CS336 Computer-Aided Geometric Design

Problem Sheet

1. Compute the space of tangent vectors for the point $[1, 1, 1]$ on the ellipsoid given by the equation:
$$9X^2 + 16Y^2 + 144Z^2 = 169$$

2. Let $f(u, v) = u^2 + u + 2v$ and $g(u, v) = v^2 + 2u + v$. Starting from the initial guess of $(1, 1)$, use the Newton-Raphson technique to compute the next two iterations.

3. Formulate a procedure for creating surfaces of revolution.

4. Consider the situation of a drafted extrude where the profile has sharp corners. Describe the geometry/topology near these sharp corners.

5. Prove that a suitable cross section of a constant-radius (say r) blend surface is actually circular of radius r. Is this also a radius of curvature for the blend surface?

6. Given two pints on a unit sphere, derive the parametrization of the great circle passing through it.

7. Let S be the unit cube and let e_1, e_2, e_3 be the edges incident at a vertex. Suppose e_1, e_2 are blended first with radius r and e_3 subsequently with radius R. Describe the geometry of all the surfaces created. Cover the cases when $r < R$ and $r > R$ separately. Describe what happens when this sequence is reversed.

8. Assume that a curve $C(t) = (x(t), y(t), z(t))$ is available in terms of its evaluators. Write pseudo-code to compute its curvature. Do the same for a surface $S(u, v)$.

9. Let P be a polygonal closed curve which may cross itself. Define the winding number for such curves and outline a procedure to compute this.

10. Let S be a solid and H be a hyperplane which cuts the solid into two parts S_1, S_2. If $S \cap H$ is simply connected, show that $genus(S) = genus(S_1) + genus(S_2)$.