25/03/2014

Recall In the last class we studied

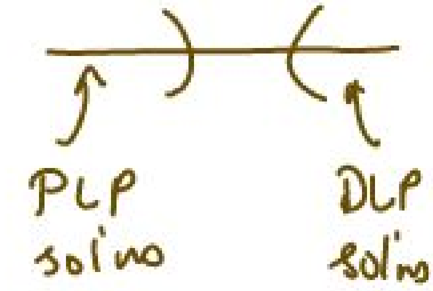
Duality

We showed that

(i) if x_0 is feasible in PLP

y_0 is feasible in DLP

then $cx_0 \leq y_0 b$



(ii) If PLP has an opt pt x_0 then $\exists y_0$ feasible in DLP s.t.

$$cx_0 = y_0 b$$

(iii) If x_0, y_0 s.t. $y_0 b = (Ax_0 - b)$

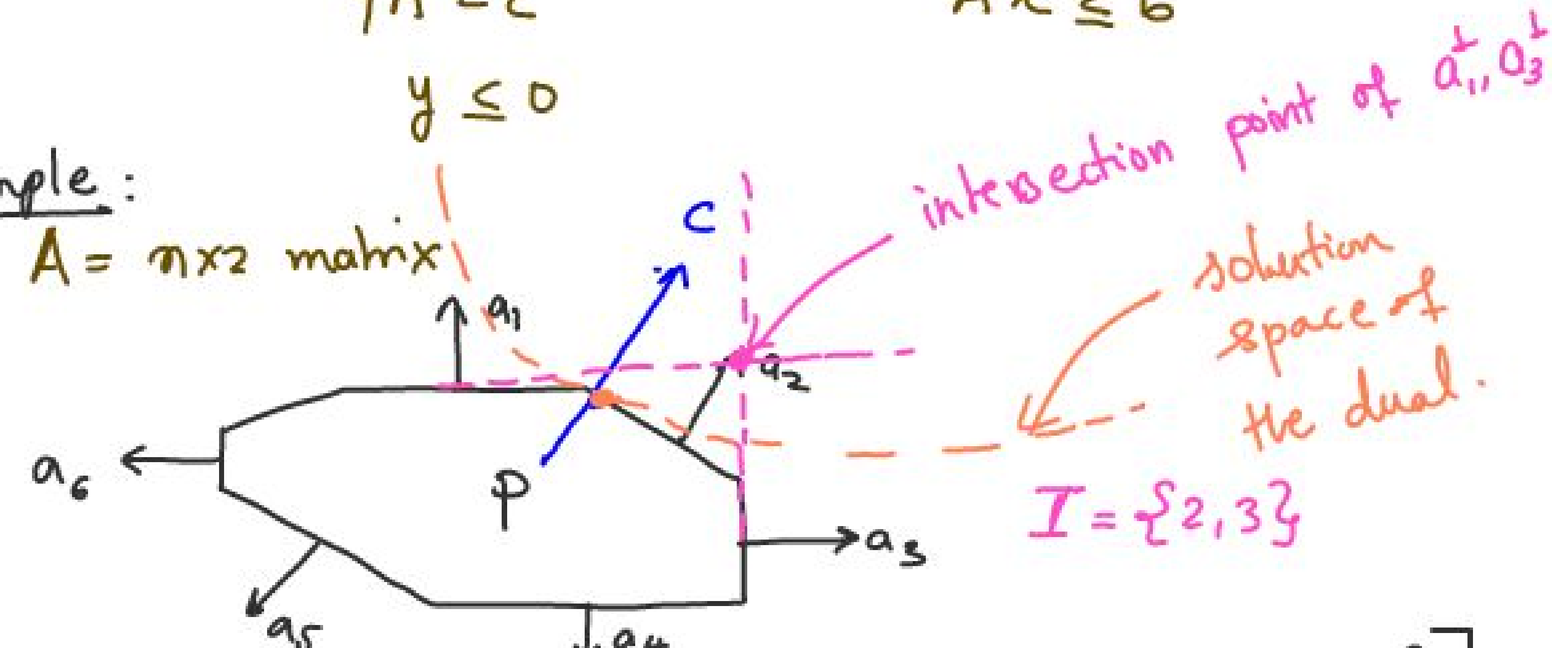
then x_0, y_0 are opt.

Today:

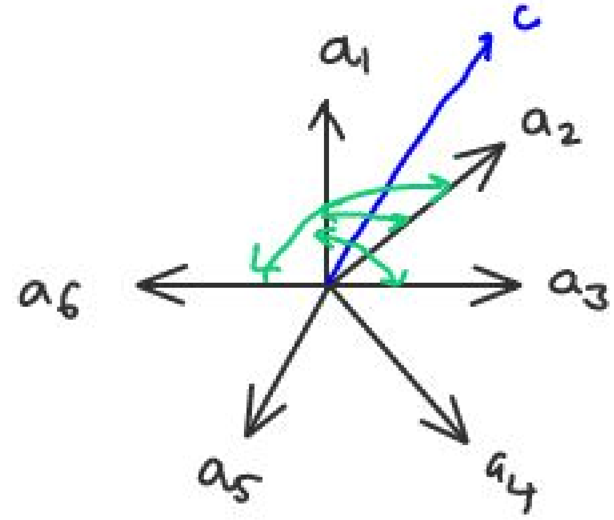
$$\min\ yb \qquad\qquad \max\ cx$$
$$yA = c \qquad\qquad Ax \leq b$$
$$y \leq 0$$

Example:

A = n×2 matrix



[Qn: Which a_i, a_j pairs include c in their cone?]



Cone (a_1, a_3) include c

cone (a_1, a_2) — " —

⋮

Cone (a_5, a_4) does not include c.

$$\underbrace{[y_1 \; y_2]}_{} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = [c]$$

$\uparrow$ A_I

$$[y_1 \; y_2] = c\, A_I^{-1}$$

$$[y_1, y_2] \cdot b = c \cdot A_I^{-1} b$$

Suppose $\exists$ I, subset of rows, s.t. $y_I \geq 0$

$y_{\bar{I}} = 0$

$y = [y_I, y_{\bar{I}}]$ then $yb = [y_I, y_{\bar{I}}] \begin{bmatrix} b_I \\ b_{\bar{I}} \end{bmatrix}$

$$= y_I \cdot b_I + \underset{\overset{\parallel}{0}}{y_{\bar{I}}} \cdot b_{\bar{I}}$$

As long as $c \in$ cone (a_i, a_j) we can express it in terms of (a_i, a_j)

Say for example $c \in$ cone (a_1, a_3) $\quad I = \{2, 3\}$

$$yb = y_I b_I = c \cdot \underbrace{A_I^{-1} b_I}_{x_I}$$

where x_I is the sol'n for $A_I x = b_I$.

Let us look at the dual.

find $|I| = 2$

$c \in$ cone (A_I)

for these $a_i, a_j \in A_I$

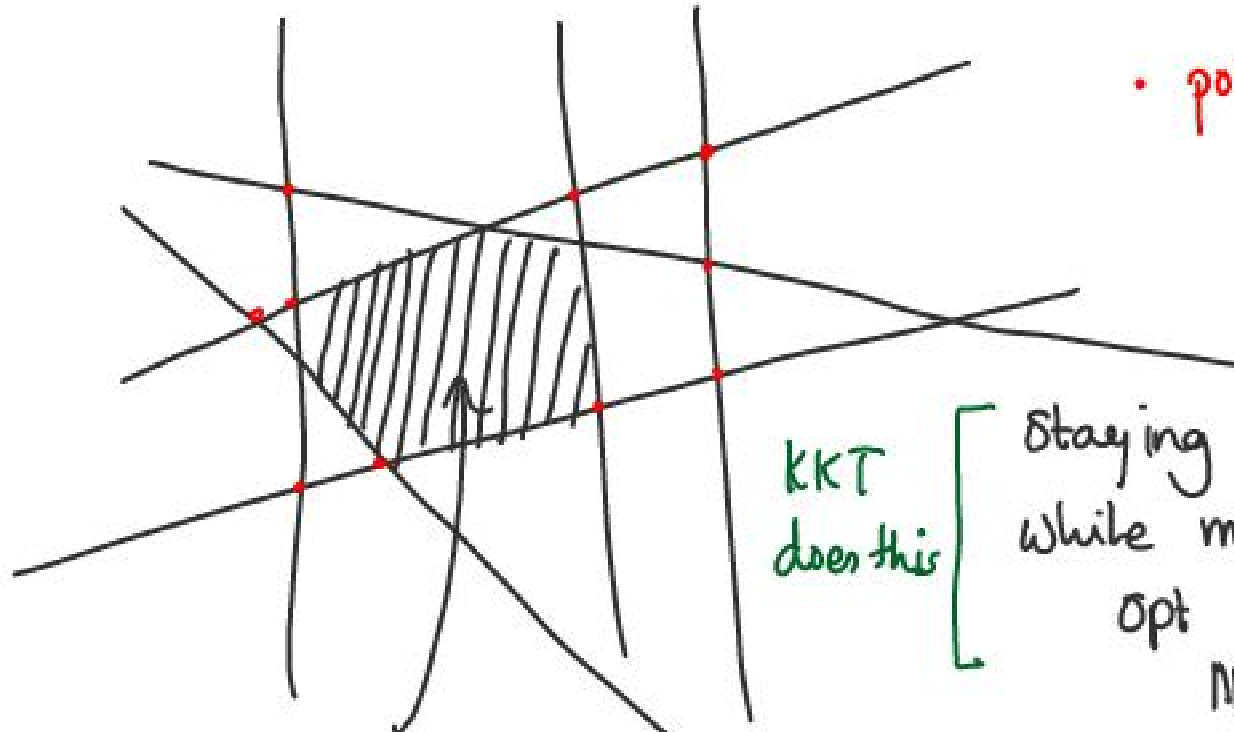
find the point where they meet. giving point b.

(Show by - - - arrows).

This point is in the dual space

($\because$ it is a feasible point)

Now evaluate the cost vector at $(A_I^{-1} b_I) = x_I$



· points of interest.

feasible

KKT does this: Staying feasible while moving towards opt

$\neq$

this is what we just described: remain opt all the time as achieve feasibility

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$$\begin{array}{ll} \min yb & \max cx \\ yA = c & Ax \le b \\ y \ge 0 & \end{array}$$

$$yA = c$$

$$[\ x\ ]\begin{bmatrix} A \end{bmatrix} = [\quad]$$

$$y \ge 0 \qquad A^T y^T = c^T$$
$$-y^T \le 0$$
$$\max -b^T y^T$$

$$\begin{bmatrix} A^T \\ -A^T \\ -I \end{bmatrix} [y_I] = \begin{bmatrix} c^T \\ -c^T \\ 0 \end{bmatrix}$$

Vars

$$\begin{array}{c} X_1 \\ X_2 \\ S \end{array} \quad A' = \begin{bmatrix} A^T \\ -A^T \\ -I \end{bmatrix}, \quad b' = \begin{bmatrix} c^T \\ -c^T \\ 0 \end{bmatrix}, \quad c' = -b^T$$

$(2n+m)\times(m)$ matrix

$\therefore$ we get $\min \{ x_1 c^T - x_2 c^T \}$

subject to $[x_1 \ x_2 \ S] A' = C'$

$x_1 \geq 0, x_2 \geq 0, s \geq 0$

Now consider $[A \ \ -A \ \ I] \begin{bmatrix} X_1^T \\ X_2^T \\ S \end{bmatrix} = -\begin{bmatrix} b \end{bmatrix}$

$[A \ \ -A \ \ -I] \begin{bmatrix} X_1^T \\ X_2^T \\ S \end{bmatrix} = -\begin{bmatrix} b \end{bmatrix}$

Basically this machinery is to make sure that unconstrained x in the program $\max\{Cx\}$ $Ax \leq b$ by x' vars wher each x' is +ve.

Let $\max\{cx\}$ $Ax \leq b$ let us replace each $x \in X$ by $x = x^1 - x^2$ where each $x^1, x^2 \geq 0$

$X_1 = \bigcup_{x \in X} \{x^1\}$ $\quad X_2 = \bigcup_{x \in X} \{x^2\}$