CS 435, Linear Optimization

Tutorial 1, 2016

1. Describe a solution space of all rectangles in $\mathbb{R}^2$, (ii) all equilateral triangles in $\mathbb{R}^2$, (iii) all lines in $\mathbb{R}^2$.

2. Recall that a sequence (x_i) is convergent iff there is an x so that for every open set O containing x, there is an N (depending on x, O) such that for all $j > N$, $x_j \in O$. Let (x_i) be a sequence. We say that (y_j) is a subsequence if $y_j = x_{n_j}$ for all j and $n_1 < n_2 <$ is an increasing sequence of positive integers.

   Show that if (x_i) is a bounded sequence then it has a convergent subsequence.

3. Recall that for a set X, a topology Ξ on X is a collection of subsets of X so that (i) $\phi, X \in \Xi$, (ii) Ξ is closed under arbitrary unions and finite intersections.

   Let Ξ be a topology on X and Γ on Y. What is the topology generated by the sets $O \times O'$ where $O \in \Xi$ and $O' \in \Gamma$. Is there an open set in $\mathbb{R}^2$ in this topology which is not of the type $O \times O'$?

4. A day is modeled as an interval $[0, 1]$. The traffic on a road at any time $t \in [0, 1]$ is given by $t(1-t)^5 + 2t^5(1-t)$. An information campaign of 3 hours is to be done on this road so as to reach as many students as possible. Represent the space of possible solutions as a domain and the desired objective as a function.

5. An ant walks at the rate of 1cm per minute on a rubber band. The rubber band extends by 1m per minute. Estimate the time required for an ant to traverse the rubber band if the starting length of the rubber band was 1m.

6. Let $S \subseteq \mathbb{R}$ be a closed subset of $\mathbb{R}$. Define $g_S : \mathbb{R} \to \mathbb{R}$ as follows. For $x \in R$, let $g(x) \in S$ be the closest point to x in S. Let $d_S = \|x - g(x)\|$. Is g_S continuous? Is d_S continuous? Plot these functions when $S = [a, b] \cup [c, d]$.

7. Let $f, g : \mathbb{R} \to \mathbb{R}$ be continuous. Show that $h(x, y) = f(x) + g(y)$ and $k(x, y) = f(x)g(y)$ are also continuous.

8. Let $u_w = \log(1+x)$ be the happiness I derive from x liters of water and $u_b(x) = x$ be the happiness that I derive from x liters of beer. If the price of water and beer (in Rs./liter) is p_w and p_b, respectively, and I have R rupees to spend today, what fraction of my money will I spend on beer and water? Suppose that the government raised through taxation, the price of beer, what would its impact be on the consumption of beer by different classes of people, i.e., with different Rs?

   How would you extend this model to allow that I may decide to split my R rupees into y for today, and $R - y$ for tomorrow and that my total happiness is the sum of the happinesses of today and tomorrow?

9. Suppose that there are two candidates who must prepare for a competitive exam with a prize M. Suppose each candidate has basic preparation α_i and spends an additional x_i units of time in preparing for the exam. The winner of the exam is the index, i for whom $\alpha_i + x_i$ is maximized. The returns to the winner is $M - x_i$ while for the loser it is $-x_i$. Define the functions $P_1(x_1, x_2)$ and $P_2(x_1, x_2)$, the winnings for each candidate carefully. Is it continuous? Supposing all of the data were known to the competitors, what would be the outcome of the exam?