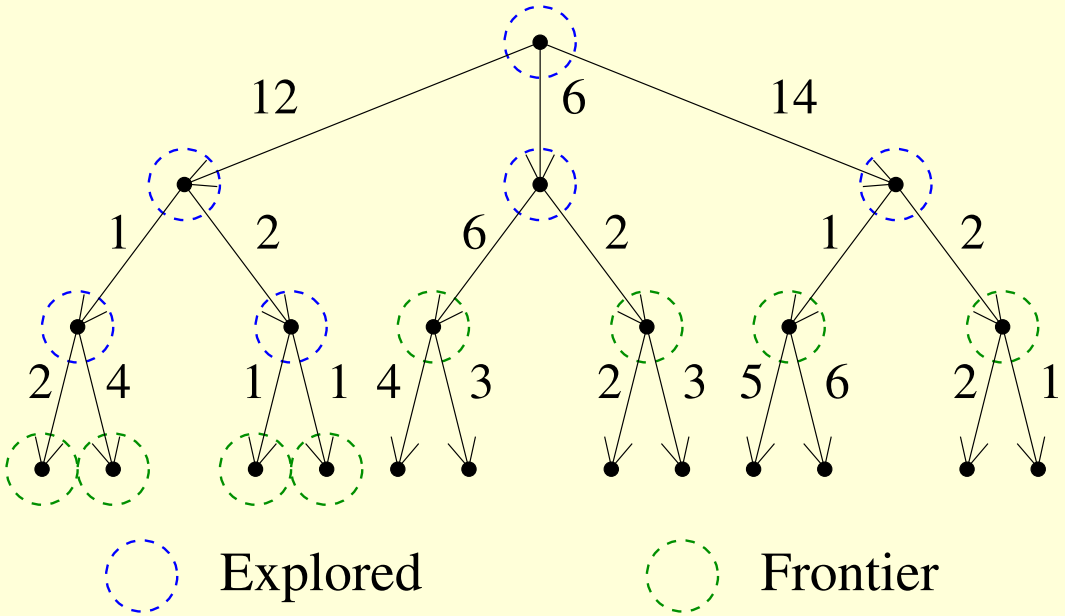

- Frontier treated like a queue (FIFO).

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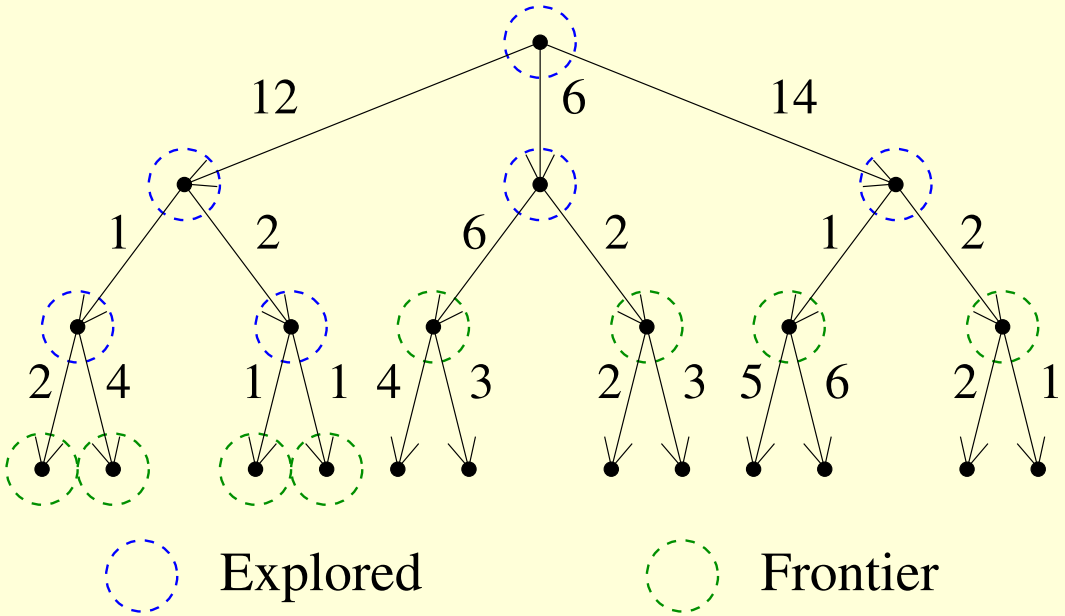

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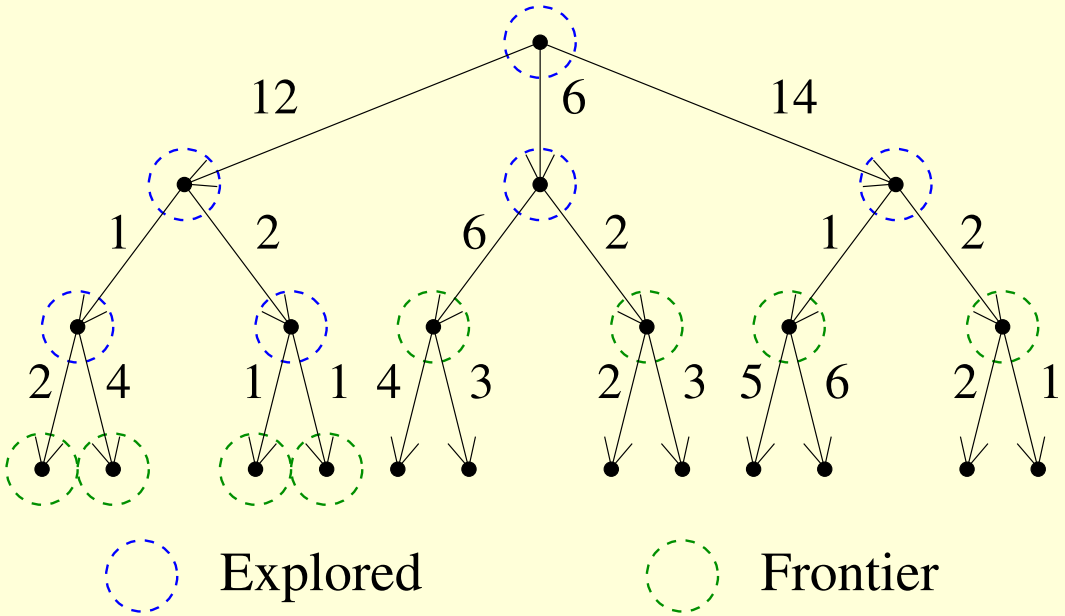

- Frontier treated like a queue (FIFO).
- Guaranteed to terminate if search depth is finite.
- Memory requirement $O(b^d)$.

Lowest-cost-first Search (LCFS)

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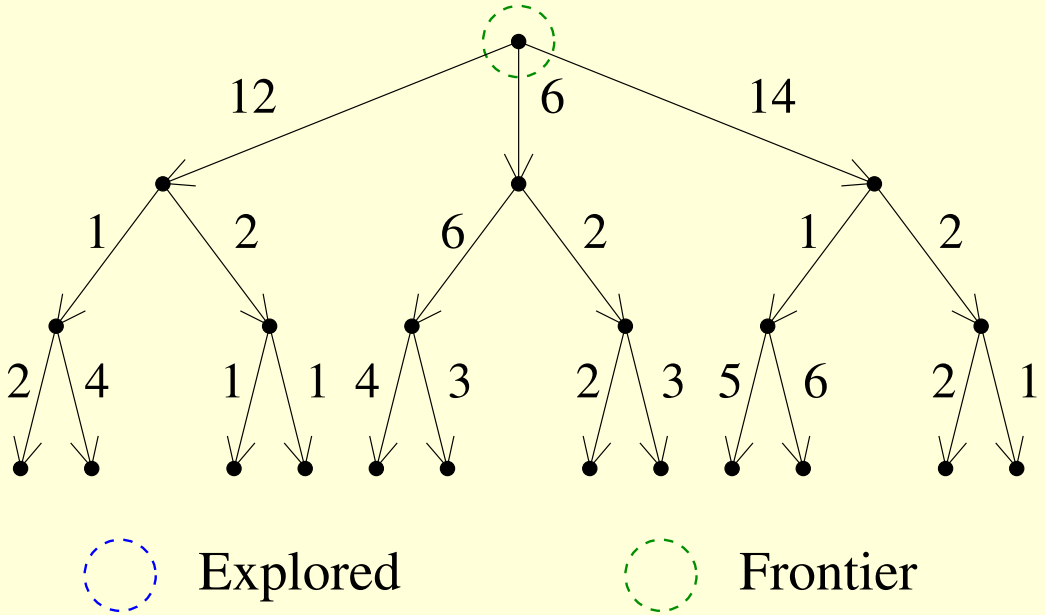

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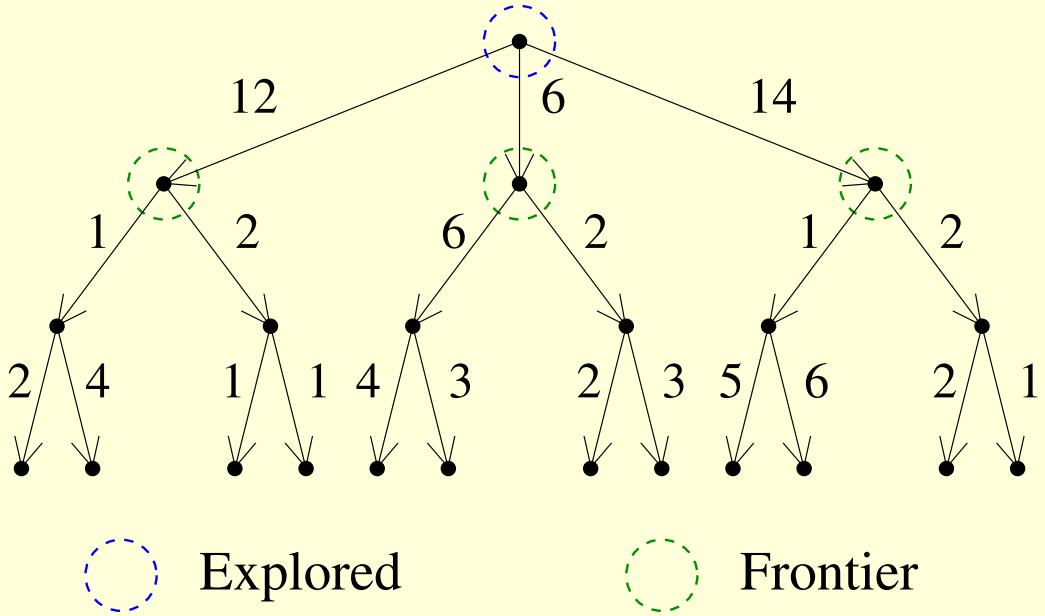

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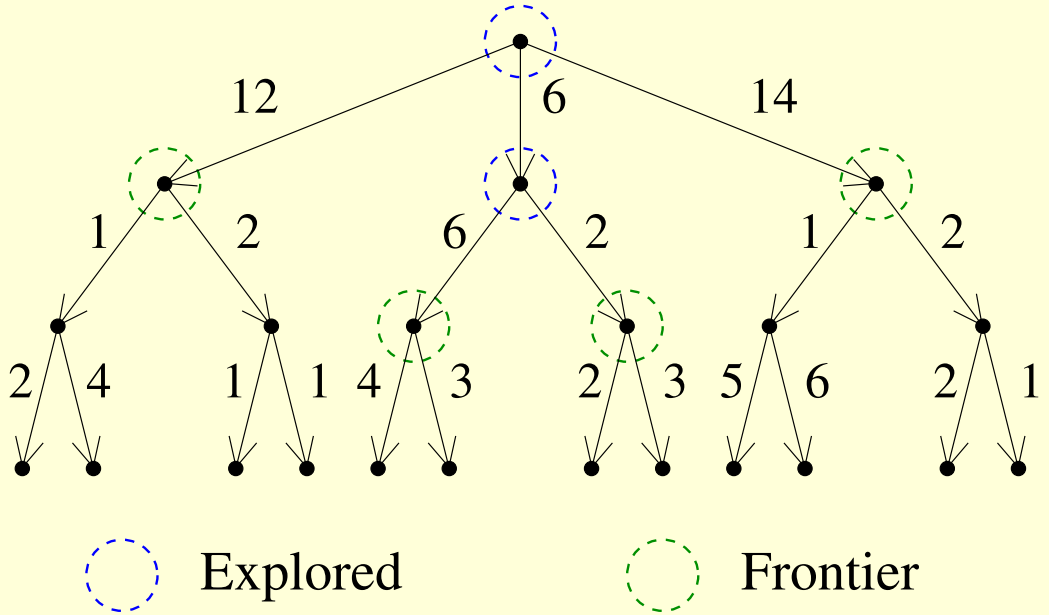

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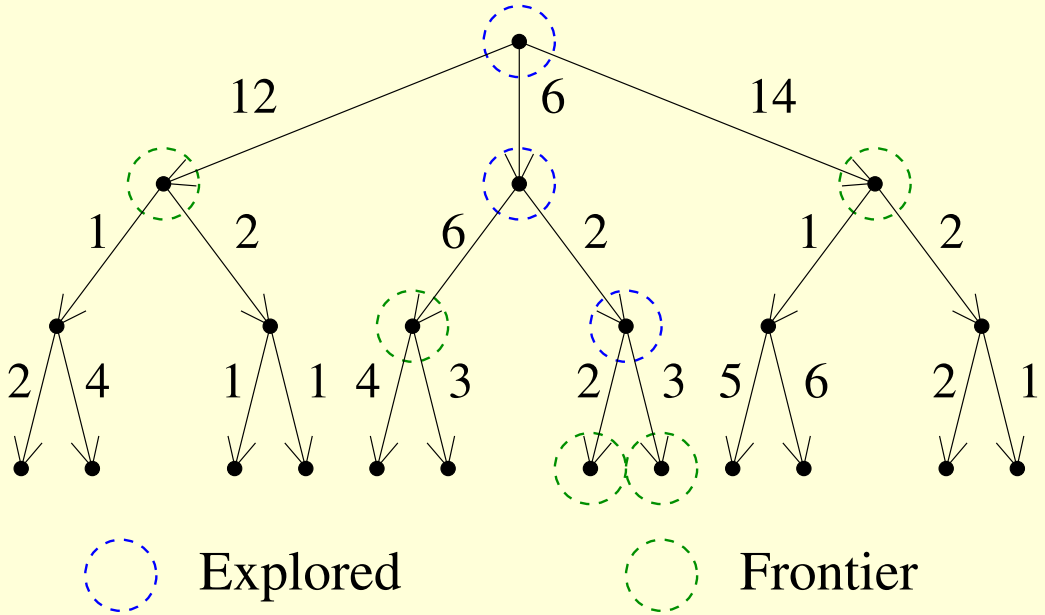

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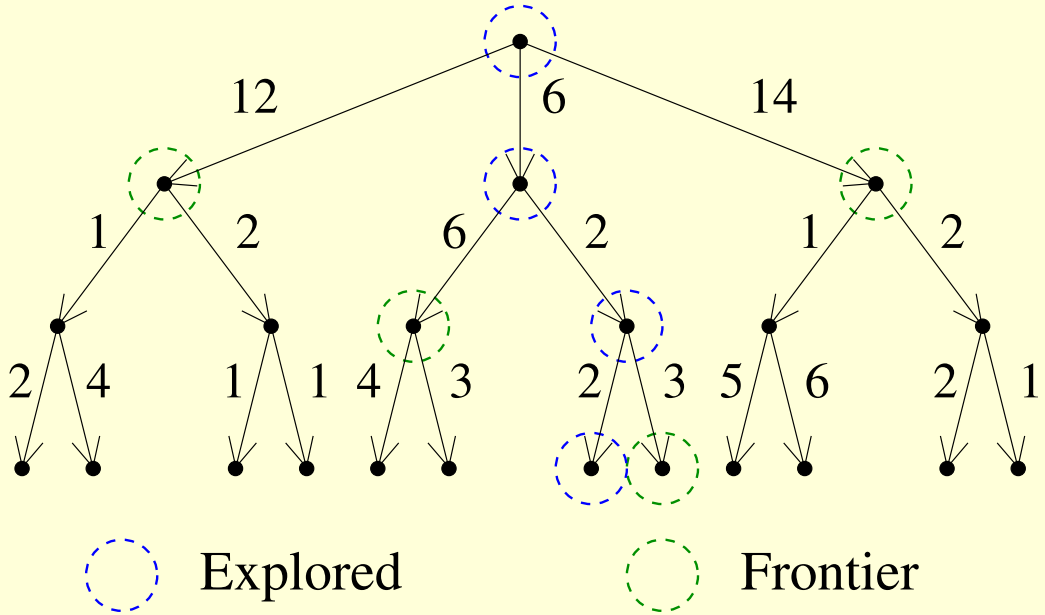

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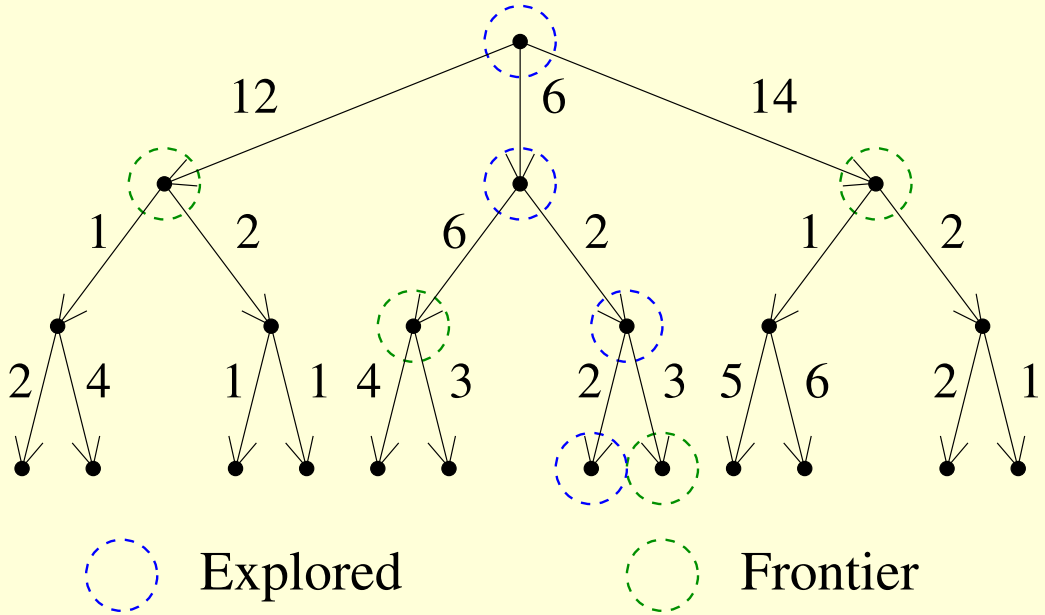

- Frontier treated like a priority queue (priority = path-cost from start state).

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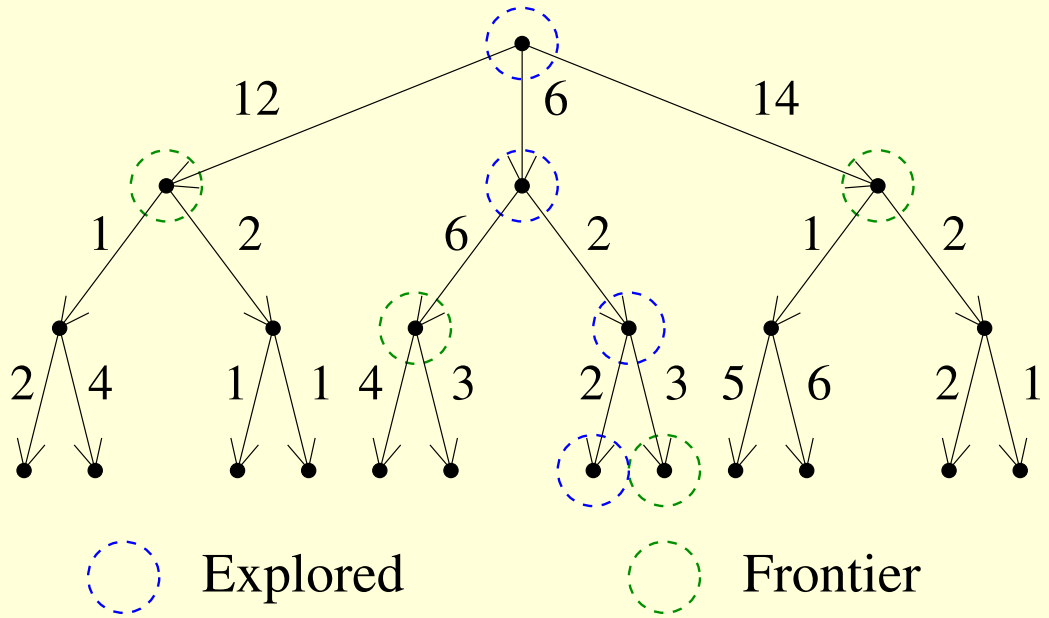

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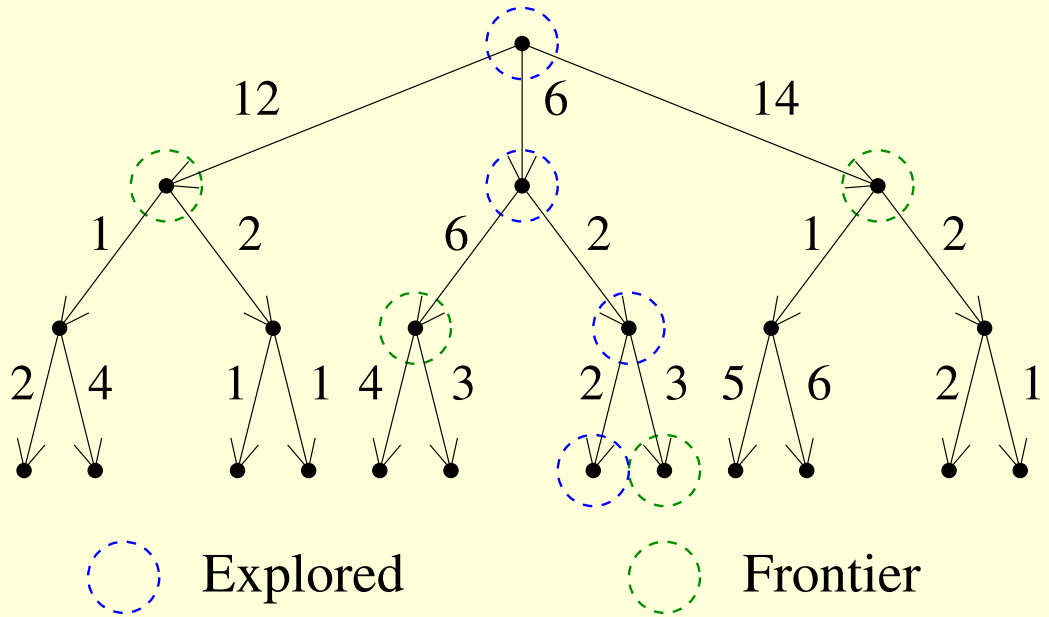

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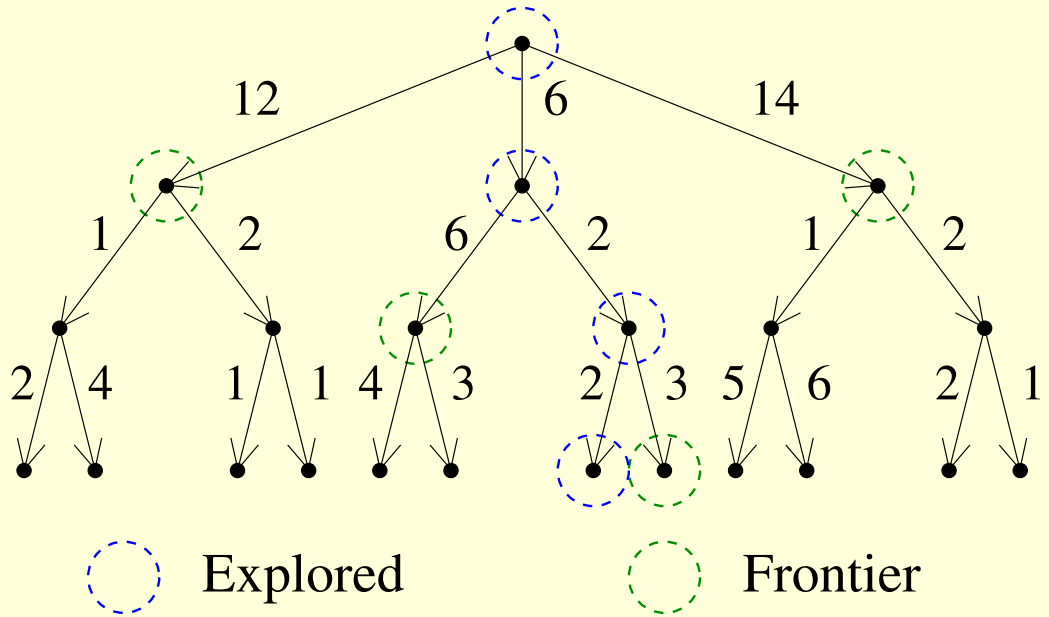

- Frontier treated like a priority queue (priority = path-cost from start state).
- Guaranteed to terminate if search depth is finite and each cost exceeds $\epsilon > 0$.
- Memory requirement depends heavily on instance.
- For node n, denote path-cost from start state $g(n)$. Will need it shortly.

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  You're right. DFS, BFS, LCFS are not considered AI. What's coming up next is!

- Is LCFS related to Dijkstra's shortest path algorithm?
  Yes. It's the same algorithm!

References

- Sections 3.1, 3.2, 3.3, 3.4, 3.5, **Artificial Intelligence: Foundations of Computational Agents**, David Poole and Alan Mackworth, Cambridge University Press, 2010. Available on-line at `https://artint.info/html/ArtInt.html`.