

Are you ready for CS 782?

CS 782 is a course meant for students with background in theoretical CS and mathematics, especially basics of groups, rings and fields.

Here is a list of questions which will help you decide if your background is right for CS 782.

Linear Algebra

1. Let V be a vector space of dimension m over $\mathbb{C}$ and $W \subseteq V$ be a subspace. Let B be a basis for W . Show that there is a basis of V which extends W .
2. Recall the definition of rank of a matrix $X \in \mathbb{C}^{m \times n}$, the space of $m \times n$ matrices. Let $GL_m(\mathbb{C})$ and $GL_n(\mathbb{C})$ be the general linear group. Show that $\text{rank}(AXB) = \text{rank}(X)$, where $A \in GL_m(\mathbb{C})$ and $B \in GL_n(\mathbb{C})$.
3. Let P^d be the $d+1$ -dimensional space of all polynomials in the indeterminant x with complex coefficients of degree less than or equal to d . Let $X = (x_1, \dots, x_n)$ be a collection of distinct points and consider the map $\phi_X : P_d \rightarrow \mathbb{C}^n$ defined as follows $\phi_X(p) = (p(x_1), \dots, p(x_n))$. Compute the kernel and image of the above map. Consider all cases. What is the speciality of the Lagrange polynomials for the data X ?
4. Use the Gauss elimination algorithm to show that every invertible $m \times m$ matrix A has a factorization $A = LPU$ where L is lower triangular, U is upper triangular and P is a permutation matrix.
5. Let A be a $p \times m$ -matrix and B be an $r \times m$ matrix. Describe an algorithm to compute $\text{rowspace}(A) \cap \text{rowspace}(B)$.

Basic Algebra

1. Let m, n be integers so that $\gcd(m, n) = 1$. Let a, b be arbitrary. Show that there is a number p in the interval $[1, mn]$ so that $p \cong a \pmod{m}$ and $p \cong b \pmod{n}$.
2. Let $\mathbb{C}[x]$ be the ring of polynomials over the variable x . State the division algorithm and use it to prove that $\mathbb{C}[x]$ is a unique factorization domain (UFD).
3. Let $R = \mathbb{C}[x_1, \dots, x_n]$ be the ring of polynomials in the variables $x_1, \dots, x_n$. Prove that R is a UFD.
4. Let $R = \mathbb{C}[x_1, x_2, x_3]$ and I be the ideal $I = (p, q_a, r_b)$ where $p = x_1^2 + x_2^2 - x_3$ and $q_a = x_3 - a$ and $r_b = x_1 - bx_3$. What is the common zero set of I . Discuss all possibilities.
5. Let R, S be finitely generated $\mathbb{C}$ -algebras. Let $\phi : R \rightarrow S$ be an algebra homomorphism. Show that the kernel is an ideal I in R and there is an injection $\phi' : R/I \rightarrow S$.
6. G be a group and $N \subseteq G$ a normal subgroup. Show that the left cosets $\{gN \mid g \in G\}$ have a group structure which matches the collection of right cosets.
7. Let $G = GL_m(\mathbb{C})$, the space of all invertible matrices. Let A be a diagonalizable matrix. Show that $Z_A = \{B \in G \mid BA = AB\}$ is a subgroup. Compute Z_A . Discuss all cases.

Theoretical Computer Science

1. Recall the proof that any sorting algorithm which uses comparisons will take time $O(n \log n)$.
2. Let $G(U, V, E)$ be a bipartite graph such that $|U| = |V|$. State Hall's condition for the existence of a complete matching. Review the alternating path algorithm to construct a maximum cardinality matching.
3. List one graph problem which is NP-complete. Reduce this problem to the satisfiability problem.