Computer Aided Geometric Design Introduction

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www.cse.iitb.ac.in/sohoni/gsslcourse

A Solid Modeling Fable

- Ahmedabad-Visual Design Office
- Kolhapur-Mechanical Design Office
- Saki Naka – Die Manufacturer
- Lucknow- Soap manufacturer

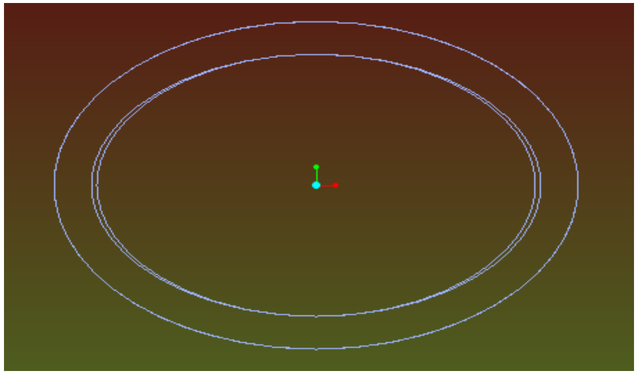
Ahmedabad-Visual Design

- Input: A dream soap tablet
- Output:
- Sketches/Drawings
- Weights
- Packaging needs

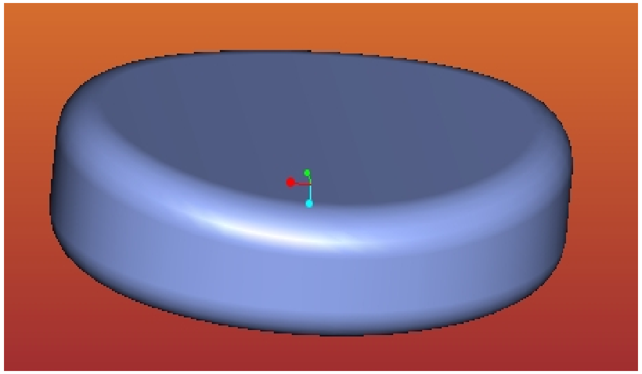
Soaps

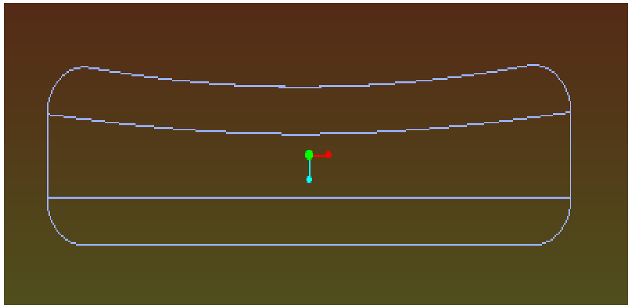
More Soaps

Ahmedabad (Contd.)

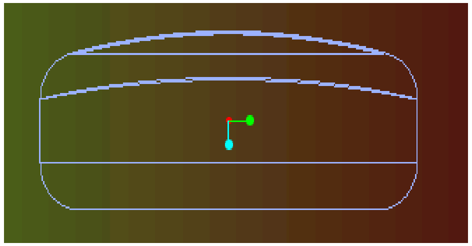


Top View





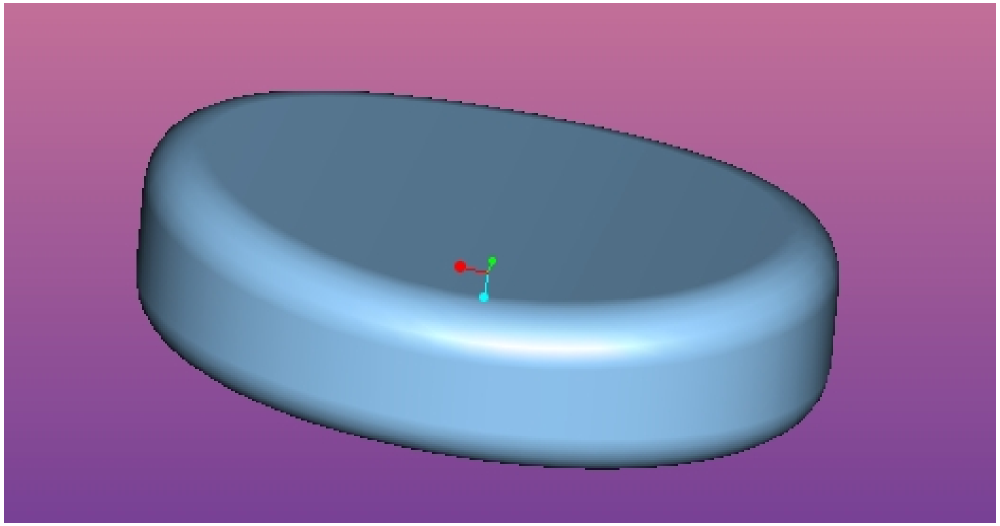
Front View



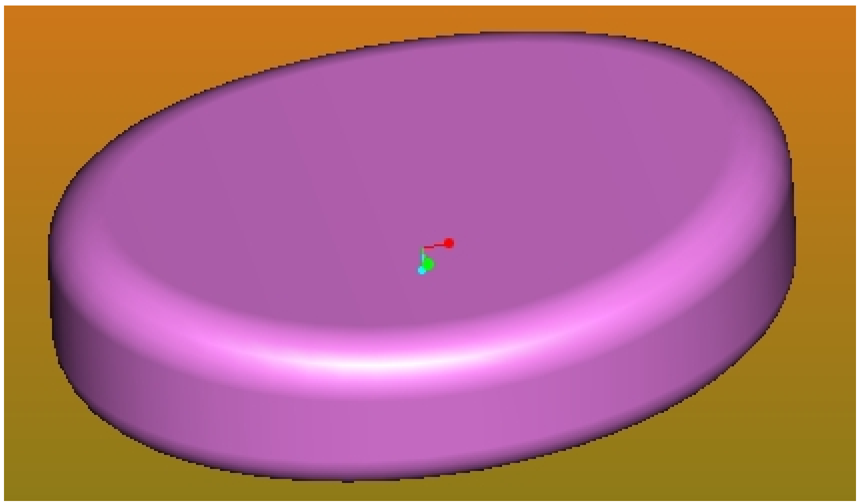
Side View

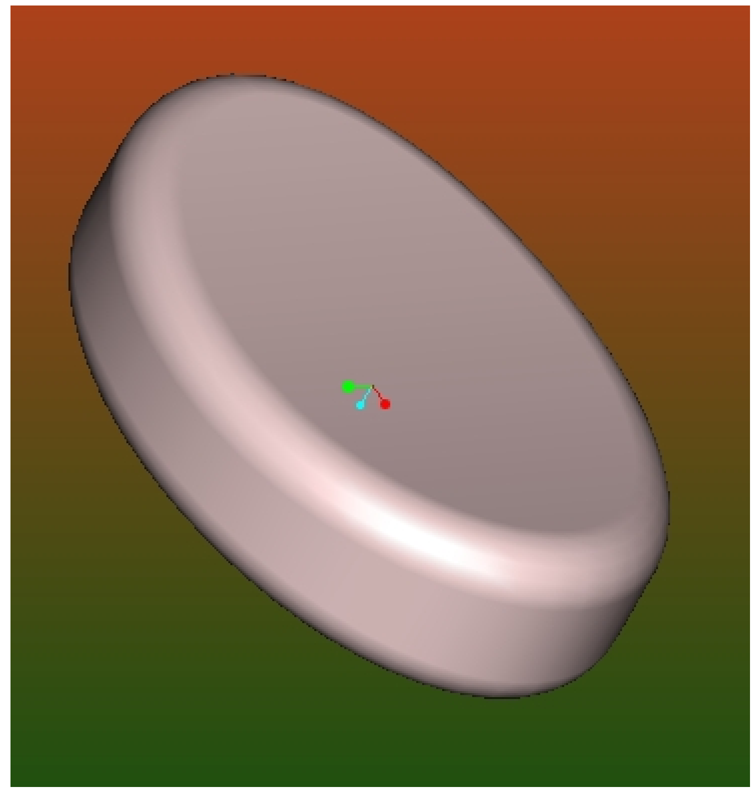
Kolhapur-ME Design Office

- Called an expert CARPENTER
- Produce a model (check volume etc.)
- Sample the model and produce a data-set



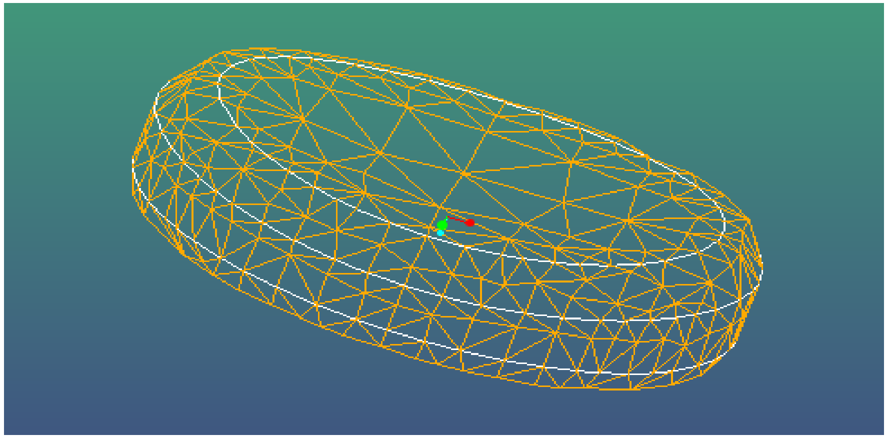
Kolhapur(contd.)

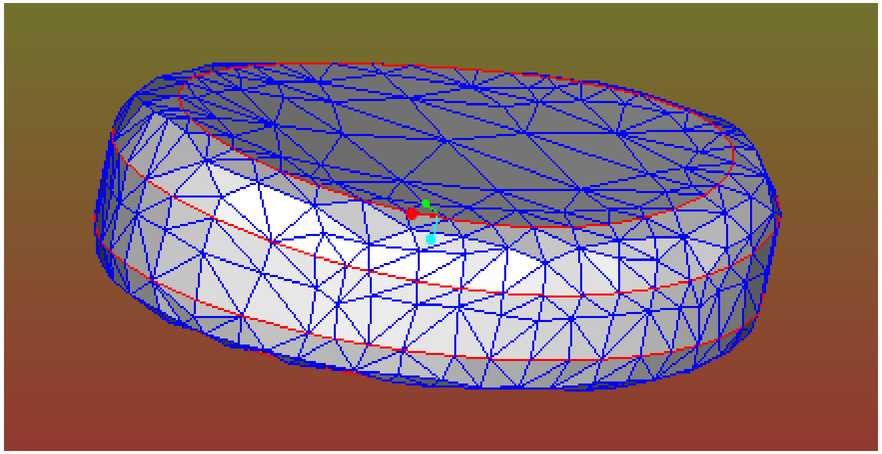




Kolhapur (contd.)

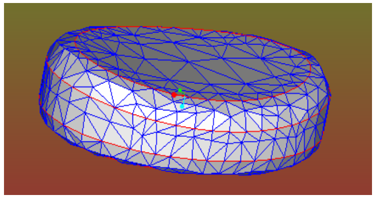
- Connect these sample-points into a faceting
- Do mechanical analysis
- Send to Saki Naka

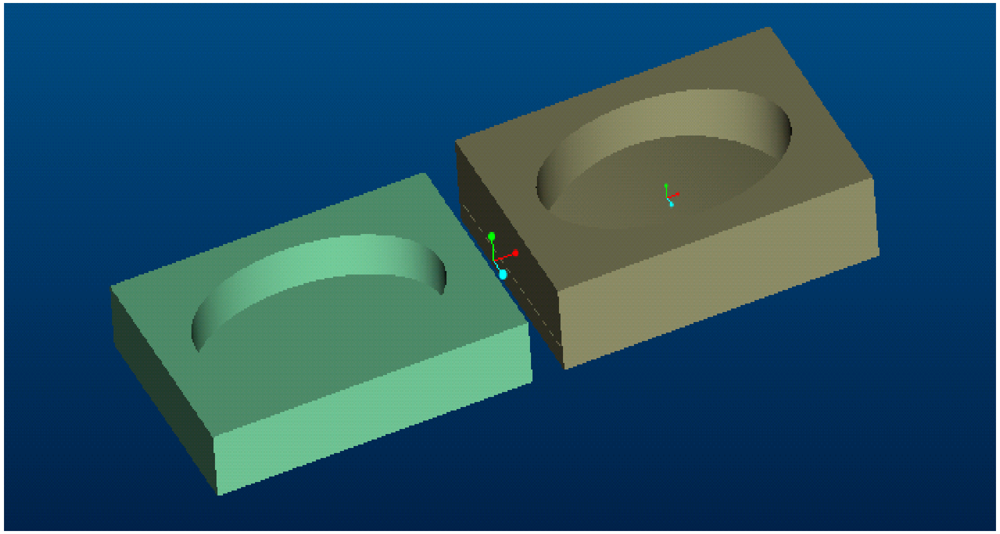




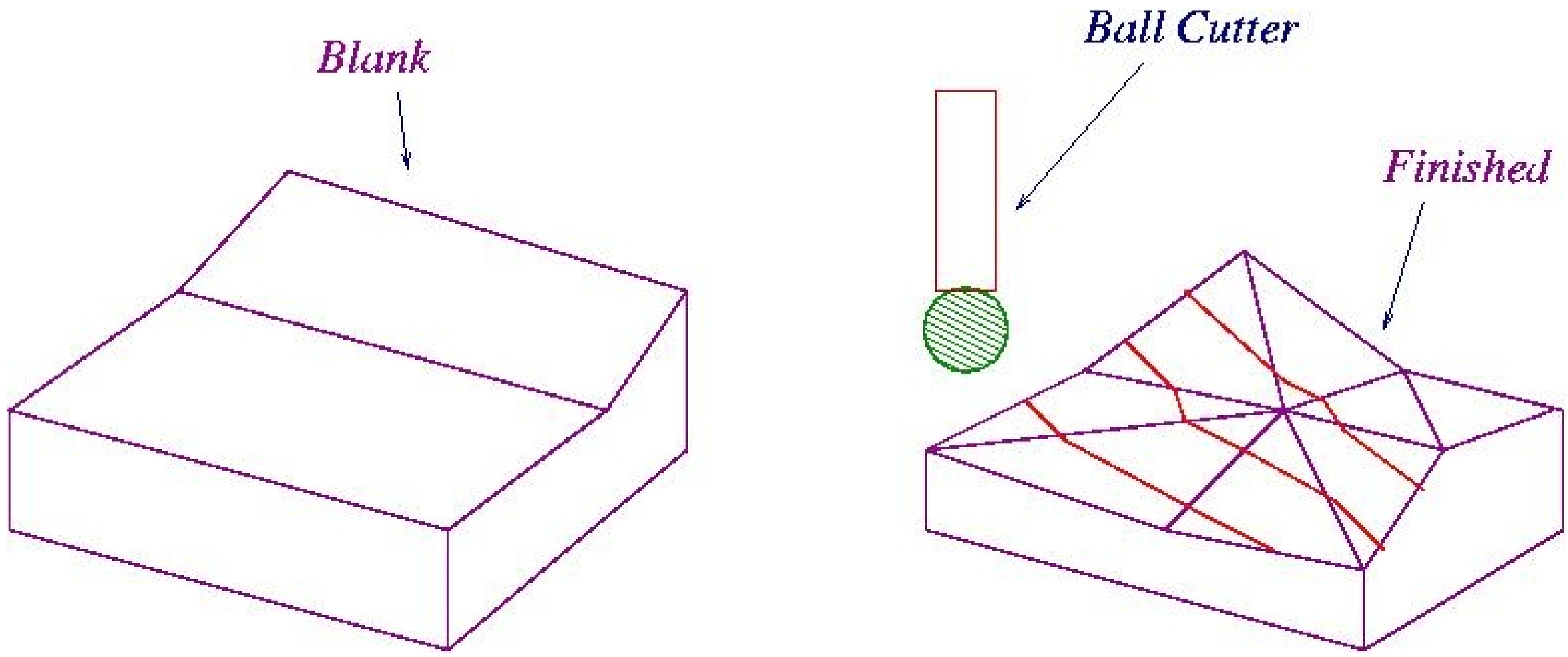
Saki Naka-Die Manufacturer

- Take the input faceted solid.
- Produce Tool Paths
- Produce Die



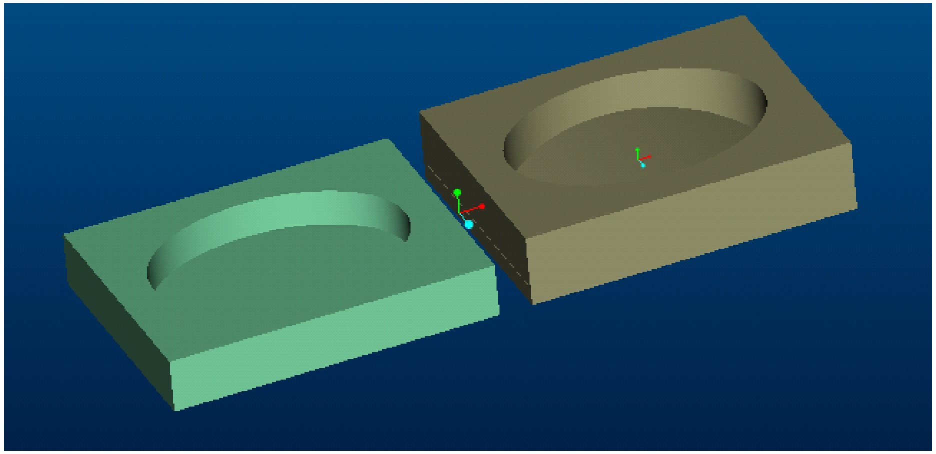


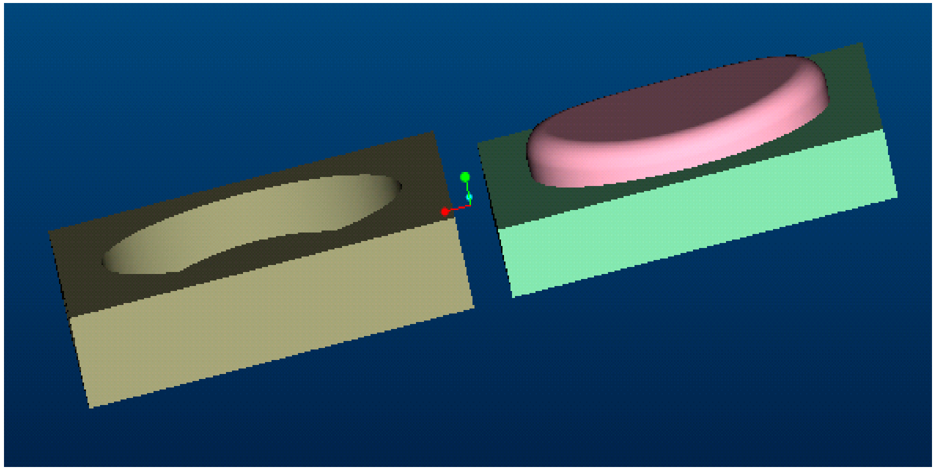
The Mechanics of it…..



Lucknow-Soaps

- Use the die to manufacture soaps
- Package and transport to points of sale





Problems began…

- The die degraded in Lucknow
- The Carpenter died in Kolhapur
- Saki Naka upgraded its CNC machine
- The wooden model eroded

But

The Drawings were there!

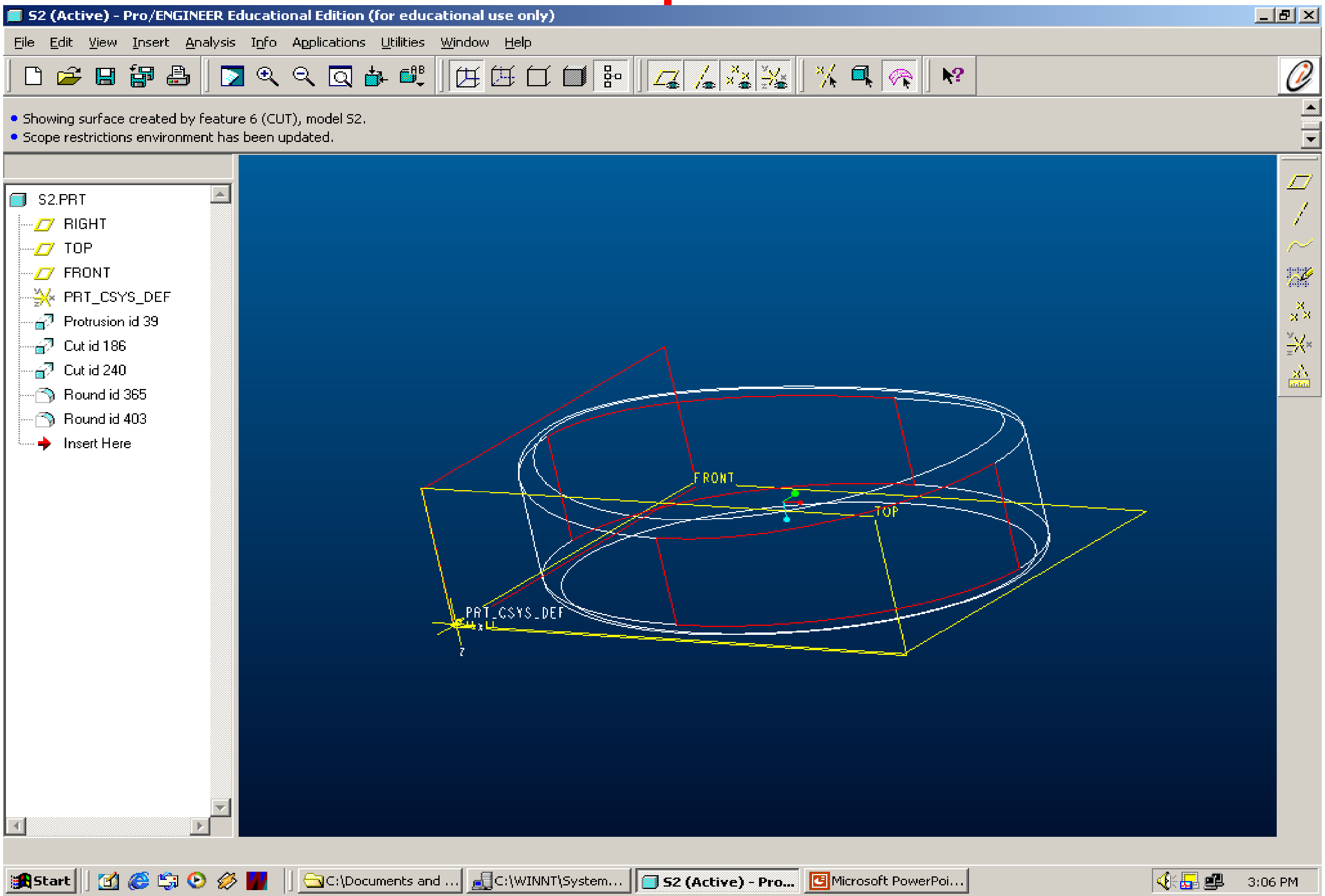
So Then….

- The same process was repeated but…

The shape was different!

The customer was suspicious and sales dropped!!!

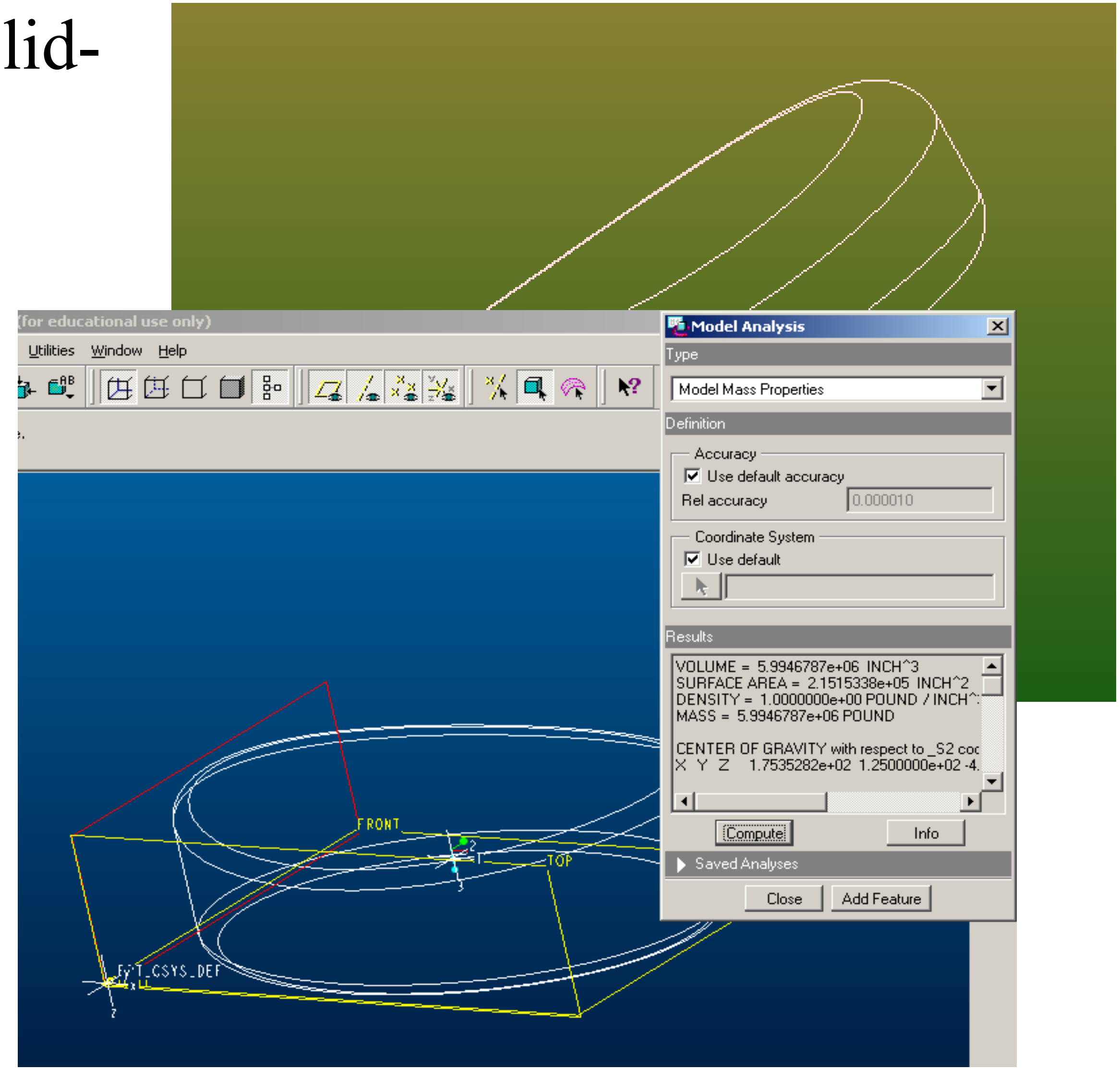
The Soap Alive !



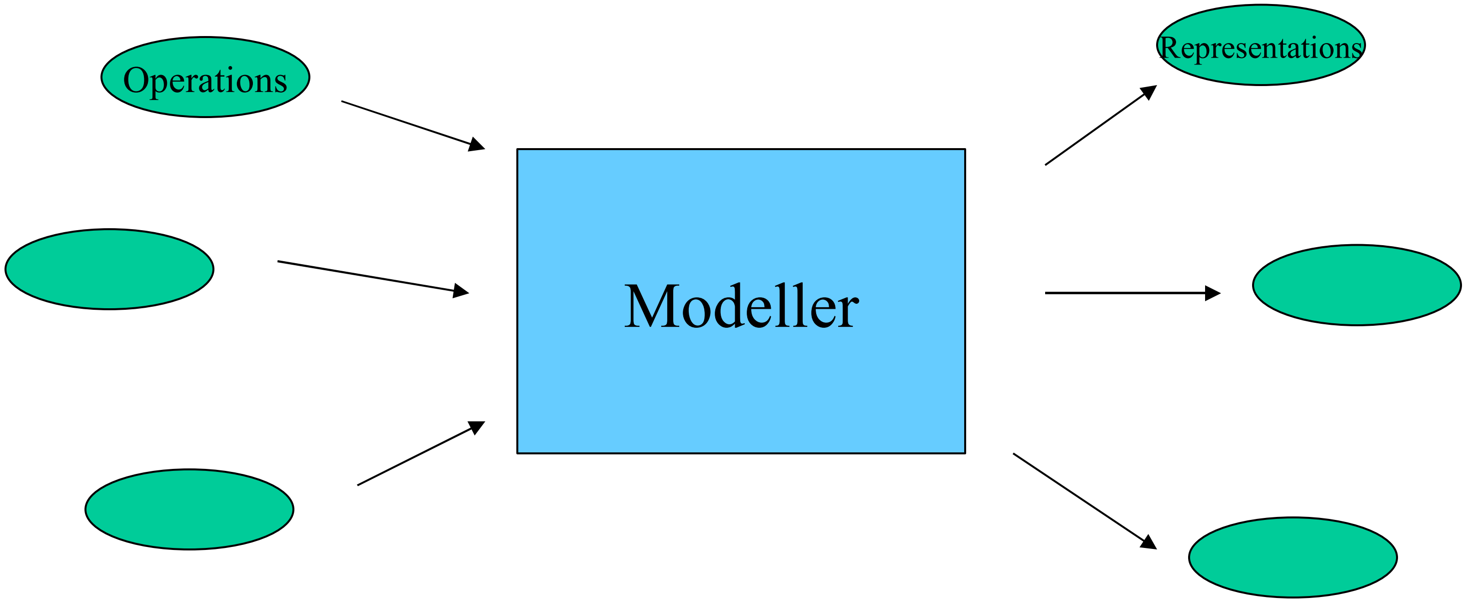
What was lacking was…

A Reproducible Solid-Model.

- Surfaces defn
- Tactile/point sampling
- Volume computation
- Analysis



The Solid-Modeller



The mechanical solid-modeller

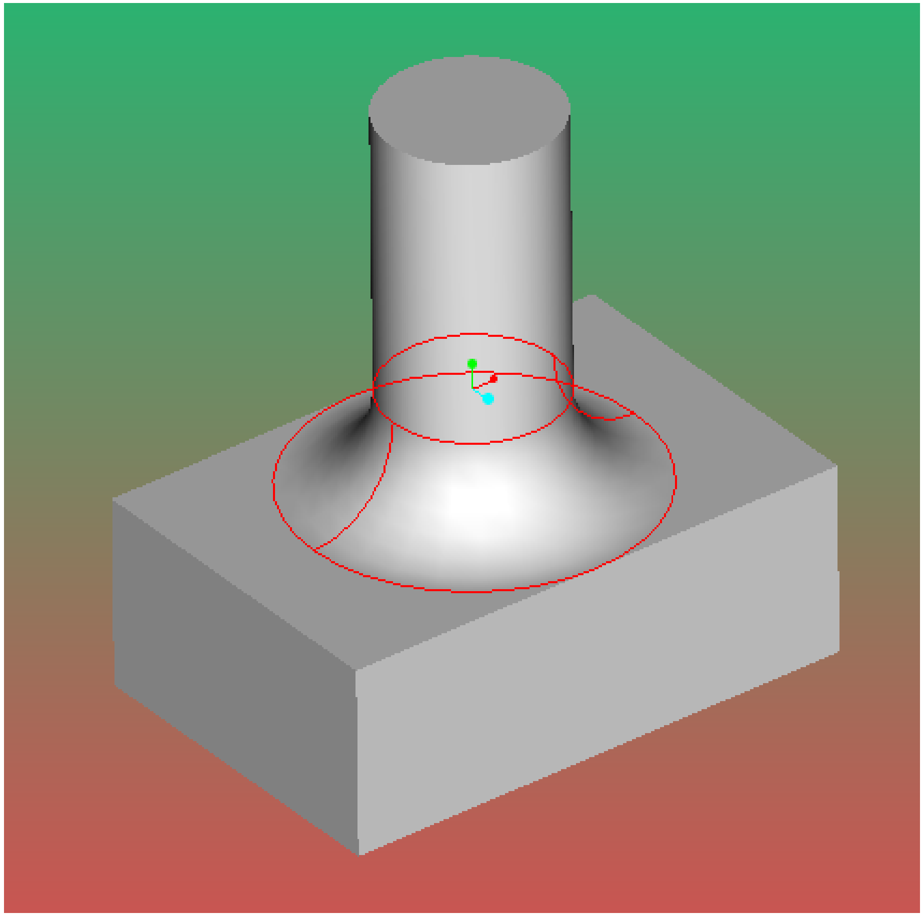
Operations

- Volume Unions/Intersections
- Extrude holes/bosses
- Ribs, fillets, blends etc.

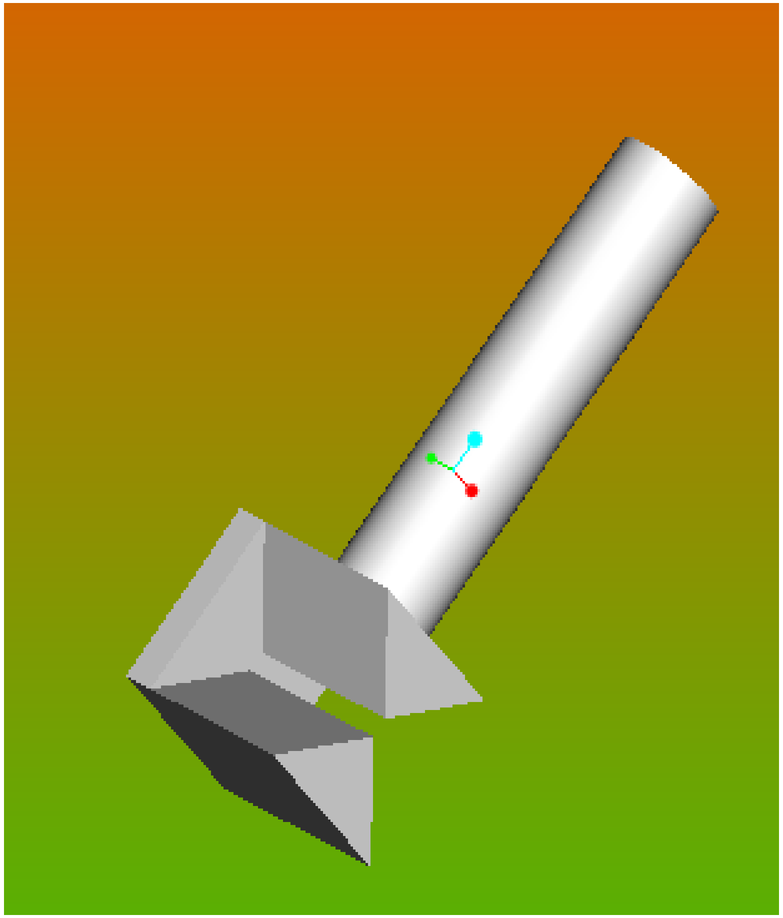
Representation

- Surfaces-*x,y,z* as functions in 2 parameters
- Edges –*x,y,z* as functions in 1 parameter

Examples of Solid Models

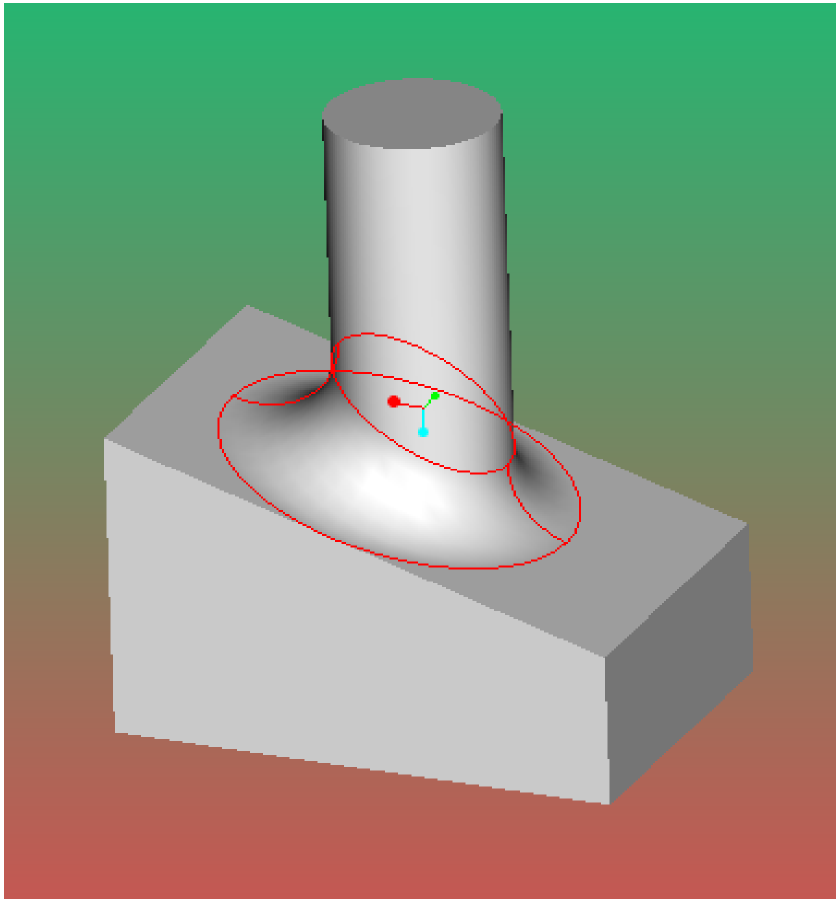


Torus

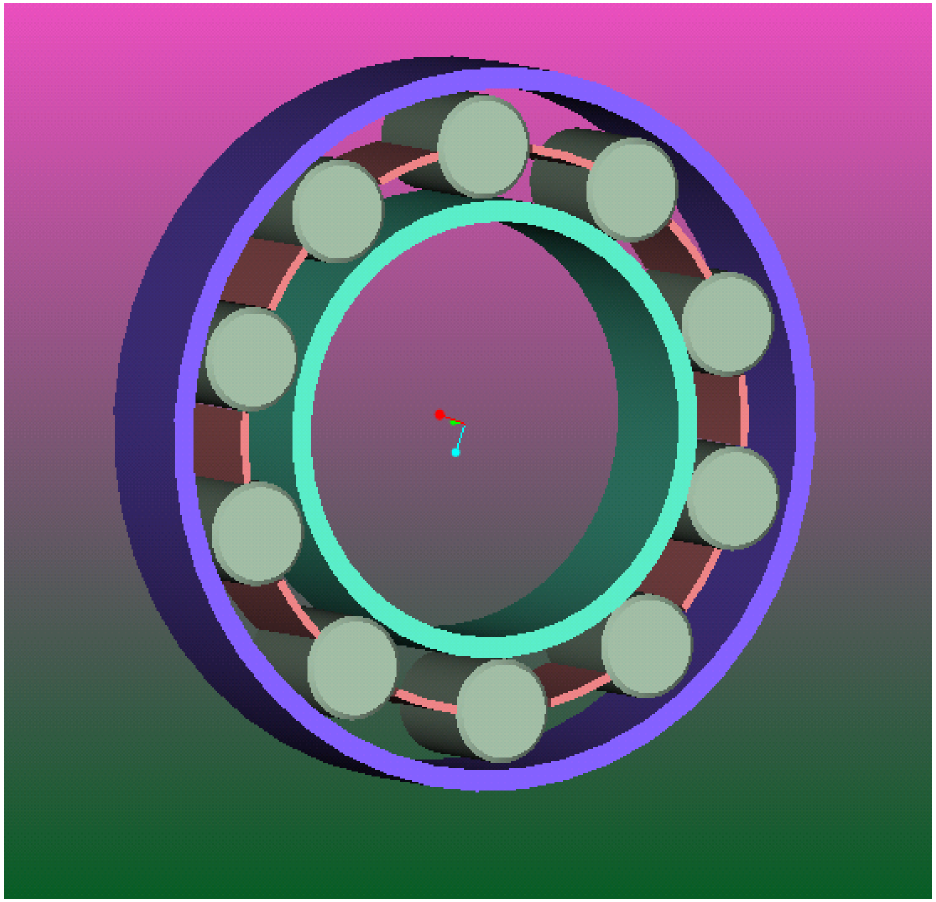


Lock

Even more examples

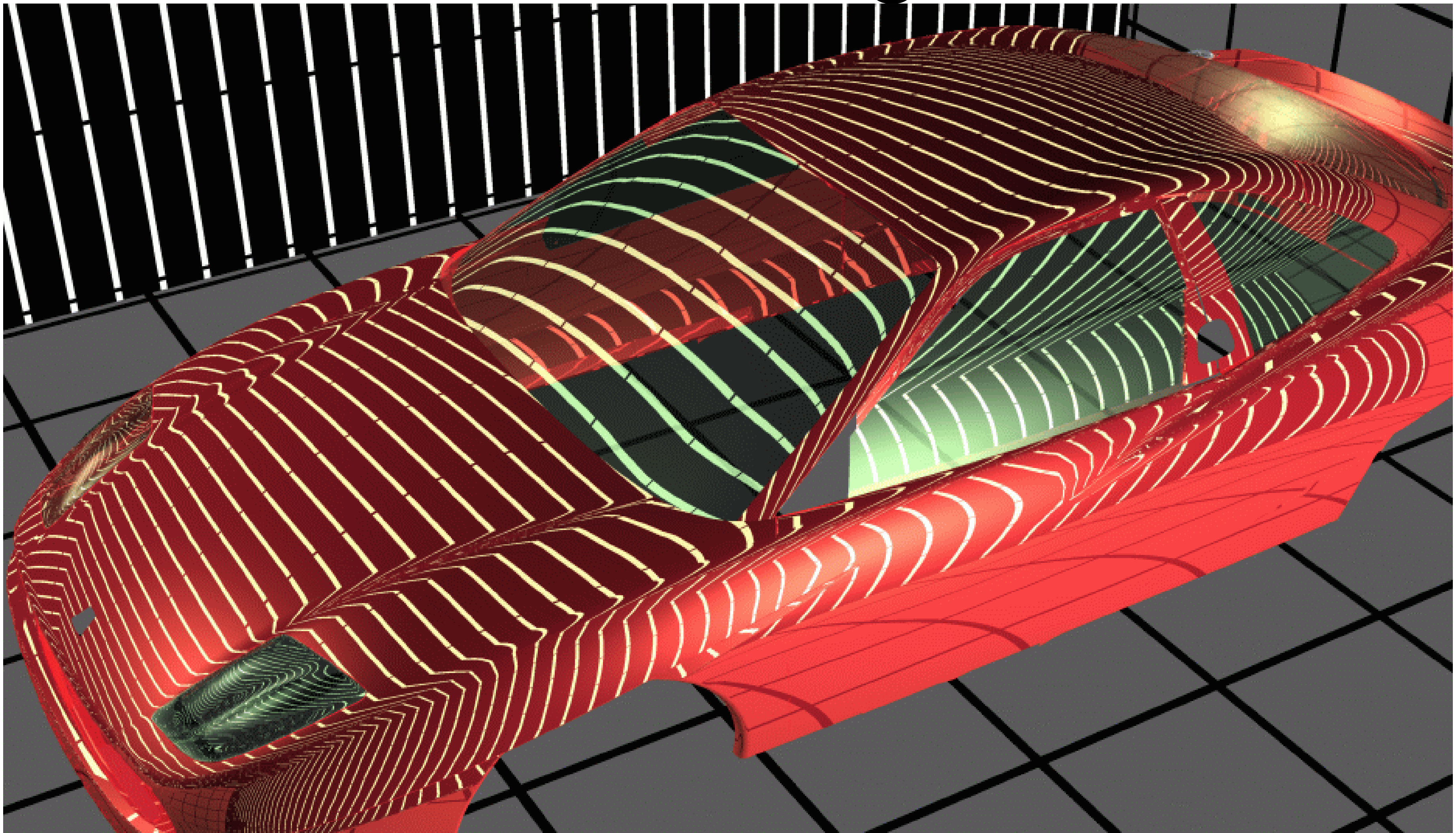


Slanted Torus

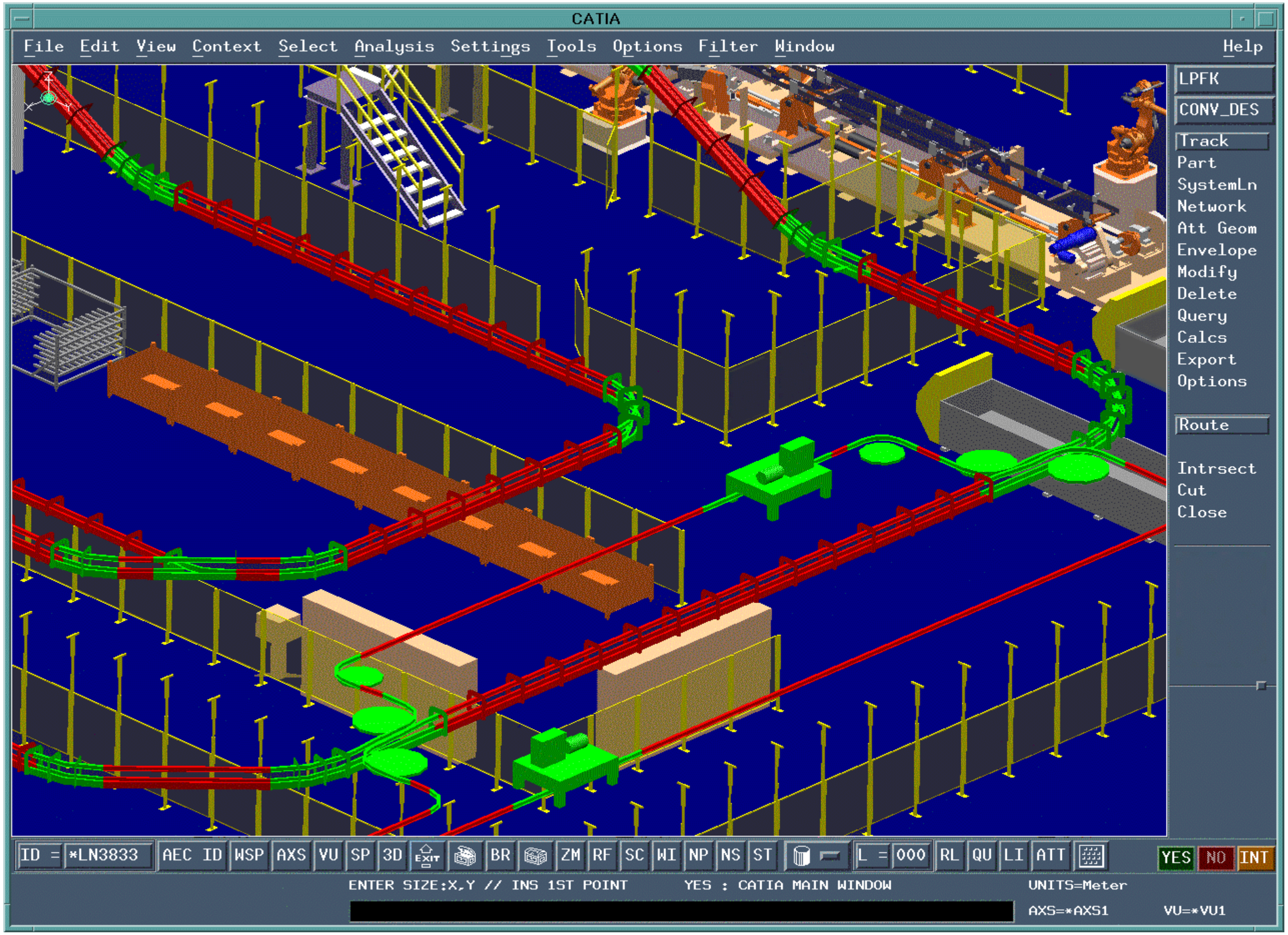


Bearing

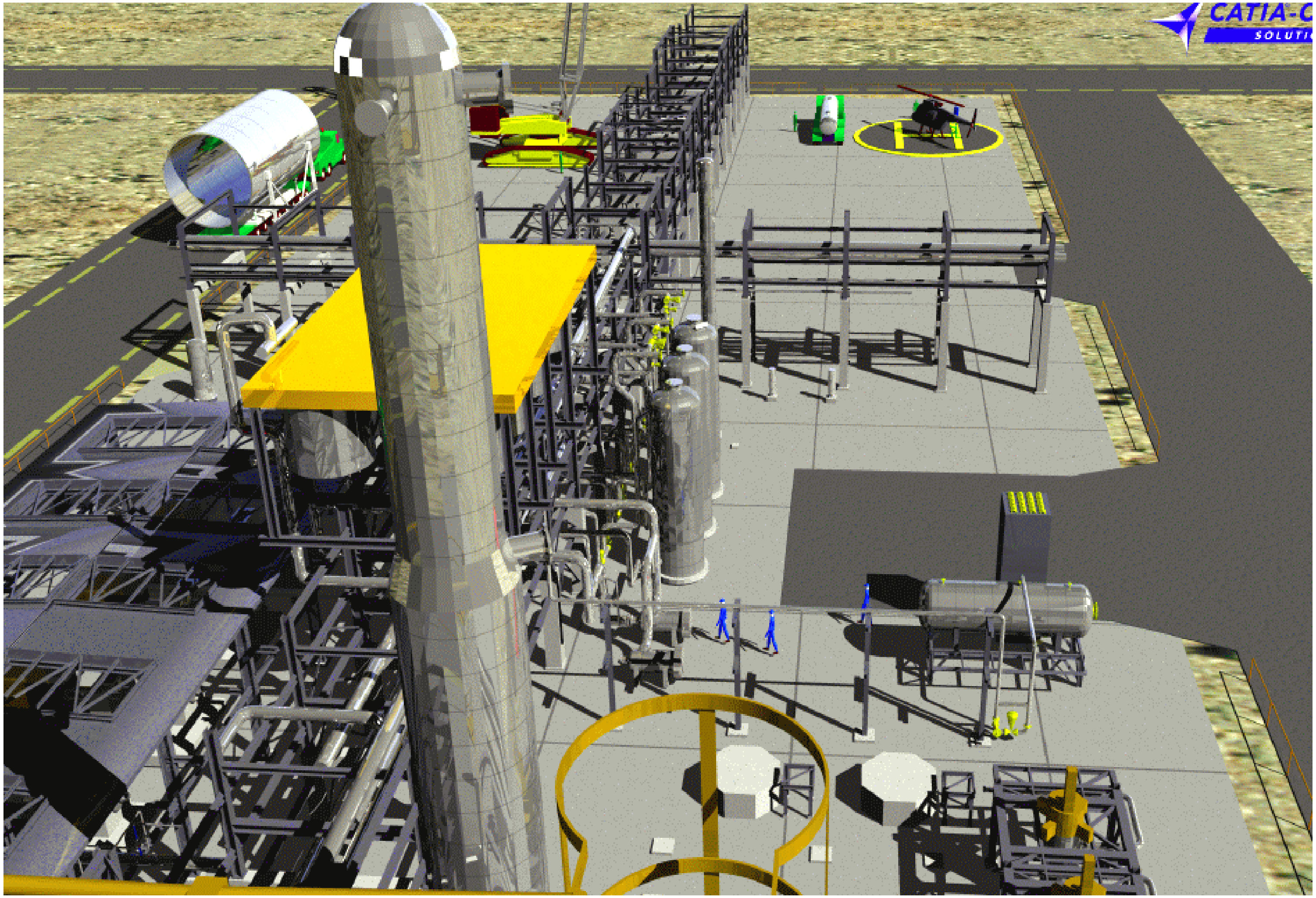
Other Modellers-Surface Modelling



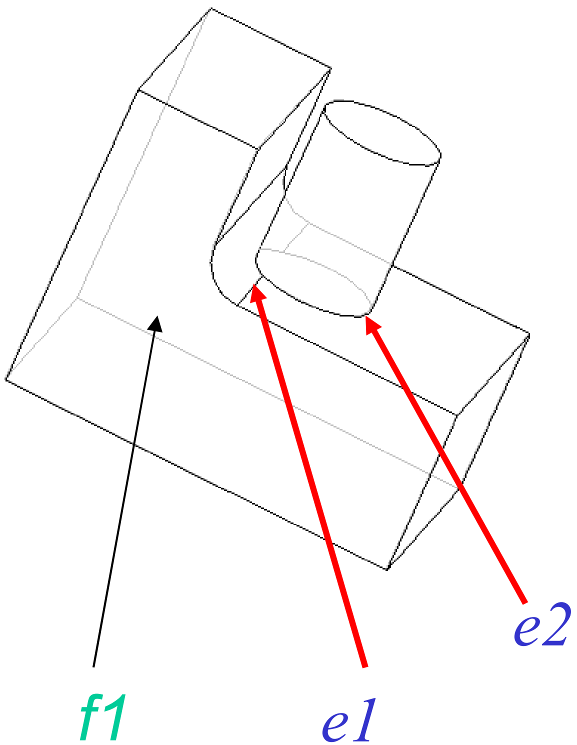
Chemical plants.



Chemical Plants (contd.)



Basic Solution: Represent each surface/edge by equations



e1: part of a line

X=1+t; Y=t, Z=1.2+t

t in *[0,2.3]*

e2: part of a circle

X=1.2 +0.8 cos t

Y=0.8+0.8 sin t

Z=1.2

T in *[-2.3,2.3]*

f1: part of a plane

X=3+2u-1.8v

Y=4-2u

Z=7

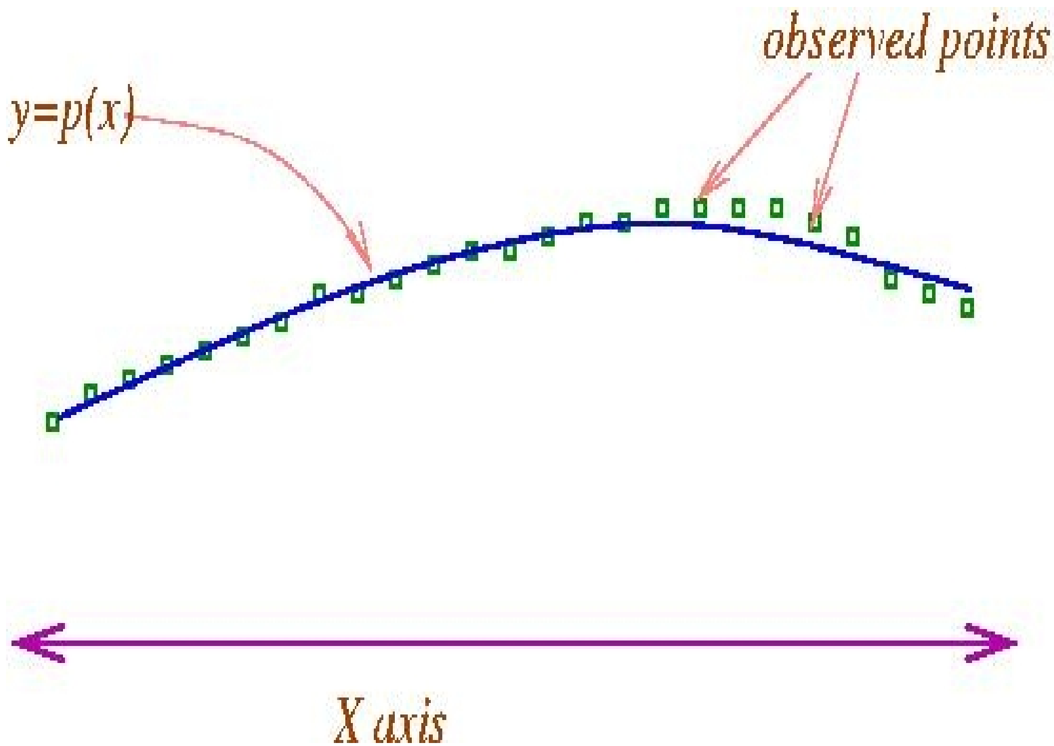
[u,v] in Box

A Basic Problem

Construction of defining equations

- Given data points

  arrive at a curve approximating this point-set.
- Obtain the equation of this curve



The Basic Process

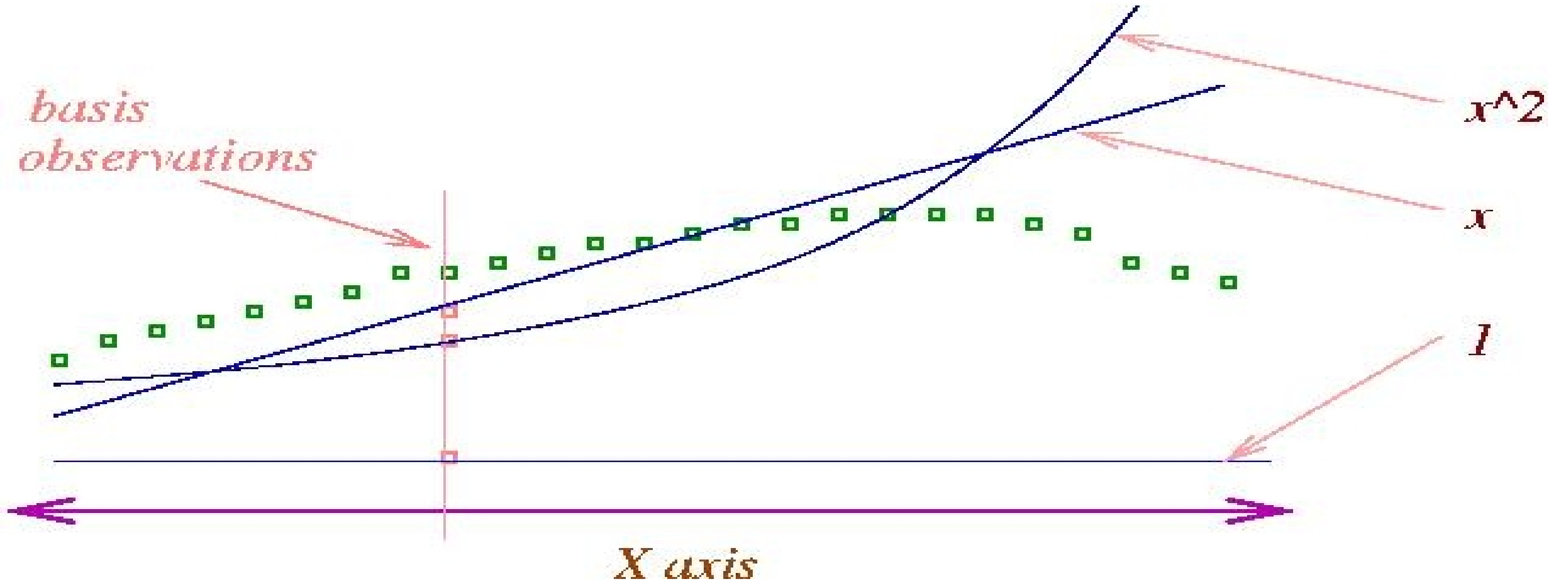
- Choose a set of basis functions
- Observe these at the data points
- Get the best linear combination

In our case, Polynomials

1, x, x^2, x^3

P(x)=a0+a1.x+a2.x^2+…

The Observations Process



v		6.1			2

1		1			1
x		1.2			3.1
x^2		1.44			9.61

=B

The Matrix Setting

We have

- The basis observations

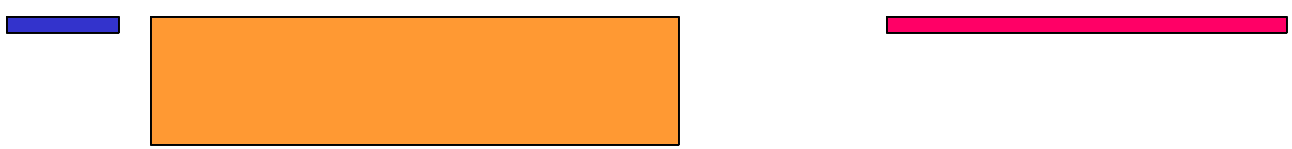
  Matrix B which is 5-by-100

- The desired observations

  Matrix v which is 1-by-100

We want:

- a which is 1-by-5 so that aB is close to v



The minimization

	v		6.1			2
a0	*1*		1			1
a1	*x*		1.2			3.1
a2	*x^2*		1.44			9.61

- Minimize least-square error (i.e. distance squared).

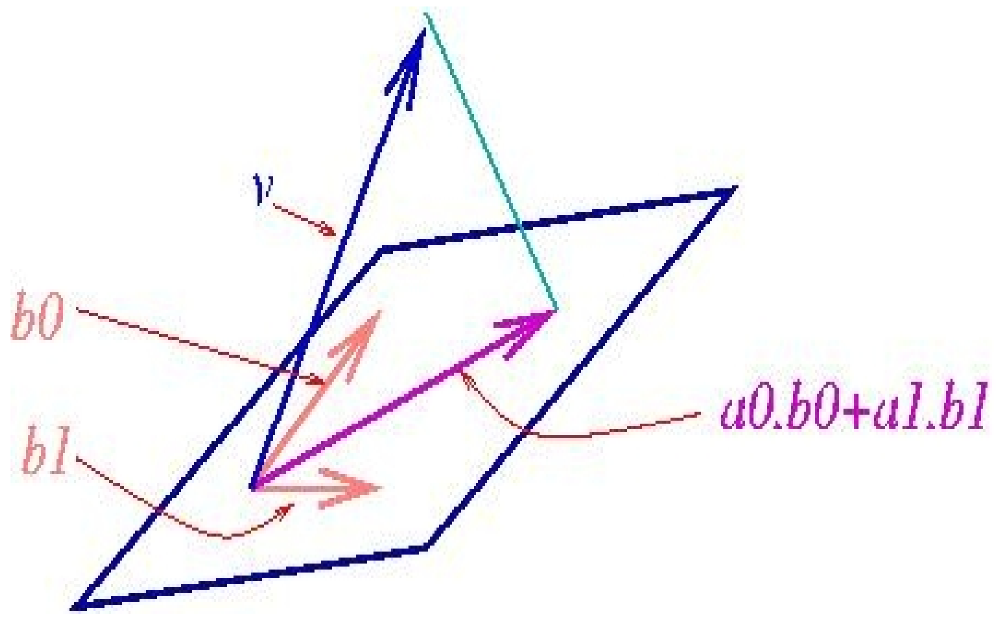
(6.1-a0.1-a1.1.2-a2.1.44)^2+…+(2-1.a0 - 3.1a1 - 9.61 a2)^2 +…

Thus, this is a quadratic function in the variables

a0,a1,a2,…

And is easily minimized.

A Picture



Essentially, projection of *v* onto the space spanned by the basis vectors

The calculation

- How does one minimize

  1.1 a0^2 +3.7 a0 a1 +6.9 a1^2 ?

- Differentiate!

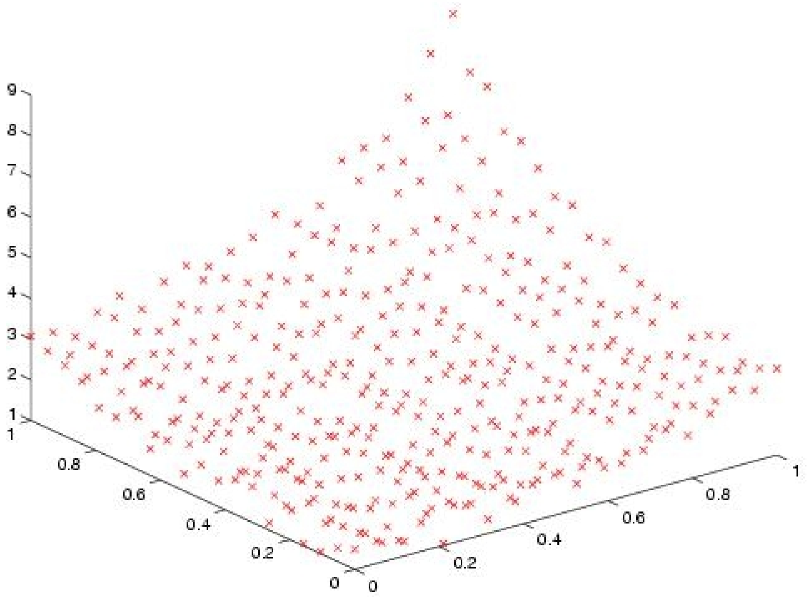
  2.2 a0 + 3.7 a1 =0

  3.7 a0 +13.8 a1=0

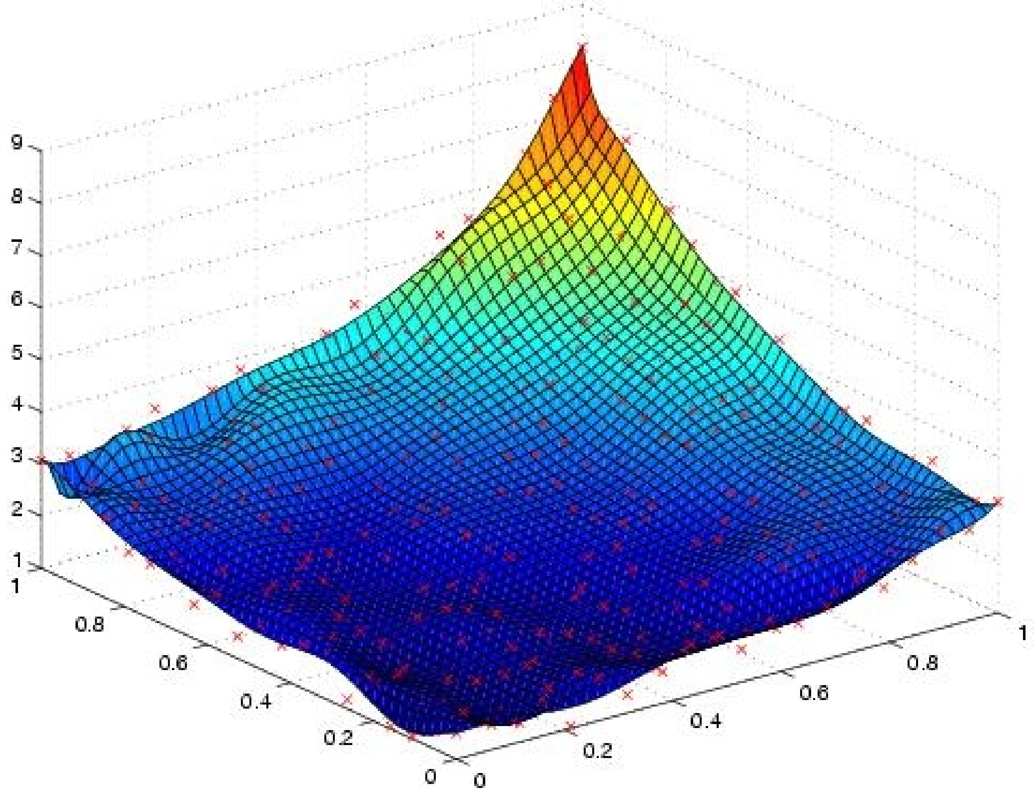
- Now *Solve* to get *a0,a1*

We did this and….

- So we did this for *surfaces* (very similar) and here are the pictures…



And the surface..



Unsatisfactory….

- Observation: the defect is because of *bad curvatures*, which is really *swings in double-derivatives!*
- So, how do we rectify this?
- We must ensure that if

p(x)=a0 +a1.x +a2.x^2 +… and

q(x)=p''(x) then

q(x)>=0 for all *x*

What does this mean?

$q(x)=2.a2+6.a3.x+12.a4.x^2+\ldots$

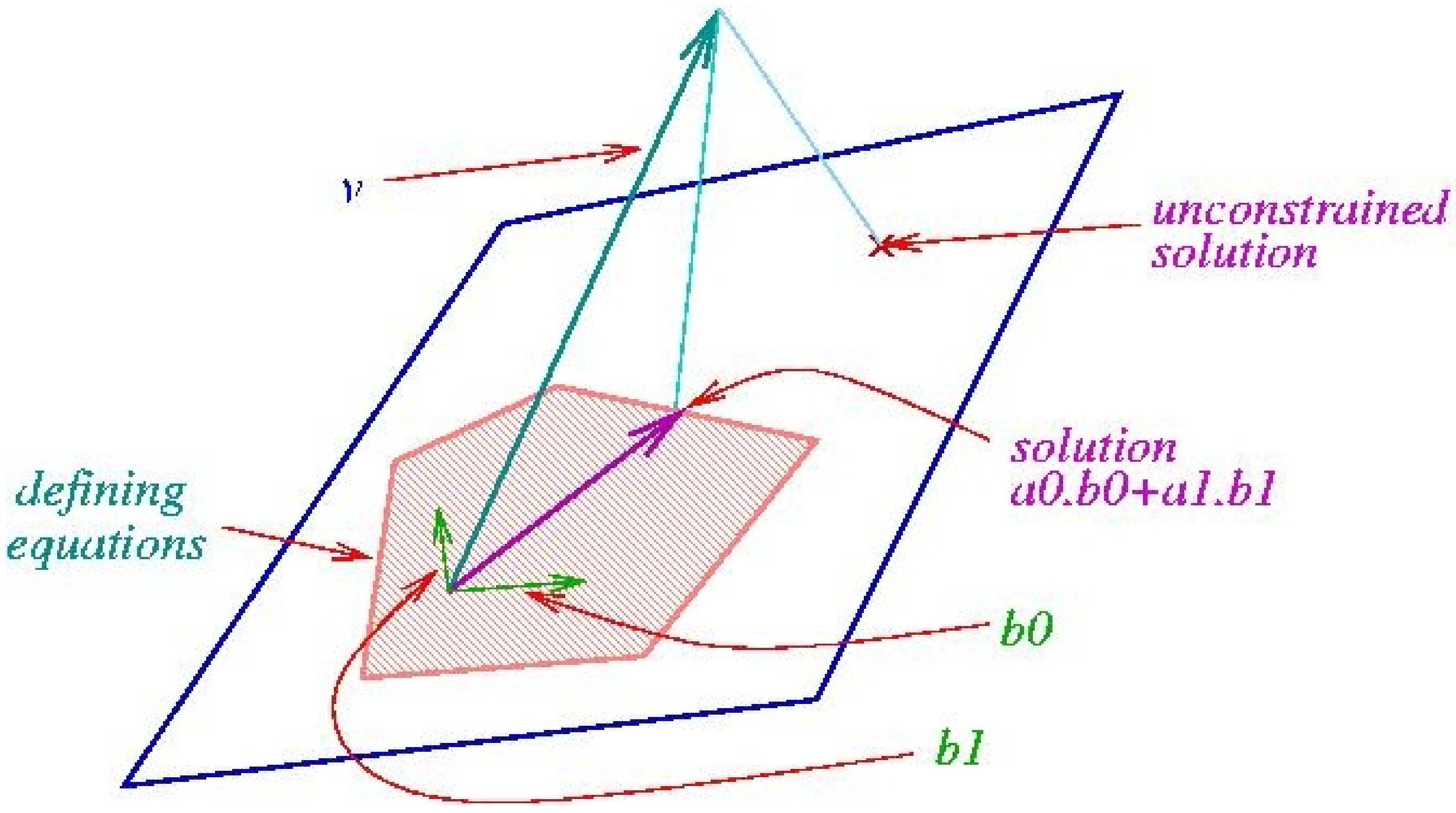
Thus $q(1)>=0, q(2)>=0$ means

$2.a2 +6.a3 +12.a4 >=0$

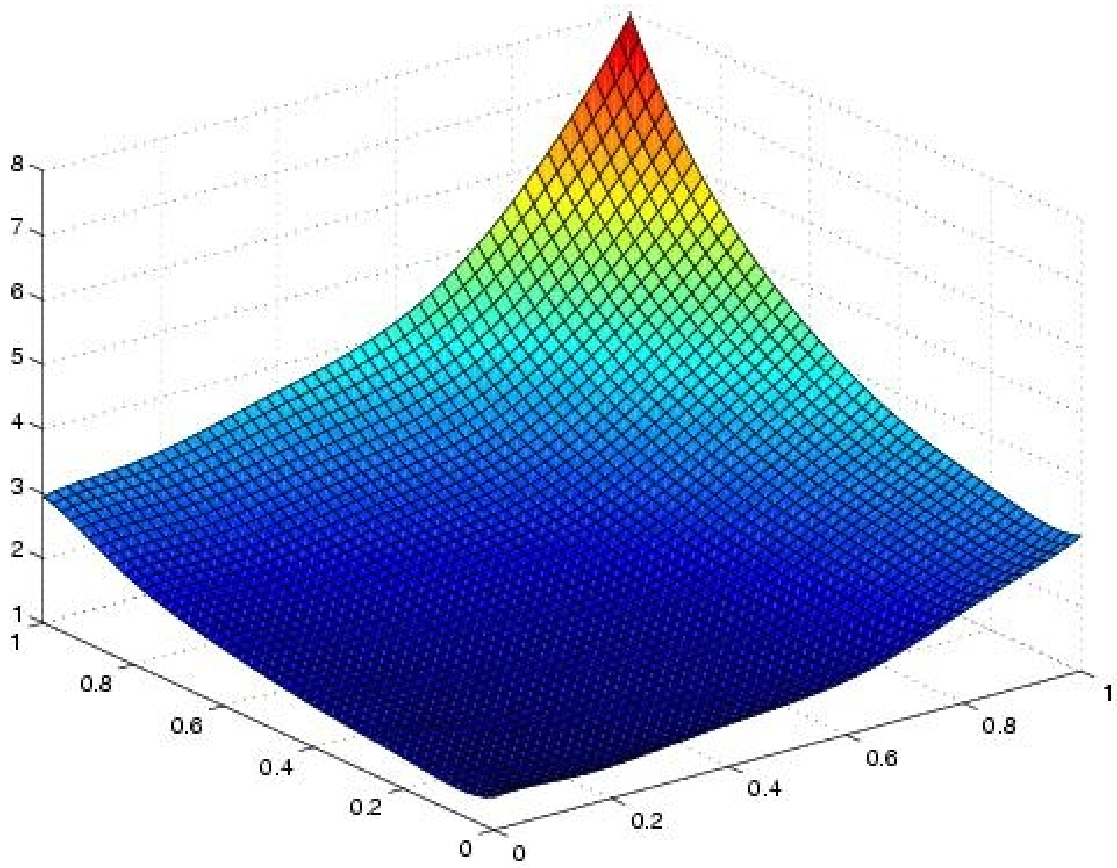
$2.a2 +12.a3+48.a4 >=0$

- Whence, we need to pose some *linear inequalities* on the variables *a0,a1,a2,…*

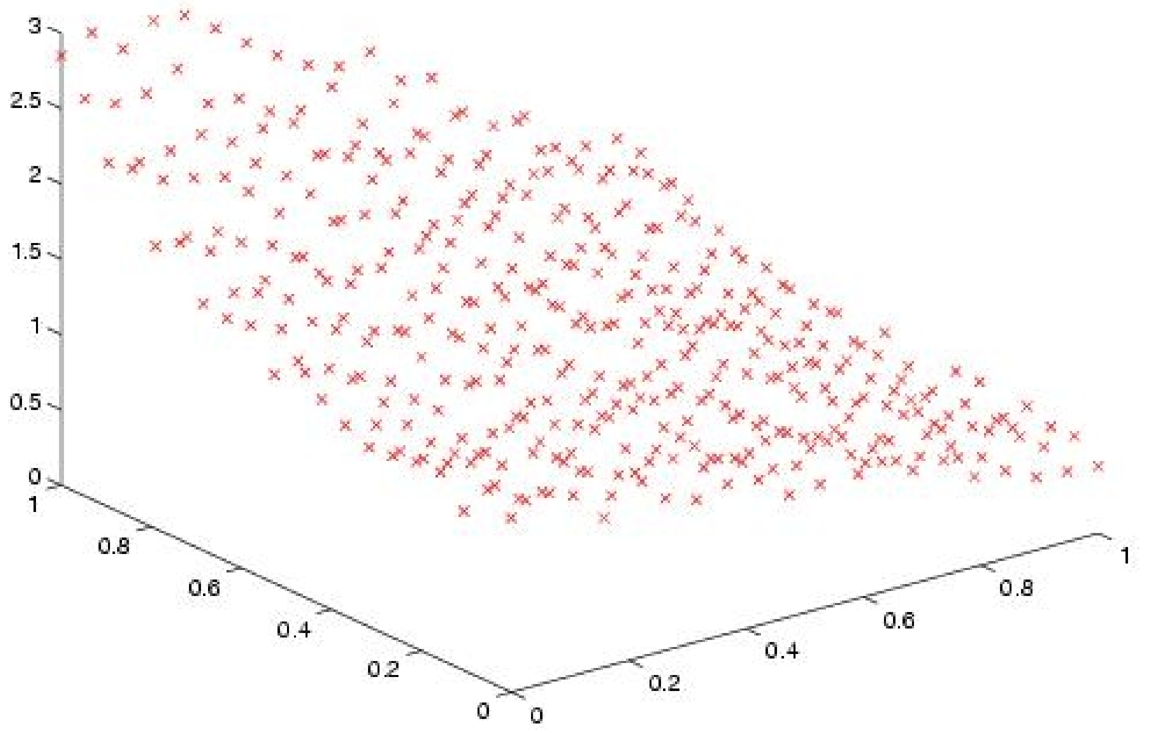
So here is the picture…



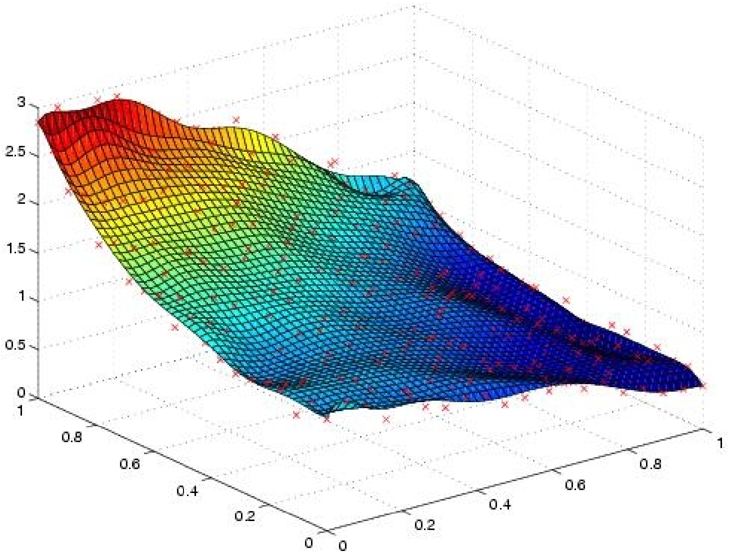
The smooth picture

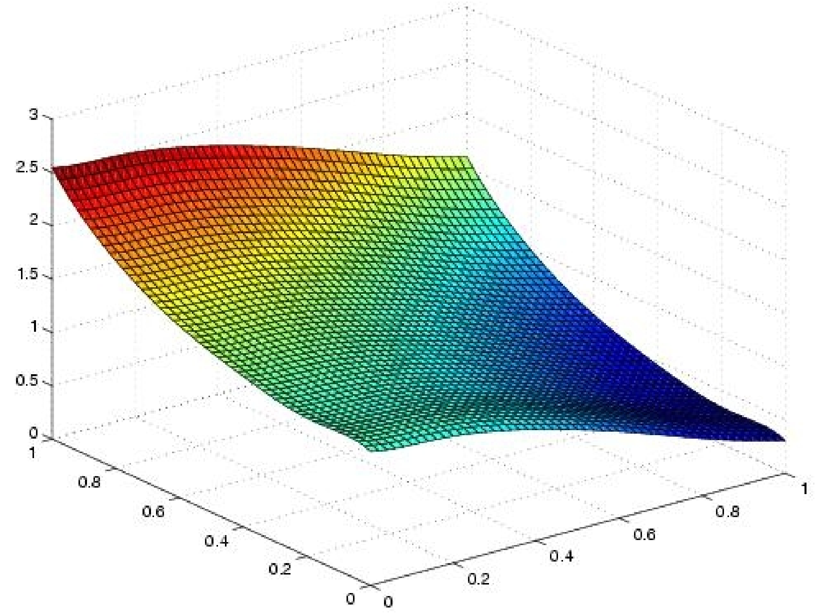


Another example



The rough and the smooth





Summary

- The Solid Modeler
- Boundary Representation
- Polynomials and Splines
- Operations
- Optimization
- Curvatures and Basic differential geometry
- Genus

Softwares

MATLAB   ProEngineer