CS101 Computer Programming and Utilization

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1. So far
2. What is sorting
3. Bubble Sort
4. Other Sorts
5. Searching

The story so far ...

- We have seen various control flows.
- We have seen multi-dimensional arrays and the `char` data type.
- We saw the use of functions and calling methods.
- We have seen structs and file input/output

This week...

Sorting and Searching

A Problem

Recall that we have the struct:

```
struct student
{
   char name[6];
   char roll[8];
   int hostel;
}
```

Suppose next that we have a list (array) of students and we wish to

- Insert into that list.
- Delete from that list.
- Check if present in that list.

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Suppose next that we have a list (array) of students and we wish to

- Insert into that list.
- Delete from that list.
- Check if present in that list.

It is then clear that we must store the array in a sorted order.
What does sorting mean?

- Every element of the lis must have a key on which the sorting will be done.
- For two keys k_1 and k_2, we must have that either $k_1 < k_2$, or $k_1 = k_2$ or $k_1 > k_2$. This is called total ordering.
- Sorting then means that arranging the in the order $s_1, \ldots s_n$ so that $k_1 \leq k_2 \leq \ldots \leq k_n$.

For our example, the alphabetical ordering is the required total order.

Sorting

Let us assume, for simplicity, we have a `struct` called `key` and a function which implements the total order:

- `int CompareKey(key k1,k2)`, which returns
  - 1 if $k_1 > k_2$.
  - 0 if $k_1 = k_2$.
  - -1 if $k_1 < k_2$.

Our problem then is to sort an array of `keys`.

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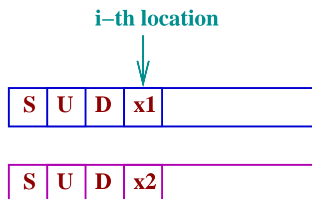
Our problem then is to sort an array of `keys`.

Let us first write this `CompareKey` for `char name[6]`.

```
#include <iostream.h>
int main()
{
  char name1[7],name2[7];
  int i;
  cout << "student names?\n";
  cin >> name1 >> name2;
  for (i=0;i<7;i=i+1)
  { if (name1[i]<name2[i])
    {
      cout << "-1 \n"; return 0;
    };
    if (name1[i]>name2[i])
    {
      cout << "1 \n"; return 0;
    };
  }; // of for
  cout << "0 \n"; return 0;
  return 0;
}
```

Sorting

Whats happening?



- If $x_1 > x_2$ return 1.
- If $x_1 < x_2$ return -1.
- If $x_1 = x_2$ then increment i.
- If $i == 7$ then return 0.

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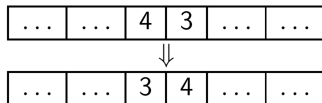
Bubble Sort

Now that key comparison is clear, let us now sort an array of integers. Obviously this algorithm can be used to sort any key.

We look at the bubble sort whose basic step is the flip(i):

- Compare two adjacent elements x_{i-1}, x_i.
- If $x_i > x_{i-1}$ then interchange.



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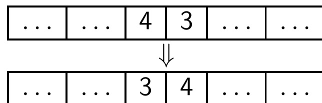
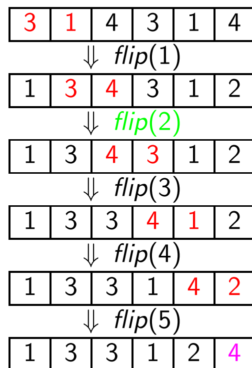


A Phase(N-1) is a sequence of flips:

$$flip(1), flip(2), \ldots, flip(N-1)$$



Bubble Sort

Thus, we see that:

- At the end of Phase(N-1), the largest element is in the last location.

We may thus run:

- Phase(N-1) which fixes the (N-1)-th element.
- Phase(N-2) which fixes the (N-2)-th element.
- $\vdots$
- Phase(1) which fixes the (1)-th element. and obtain:

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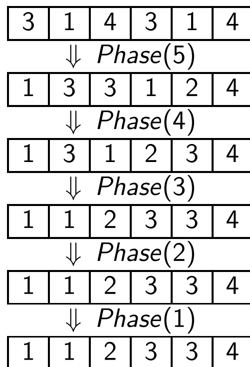

This sorts the array

Bubble Sort sort.cpp

```
#include <iostream.h>
void sort(int c[],int N)
{ int i,j,temp;
   for (i=N-1;i>=1;i=i-1)
    // beginning Phase (i)

    for (j=1;j<=i;j=j+1)
    // beginning flip (j)
     if (c[j]<c[j-1])
      { temp=c[j]; c[j]=c[j-1];
        c[j-1]=temp;
      };


}
```

- i is counting phase.
- j is counting flip.
- array c is passed by reference.

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Question How many steps does it take to sort an array of size N

Answer $(N-1)+(N-2)+\ldots+1 = N(N-1)/2$

Thus it takes quadratic, i.e., $O(N^2)$ time to bubble-sort.

Other Sorts

There are faster ways to sort:

- Merge-Sort, Heap-Sort, $O(N \log N)$.
- Quick-Sort, expected time $O(N \log N)$.

All of these are fairly simple but clever. We will look at Merge-Sort though, not in detail.

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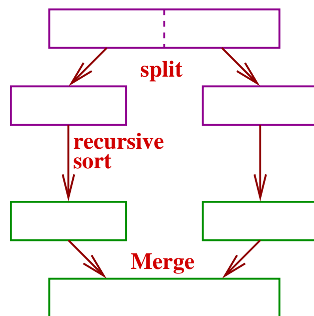
- Split the given array A into two equal halves A_1 and A_2.
- Recursively, sort A_1, A_2 to get sorted B_1, B_2.
- Merge B_1, B_2 to get sorted B.



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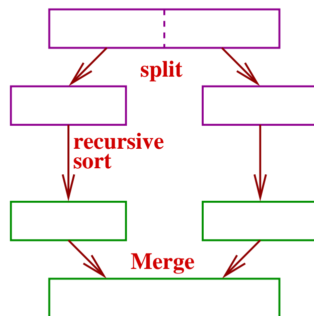

How much time does Merge-Sort take?

Merge-Sort

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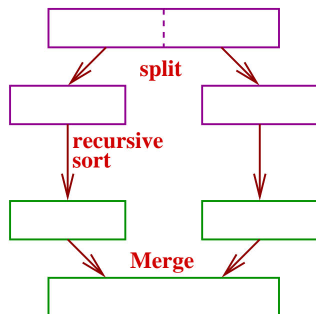

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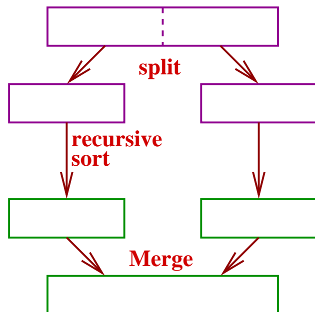


Lets say $T(N)$ is the time taken to merge-sort an N-array.

- Split-ting an N-array into two equal parts is easy. At most a single for loop.
- Recursive Merge: this should take time $2 \star T(N/2)$.
- Merge is the operation of merging two sorted arrays into a single sorted array. We will see that this takes time $2N$.

Thus we have:

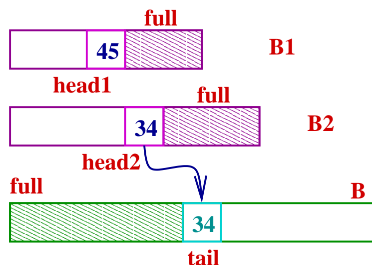
$$T(N) = 3N + 2T(N/2)$$

We may expand this to check that $T(N) = O(N \log N)$.

Merge

Let us look at the merge-operation:

- Merge merges two sorted arrays into a single sorted array.



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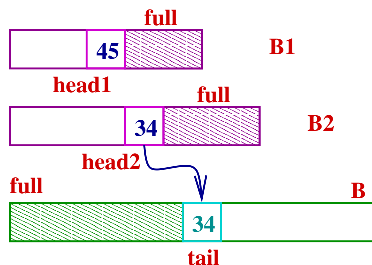

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```
void merge(int B1[],B2[],B[],int
{
  int tail=0, head1=0, head2=0;
  while (head1<N1 && head2<N2)
    if B1[head1]<B2[head2]
    {  B[tail]=B1[head1];
     head1=head1+1;
    }
    else
    {  B[tail]=B2[head2];
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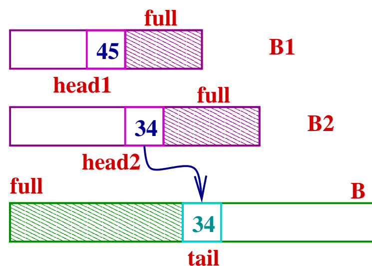

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    };
    tail=tail+1;
  } // of while
  if (head1==N1) {  push B2}
  else {push B1};
}
```

Search

- Check if the integer n occurs in a sorted array B of size N.

The simplest way is to

- Start at the beginning and stop at the end. .

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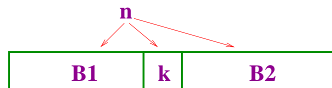

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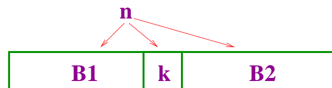
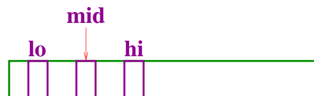
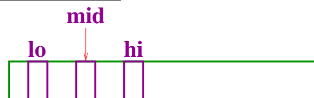




Slowly...



- First ensure that c[hi]!=ip, c[lo]!=ip. .
- Now enter the infinite while loop.
  - Compute mid and check that c[mid]!=ip.
  - Check that lo,hi have a gap.
- Now, redefine lo,hi.

```
int search(int c[],int N,
                      int ip)
{
  int lo=0, hi=N-1, done=0,mid;
  if (c[lo]==ip) return (lo);
  if (c[hi]==ip) return (hi);
  while (done==0)
   {
     mid=(lo+hi)/2;
     if (c[mid]==ip) return (mid
     if (hi-lo<2) return (-1);
     if (c[mid]<ip)
     {  lo=mid; }
     else
       hi=mid;
   }
}
```