Disk Bezier curves

Slides made by:-

Mrigen Negi

Instructor:-

Prof. Milind Sohoni.

A disk in the plane R 2 is defined to be the set

$$<P> := \{x \mid |x - c| \le r;\ c \in R^2,\ r \in R^+\}.$$

we also write <P> = <c ; r>,

The following operations are defined for disks $<c\,;r>$, $<c_i\,;r_i>$

$a<c\,;r> = <ac\,;|a|r>$,

$<c_1\,;r_1> + <c_2\,;r_2> = <c_1 + c_2\,;r_1 + r_2>$

We get equations

$$\sum_{i=1}^{n} <c_i;r_i> = <\sum_{i=1}^{n} c_i;\sum_{i=1}^{n} r_i>$$

And

$$\sum_{i=1}^{n} a_i <c_i;r_i> = <\sum_{i=1}^{n} a_i c_i;\sum_{i=1}^{n} |a_i|\, r_i>$$

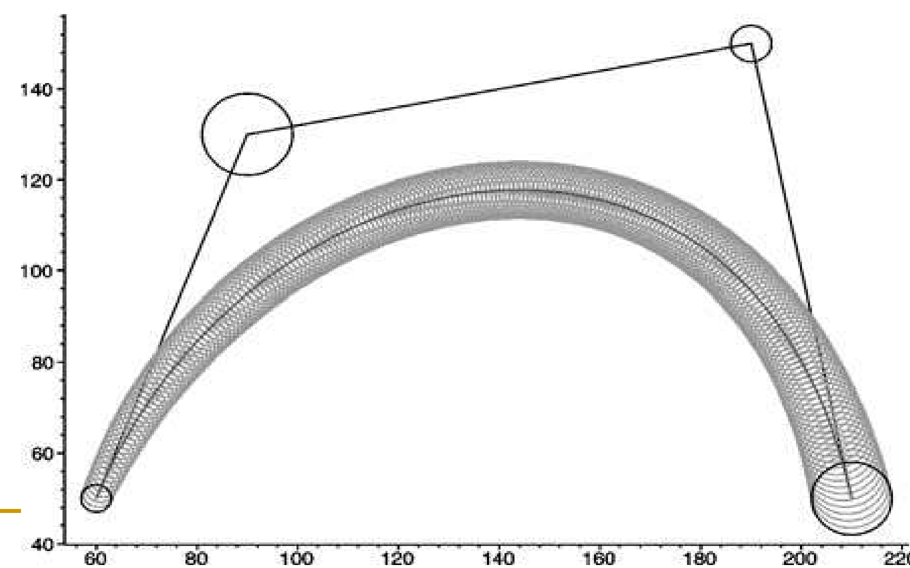
A planar disk Bezier curve is then defined as

$$<Q>(t) = \left\{ \sum_{k=0}^{n} <P_k> B_k^n(t), t \in [0,1] \right\}$$

$$<Q>(t) = \sum_{k=0}^{n} <c_k; r_k> B_k^n(t) = < \sum_{k=0}^{n} c_k B_k^n(t); \sum_{k=0}^{n} r_k B_k^n(t) >, t \in [0,1]$$

the center curve of the disk Bezier curve <Q>(t) is a Bezier curve with control points $\{ c_k \}$

the radius of <Q> (t) is the weighted average of the radii $\{ r_k \}$ of the control points.



The disk Bezier curve is a fat curve with variable width which is a function of the parameter t given by

$$weight(<Q>(t)) = 2\sum_{k=0}^{n} r_k B_k^n(t)$$

The disk Bézier curve <Q>*(t)* can also be written as

$$<\mathbf{Q}(t)> = (x(t), y(t))\ r(t)$$

$$x(t) = \sum_{i=0}^{n} x_i B_i^n(t)$$

$$y(t) = \sum_{i=0}^{n} y_i B_i^n(t)$$

$$r(t) = \sum_{i=0}^{n} r_i B_i^n(t)$$

$$C(t) = (x(t), y(t)) = \sum_{i=0}^{n} C_i B_i^n(t)$$

and r(t) are called the *center curve* and the *radius* of the disk Bézier curve (**Q**)(*t*) respectively.

De Casteljau algorithm.

For any $t_0 \in [0,1]$, $(P)(t_0)$ can be computed as follows:

Set $(P_i^0) = (P_i)$, i=0,1,2,…n

For $k = 1, 2, \ldots, n$ do

For $l = 0, 1, \ldots, n - k$ do

$$(P_l^k) = (1 - t_0)(P_l^{k-1}) + t_0 (P_{l+1}^{k-1})$$

End l

End k

Set $(P)(t_0) = (P_0^n)$

an obvious generalization of the real de Casteljau algorithm.

Envelop of disk Bezier curves

Let the Bezier curve be thought of as the envelope of a set of curves parametrized by t. If this is written as *F (x* , y, t) = 0 then the envelope is found by solving

F(x,y,t)=0 and $\frac{\partial F(x,y,t)}{\partial t}=0$

Since $$F(x,y,t)=[x-c_x(t)]^2+[y-c_y(t)]^2-r^2(t)=0$$

We have

$$[x-c_x(t)]^2+[y-c_y(t)]^2=r^2(t)$$

$$c_x'(t)[x-c_x(t)]+c_y'(t)[y-c_y(t)]=-r(t)r'(t)$$

$R = r(t)$, $X = x - c_x$, $Y = y - c_y$

$$X^2 + Y^2 = R^2$$

$$c'_x X + c'_y Y = -RR'$$

$$Y = \frac{-c'_y \pm c'_x\sqrt{c'^2_x + c'^2_y - R'^2}}{c'^2_x + c'^2_y} R \quad (1)$$

$$X = \frac{-c'_x \pm c'_y\sqrt{c'^2_x + c'^2_y - R'^2}}{c'^2_x + c'^2_y} R \quad (2)$$

Now substituting (2) to (1) we get

solution of the above system of equations is

$$y_0 = c_y(t) + \frac{-c_y' r' \mid c_x' \sqrt{c_x'^{2} \pm c_y'^{2} - R'^{2}}}{c_x'^{2} + c_y'^{2}} R$$

$$x_0 = c_x(t) + \frac{-c_x' r' \mid c_y' \sqrt{c_x'^{2} \pm c_y'^{2} - R'^{2}}}{c_x'^{2} + c_y'^{2}} R$$

We might have a particular case when r0 = 'rl . . . . rn= constant. Then r(t) = r in which case

$$x_0 = c_x(t) \frac{c_y'(t)}{\sqrt{c_x'^{2} + c_y'^{2}}} r$$

$$y_0 = c_y(t) \mid \frac{c_x'(t)}{\sqrt{c_x'^{2} + c_y'^{2}}} r$$

Let

$$q(t) = \frac{1}{\sqrt{c_x'^{2} + c_y'^{2}}} (-c_y' \quad c_x')^T$$

Then c ' (t) q (t) = 0, II q (t)|| = 1. This means that the two envelopes of the disk Bezier curve can be written as

Q1(t) = c(t) + rq(t),

Q2(t) = c(t) - rq(t).

Subdivision

Let c ? $(0, 1)$ be a real number. Then the disk Bézier curve can be subdivided into two segments:

$$< Q(t) >= \begin{cases} \sum_{i=0}^{n} (P_0^i) B_i^n(t/c) & \text{for } 0<=t<=c \\ \sum_{i=0}^{n} (P_i^{n-i}) B_i^n\left((t-c)/(1-c)\right) & \text{for } c<=t<=1. \end{cases}$$

Degree elevation

The degree n disk Bézier curve can be represented as a degree n + 1 disk Bézier curve as follows

$$<Q(t)> = \sum_{i=0}^{n} (P_i) B_i^n(t) = \sum_{i=0}^{n+1} (P'_i) B_i^{n+1}(t)$$

where the control disks for the degree elevated curve are

$$(P'_i) = \frac{i}{n+1}(P_{i-1}) + \frac{n-i+1}{n+1}(P_i)$$