PhD Admissions Written Test (Basic)

May 11, 2016 (Wednesday), 9:00 am-11:00 noon (Duration: 2 hours)

Important Instructions

- The paper consists of 31 "fill in the blank(s)" type questions with a total of 36 blanks to fill. All questions are compulsory.
- Each question may have a single blank or multiple blanks. The first blank will be represented by _____(i)_____ , second (if any) by _____(ii)_____ and so on.
- Each blank carries 1 mark. Answer given must be absolutely correct to get 1 mark. Wrong or partially correct answers will get 0 (No negative marks).
- **The answers for the blanks SHOULD ONLY be filled in the answer sheets provided. For example, please write your answer for the second blank in the 14^{th} question against '14(ii)' in the answer sheet. Please do NOT write your answers in the question paper. Answers written in question papers will NOT be evaluated.**
- For rough work you may use the question paper or the rough sheet provided.
- Before leaving the exam hall you will need to turn-in your answer paper, your question paper and all rough sheets used back to the invigilators.
- You cannot carry any electronic gadgets like cellphones, calculators etc. In case you have them please deposit them with the invigilators and remember to collect them back before leaving the exam hall. The invigilators are not responsible for loss of any items.
- Please do not ask for any further clarifications in the questions. While the exam is in progress avoid moving out of the exam hall for any reason.
- Please do not use any unfair means like copying, consulting etc. during the exam. Students employing unfair means will be disqualified from the examination and the admissions process. The invigilator's decision is final in such cases.
- **Cutoff for Interview: Candidates will be shortlisted for interviews in the General Category only if they score 9 marks or more AND they are in the top 36 ranks. Other categories will be as per Institute norms based on this.**

Questions (Do not write answers here. Use Answer Sheet.)

1. The number of distinct rearrangements of the 6 letters in the name `RAJAJI` is _____(i)_____ .

2. Consider a coin that can move in at most two ways when it is placed on any square of the usual (8 x 8) chessboard (64 squares). From its current square it can move either to the adjacent square above (if it is not in the top most row) or to the adjacent square on the right (if it is not in the rightmost column). The number of distinct paths in which the coin can move from the lower left corner square to to the top right corner square is _____(i)_____ .

3. Let $S(n)$ be the sum of the digits of positive number n written in decimal notation. For example, $S(725) \; = \; = 7 + 2 + 5 \; = \; 14$. Then

$$\sum_{n=0}^{999} S(n) \; = \; \underline{\qquad (i) \qquad}$$

.

4. The co-efficient of x^3y^5 in the expansion of $(1 + xy + y^2)^n$ is _____(i)_____ .

5. Given a vector $\boldsymbol{x} \in \mathbb{R}^n$, we define $\|\boldsymbol{x}\|_1 \triangleq \sum_{i=1}^n |x_i|$ and $\|\boldsymbol{x}\|_2 \triangleq \sqrt{\sum_{i=1}^n x_i^2}$. Now consider the following statements:

   (a) $\|\boldsymbol{x}\|_2 \leq \|\boldsymbol{x}\|_1$ (b) $\|\boldsymbol{x}\|_1 \leq \|\boldsymbol{x}\|_2$ (c) $\|\boldsymbol{x}\|_1 \leq \sqrt{n}\|\boldsymbol{x}\|_2$

   (d) $\|\boldsymbol{x}\|_2 \leq \sqrt{n}\|\boldsymbol{x}\|_1$ (e) None of these statements are true, as it all depends upon the values in $\boldsymbol{x}$.

   The indices of all true (and no false) statements are _____(i)_____ . (For example, if you think statements (a) and (b) are true and the rest false, write down '(a),(b)' as your answer).

6. Consider the matrix $\boldsymbol{A} = \begin{pmatrix} 1 & 1 & 7 \\ 2 & -4 & 14 \\ 3 & 1 & 21 \end{pmatrix}$ and let $\boldsymbol{v_A} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ be vector not containing all 0s.

   The product $\boldsymbol{A}\ \boldsymbol{v_A}$ is $\mathbf{0}$ (all 0s) if we set x, y, z as _____(i)_____ . . If you feel no such vector can exist, write 'such a vector cannot exist'.

7. Consider seven arbitrary lines drawn on a plane. The maximum number of regions into which the plane can be divided by these seven lines is _____(i)_____ .

8. Consider all permutations of the 16 numbers from 1 to 16 which satisfy the property that every number is placed such that it is either bigger than ALL numbers preceding it or it is smaller than ALL numbers preceding it. The number of such permutations is _____(i)_____ .

9. A cube has its vertices at $A(0,0,0)$, $B(2,0,0)$, $C(2,2,0)$, $D(0,2,0)$, $E(0,0,2)$, $F(2,0,2)$, $G(2,2,2)$ and $H(0,2,2)$. (Assume that for the points, the first coordinate is the x coordinate, the second coordinate is the y coordinate, and the third coordinate is the z coordinate.) This cube is transformed with the matrix below

$$T = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

   In the transformed cube, the coordinates of vertex G is _____(i)_____ .

10. We wish to initialize integer array A of size 100 such that $A[i]$ contains i^3 using the code below. For example, $A[3]$ should contain 27 after the code is executed. The blank to be filled in below is _____(i)_____ .

```
A[0] = 0, n = 1, s = 1, t = 6;
while (n != 100){
   A[n] = A[n-1] + s;
   s = s + t;   t = __(i)___;  n = n+1;}
```

11. The following program prints _____(i)_____ .

```
void g(char *s, int len) {
      if(len == 0) return;
      char temp = s[0]; s[0] = s[len-1]; s[len-1] = temp;
      g(s+1, len-2);}

int main() {char s[] = "hello";   g(s, strlen(s));  printf("%s", s);}
```

12. Let A be an integer array of size N. The function max returns the maximum of its two arguments. The two code fragments shown below are equivalent (i.e. n, r, s have the same value at the end) if we fill the blank with _____(i)_____ .

```
n = 0, r = 0, s = 0;
while (n != N) { s =  max(s + A[n], 0);
                 r = max(r, s);
                 n = n+1;}
```

```
n = -1, r = 0, s = 0;
while (n != N){ r = max(r, s);
                 n = n+1;
                if(____(i)____)
                  s =  max(s + A[n], 0);}
```

13. When called with $x = 10! + 9!$ and $y = 8!$ the function f below returns _____(i)_____ .

```
int f(x,y) { int count=0;
             while(x > y) { x=x-y; count++; }
             while(y > x) { y=y-x; count++; }
             return count;}
```

14. Suppose G is a complete undirected graph with 5 vertices (K_5) whose 10 edges are given distinct edge weights from 1...10. Let $MST(G)$ be a minimum weight spanning tree of G. Then $MST(G)$ **must** contain edges with the following weights _____(i)_____ (fill ALL such weights). $MST(G)$ **cannot** contain edges with the following weights _____(ii)_____ (fill ALL such weights).

15. Let A be an array containing n integers. It is required to find 3 indices i, j, k such that $i < j < k$ and either $A[i] \leq A[j] \leq A[k]$ or $A[i] \geq A[j] \geq A[k]$, if such indices exist. The asymptotic time complexity of the fastest algorithm for this problem, assuming the array is already available, is $\Theta($_____(i)_____$)$.

16. Let $a_0, a_1, \ldots, a_{n-1}$ be a sequence of integers. Let $L[i, j]$ denote the length of the longest subsequence of $a_i, \ldots, a_j$ that is a palindrome. Note that a palindrome is a sequence that is equal to its reverse,

and a subsequence of a sequence need not contain consecutive elements of the sequence. Complete the following recurrence relation for computing $L[i, j]$.

$$
\begin{aligned}
L[i,j] &= 0 \text{ if } j < i \\
L[i,j] &= 1 \text{ if } j = i \\
L[i,j] &= \underline{\quad (i) \quad} \text{ if } j > i \text{ and } a_j = a_i \\
L[i,j] &= \underline{\quad (ii) \quad} \text{ if } j > i \text{ and } a_j \neq a_i
\end{aligned}
$$

17. Let A be a sorted array of distinct integers of length n. Design an algorithm to find an index i such that $A[i] = i$ if such an index exists. If there are more than one such indices, you may output any one. If no such index exists then the algorithm outputs -1. The asymptotic time complexity of the fastest algorithm for this problem, assuming the array is already available, is $\Theta(\underline{\quad (i) \quad})$.

18. Let $x_1, x_2, ...x_8$ be 8 propositional variables. Let $\cdot$ represent AND connective $\oplus$ represent the Exclusive-or connective.

    The number of satisfying assignments of the formula $x_1 \oplus x_2 \oplus ...x_8$ is $\underline{\quad (i) \quad}$.

    The number of satisfying assignments of the formula $(x_1 \cdot x_2) \oplus (x_3 \cdot x_4) ... \oplus (x_7 \cdot x_8)$ is $\underline{\quad (ii) \quad}$.

19. The length of the longest word that can be accepted by any DFA which satisfies the following two properties is $\underline{\quad (i) \quad}$.

    - It has 4 states
    - It does not accept any word of length 8 or 9 or 10 or 11 or 12?

20. For any language L, and for any $k \geq 0$, let L^k denote the language obtained by concatenating any k-strings from L (note that $L^0 = \epsilon$). Suppose L_1 is a context-free language and R is a regular language over the same alphabet Σ of size at least 3. The smallest value of k for which $(R \setminus L_1)^k \cup (L_1 \setminus R)$ is not necessarily context-free is $\underline{\quad (i) \quad}$.

    Your answer should be a non-negative integer or ∞ if there is no such k, i.e., the language is always context-free for all k.

21. Consider a memory system with a cache access time of 10ns and a memory access time of 110ns. Assume the memory access time includes the time to check the cache. If the effective access time is 10% greater than the cache access time, the hit ratio H is $\underline{\quad (i) \quad}$.

22. Out of the following actions by a process running on a system the actions $\underline{\quad (i) \quad}$ (fill ALL possible correct choices) will *always* result in a context switching out of the running process.

    (a) Servicing an interrupt. (b) A blocking system call.

    (c) A non-blocking system call. (d) The system call *exit*, to terminate the current process.

23. Out of the following schedulers starvation can be caused by $\underline{\quad (i) \quad}$ (fill ALL possible correct choices).

    (a) Priority based Scheduling (b) round robin scheduling with varying time-slice

    (c) First come first served scheduling (d) Dynamic shortest job first scheduling.

24. An OS employs the Bankers algorithm to control allocation of 7 instances of a resource. Processes P1 , P2 , and P3 need a maximum of 7, 3 and 5 instances of the resource, respectively. The instances currently allocated to processes P1 , P2 and P3 are 2, 1, 1, respectively. The OS can now safely allocate _____(i)_____ instances to P1.

25. A time sharing system uses a round-robin scheduling policy with a time slice of 55 msec. Its scheduling overhead is 3 msec. Operation of 10 identical processes is initiated in this system at time 0 seconds. The nature of each of these processes is as follows: It has a loop that executes 100 times. In each iteration, it computes using the CPU for 50 msec and immediately following that it initiates an I/O operation that lasts for 200 msec. The third process initiates the I/O operation in its 5th iteration at time t msec. It will initiate the I/O operation in its 6th iteration at _____(i)_____ msec.

26. A 1 Mbyte file needs to be transferred from computer A to computer B over a 1 Mbps link with a one way propogation delay of 10ms. The total time taken to transfer the file (in sec) is _____(i)_____ .

27. Compute the reliability of a path between two hosts A and B in the two scenarios below.

    A and B are connected parallelly to each other by two separate wires. Each wire has a reliability of 0.99. In this case, the reliability is _____(i)_____ .

    A and B are connected via an 100% reliable intermediate router R (i.e. over 2 hops). A-R and R-B are connected by a single wire, each of reliability 0.99. Assume the router is 100% reliable. In this case, the reliability is _____(ii)_____ .

28. Assume a TCP connection uses packet size 1 KB And it uses slow start to grow the window. The number of RTTs it takes for the window size to exceed 4 MB is _____(i)_____ .

29. Consider the following on-line learning method to estimate the expected value of a real-valued random variable X. We begin with an initial estimate μ_0, and then for $t = 1, 2, \ldots,$

    - Obtain x_t as an independent identical distribution (i.i.d) sample of X, and
    - Revise our estimate as $\mu_t \leftarrow (1 - \alpha_t)\mu_{t-1} + \alpha_t x_t$.

    α_t is the learning rate at step t. Assume X is normally distributed with mean μ and variance σ^2. If we choose $\alpha_t = \frac{1}{t}$, then $\mathbb{E}[\mu_t]$ is _____(i)_____ .

30. Consider $x(i)$ for $i = 1, 2, 3, 4, 5$ to be the sequence $0, 1, 1, 1, 0$. The output $y(i)$ for $i = 3, 4, 5$ on passing x

    - through a median filter of size 3 is _____(i)_____
    - through a normalized box filter of size 3 is _____(ii)_____

31. Suppose you are given relations $r(A, B)$ and $s(A, C)$. Suppose that r has 10000 tuples, and s has 5000 tuples. Suppose attribute $r.A$ has 1001 distinct values, and $s.A$ also has 1001 distinct values. The maximum possible size of the join result is _____(i)_____ .