CS 435, Linear Optimization

Tutorial 3, 2016

1. Modify the `closest.sci` algorithm so that it maintains a convex combination for the points x_n. Is there a way to guess the limit point?

2. Let D be given by the following system:

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \leq \begin{bmatrix} 3 \\ 0 \\ 3 \\ 0 \\ 3 \\ 0 \\ 5 \\ 5 \\ 5 \\ 6 \end{bmatrix}$$

   Start with $x_0 = (0.1, 0.1, 0.1)$. Check that x_0 is feasible. Experiment `mylp.sci` with various cost vectors.

3. Modify `mylp.sci` to find the closest point to p in a domain given as $D = \{x | Ax^T \leq b\}$.

4. Given the space $S = \{x \in \mathbb{R}^n | Ax^T = b\}$ and a point $p \in \mathbb{R}^n$, find an expression for the closest point to p in S.

5. We are given vectors $A = \{a_1, \ldots, a_m\} \subseteq \mathbb{R}^n$ and a vector $c \in \mathbb{R}^n$. Use `mylp` to determine if $c \in cone(A)$. Assume that `mylp` works for arbitrary inputs. How will you determine if $cone(A) = \mathbb{R}^n$.

6. Let P be in primal form, i.e., in the form $\max cx^T, Ax^T \leq b$. Look at the dual DP, convert it into an equivalent primal form and compute its dual DDP. Show that DDP is equivalent to P. Alternately, express P is dual form and compute its primal. Show equivalence.

7. Use `mylp` to solve the householder's problem. See Diary. Verify that the point returned by `mylp` is indeed optimal. What are the dual values? Modify `mylp` to output dual values.

8. Given a directed graph $G(V, E)$ with m edges and n vertices, let B be its edge-vertex adjacency matrix. What is $rank(B)$? What are the spaces $X = \{x | Ax^T = 0\}$ and $Y = \{y | yA = 0\}$. What would be a nice basis for the space Y?

9. For the shortest-path problem, construct the primal as done in class. What would be the set of optimal points for the primal? Why is there a multitude of optimal points and how would you circumvent this?

10. Consider the problem of building models for data. Let us suppose that we have the data $\{(x_i, y_i) | i = 1, \ldots, n\}$. Suppose that for this we want to build a model $y = ax + b$. Write this as a non-linear optimization problem. If there are no contraints on a, b, how would you solve this? How would you generalize to higher degrees?

11. Suppose that there was also the constraint that $a \geq 0$. Do a region-wise analysis to identify conditions under which the optimal point would be in these regions.

12. How would you moidfy `mylp.sci` to account for any quadratic function? Describe the strategy in words and then its implementation.