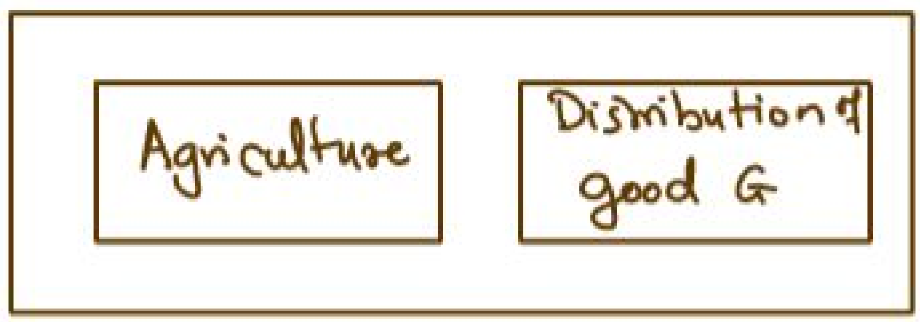
$f: D \rightarrow \mathbb{R}$ optimize f over D.

Paper by Anne Kruger "The PE on rent seeking".

<u>Classical System</u>



L_A: fraction employed in agriculture.

$1 - L_A = L_D$: —— " —— distribution.

Let M be the number of imported color TV.

Price seen by society $= P_m$: price seen by society.

$$P_m = 1 + P_D$$

$\uparrow$ distribution costs.

Assuming that there are no restrictions on imports

<u>Q1</u>: What is M : total imports?

<u>Q2</u>: What is L_A, L_D (recall $L_A + L_D = 1$)

<u>Q3</u>: What is P_D?

Let us try to answer these three questions.

Agricultural productivity $= f(L_A)$

$= A$.

Demand for G (obtained by applying "clearing market" argument, we get $d(A, P_M)$ $A\uparrow$, $P_M \downarrow$

Eq'n 1: (obtained by saying supply = demand)

$$\underbrace{d(f(L_A)}_{\text{var 1}}, \underbrace{1+P_D)}_{\text{var 2}} = \underbrace{M}_{\text{var 3}}$$

Eq'n 2: # people needed to distribute M goods is $1-L_A$.

Eq'n 3: Wages are equal (ie. people are free to move from distribution to agriculture)

Wages at A: $\partial A / \partial L_A$.

Wages at D: $\frac{\text{Revenues}}{\text{Employer}} = \frac{P_D \cdot M}{1-L_A}$.

Possible f functions:

1. $f(L_A) = \sqrt{L_A}$

$d(A, P_M) = \frac{s \cdot A}{1+P_D} = \frac{sA}{P_M}$

scaling factor

% of Agricultural input spent on distribution sector

∴ Now this is how Eq'n 1, 2, 3 look:-

Eq'n 1: $\frac{s\sqrt{L_A}}{1+P_D} = M$

Eq'n 2: $1-L_A = kM$

Eq'n 3: $\frac{1}{2\sqrt{L_A}} = \frac{P_D M}{1-L_A}$

2. Suppose M is restricted to a constant $\overline{M}$.

$$\frac{8\sqrt{L_A}}{1+P_D} = \overline{M}. \quad \text{— } \underline{\text{Eq'n 1}}$$

$$1 - L_A = k\overline{M} \quad \text{— } \underline{\text{Eq'n 2}}$$

However, Eq'n 3 no more holds. ($\because$ $\overline{M}$ is a constant.

$\therefore$ $L_D = 1 - L_A$ is fixed, assuming k & $\overline{M}$ are fixed.

$\therefore$ No free movement between distributn & agriculture.)

See $\overline{M}$ related entries in Table 2.

We obsere that wages of distributn workers increase (almost 10 times that of wages of agri workers).

One possible response to this wage differential is:

Suppose $\overline{m}$ is an artifact of the system (i.e. industry can not produce more than $\overline{m}$). Now due to this rent may get created. But industry takes note of this situation increases productn & raises $\overline{m}$ to nullify rents.

$\therefore$ The game goes back to being where there is no $\overline{m}$.

The other possibility is :-

3rd segment of populat'n will emerge which is seeking to become rentiers. To model this :-

Anne Kruger introduces a new variable

L_A'', L_R'', L_D''

↑ rent seekers.

<u>Eq'n 1</u> : $\bar{M} = \dfrac{s\sqrt{L_A''}}{1+P_D}$

<u>Eq'n 2</u> : $L_D'' = kM$

Average wages in Agriculture

= Average wages in Distribut'n.

$$\frac{1}{2\sqrt{L_A''}} = \frac{P_D''\cdot M}{1-L_A''}.$$

<u>Game theory</u>

agents have private values for goods

a_1	3 Rs	v_1
a_2	5 Rs	v_2
⋮	⋮	⋮
a_n	.	v_n

Let a_1 bid b_1
a_2 bid b_2
⋮
a_n bid b_n

$\boxed{\text{Mechanism : Auction}}$

M: selects the highest bid
allots pumpkin
takes payment.

if k bids are equal then uniformly randomly allots pumpkin.

Given the choices $b_1 \in S_1, \ldots, b_n \in S_n$

Say $n = 3$

<u>Example 1</u> $u_1(2,2,3) = 0$, $u_2(2,2,3) = 0$, $u_3(2,2,3) = v_3 - b_3 = 3$

<u>Example 2</u> $u_1(4,2,3) = v_1 - b_1 = 0$, $u_2(4,2,3) = 0$, $u_3(4,2,3) = 0$

<u>Example 3</u> $u_1(3,2,3) = \frac{1}{2}(1) + \frac{1}{2}(0) = 1/2$

$u_2(3,2,3) = 0$

$u_3(3,2,3) = \frac{1}{2}(3) + \frac{1}{2}(0) = 3/2$

So far our optimization problem only consisted of a single player. In multi-player setting, what is the correct optimization problem?

<u>Goal</u>: Each agent i wishes to maximize u_i

"Optimal?" : Nash Equilibrium

$$\bar{s} = (s_1, s_2, \ldots, s_n) \in S_1 \times S_2 \times \cdots \times S_n$$

$$u_1(s_1, s_2, \ldots, s_n) \geq u_1(s_1', s_2, \ldots, s_n) \quad \forall s_1' \in S_1$$

$$\vdots$$

$$u_i(s_1, \ldots, s_i, \ldots, s_n) \geq u_i(s_1, \ldots, s_i', \ldots, s_n) \quad \forall s_i' \in S_i$$

$$\vdots$$

$$u_n(s_1, s_2, \ldots, s_n) \geq u_n(s_1, s_2, \ldots, s_n') \quad \forall s_n' \in S_n$$

$$\text{BestResp}_1 : S_2 \times \cdots \times S_n \longrightarrow S_1$$

$$\text{BestResp}_1(s_2, \ldots, s_n) = \{ s_1 \in S_1 \mid \max u_1(s_1, \ldots, s_n) \}$$

$\therefore$ Nash equilibrium is the state where each player is playing the best response.

<u>Imp to note</u>: If $\bar{s}$ is nash equilibrium then it need <u>not</u> be that $u_1(\bar{s}) + \cdots + u_n(\bar{s})$ is maximized.

Yet another way to define an optimization problem:

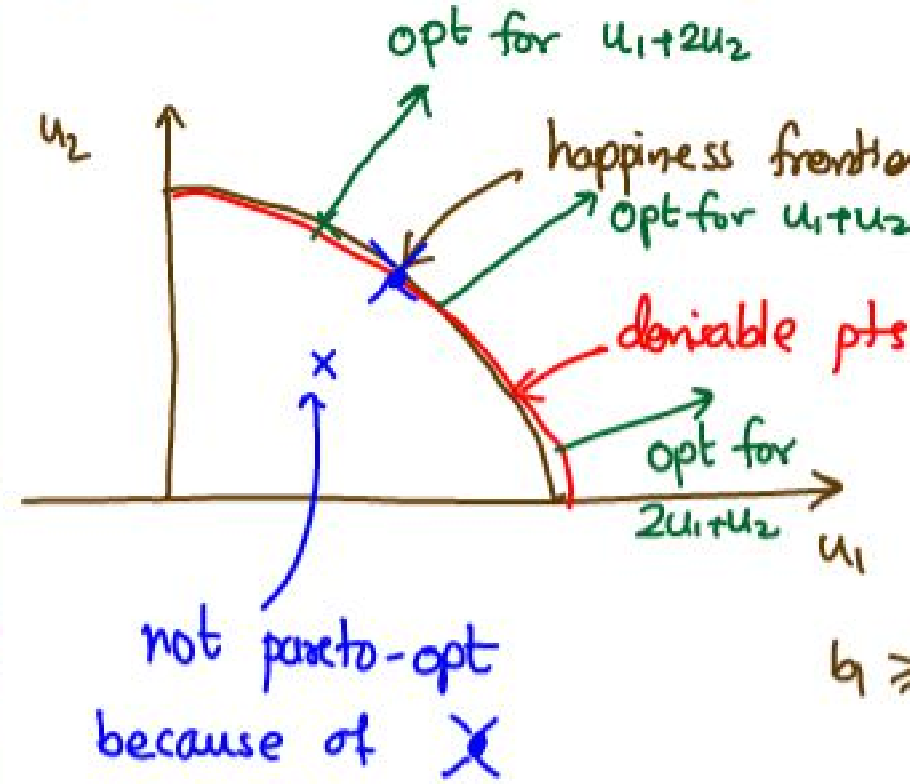
<u>Social Welfare</u>: $f : u_1(\bar{s}) + u_2(\bar{s}) + \cdots + u_n(\bar{s})$

Max f $\quad \bar{s}_0$ be the opt.

Now $\underbrace{U(\bar{s}_0)}_{\text{social welfare opt}} - \underbrace{U(\bar{s})}_{\text{nash equilibrium}} \longrightarrow$ Gap of anarchy.

Social Welfare as the choice for opt <u>may not</u> make sense: It may bag together or correlate agents which may not be correlated in their strategy space.

$S_1 \times S_2 \quad (u_1(s_1, s_2), u_2(s_1, s_2))$



Name of the game

NE may not be pareto-opt.
But pareto-opt may want to be achieved.
$\therefore$ The fns need to be designed "cleverly" so that they extend and make pareto-opt the equilibrium

Desirable outcome is pareto-optimal

(a_1, a_2) is pareto-opt for set D if there is no (b_1, b_2) s.t. $b_1 \geq a_1$, and $b_2 \geq a_2$.

Imp to Note: ① NE may not exist, may not be unique
② An NE $\bar{s}$ may not be pareto-opt.

Example [Prisoners' Dilemma].

$S_1 = \{\text{Fink, Don't Fink}\} \qquad S_2 = \{\text{Fink, Don't fink}\}$

u_1	F	D
F	-5	-1
D	-10	-2

u_2	F	D
F	-5	-10
D	-1	-2

is (D,D) the NE? : $BR_1(?, D) = F$

$BR_2(D, ?) = F$

$\therefore$ (D, D) is not a NE.

is (F, F) a NE? : $BR_1(?, F) = F$.

$BR_2(F, ?) = F$

$\therefore$ If players play (D, D) this is not a NE.

This a social agreement based on some faith.

Example

$(v_1, v_2, \ldots, v_n)$: private values.

$(b_1, b_2, \ldots, b_n)$: bids

Highest bid, highest price auctions:

Expected Output : $(v_1, \ldots, v_n)$ is NE.

But it is not: Say $v_1 \leq \ldots \leq v_n$

for this payoff $(0, \ldots, 0)$

But if n^{th} player knows that others have played

then $(v_1, v_2, \ldots, v_{n-1}, v_n - \epsilon)$ gives payoff

$(0, 0, \ldots, 0, \epsilon)$

Truthfulness is not NE, in fact there is no NE.

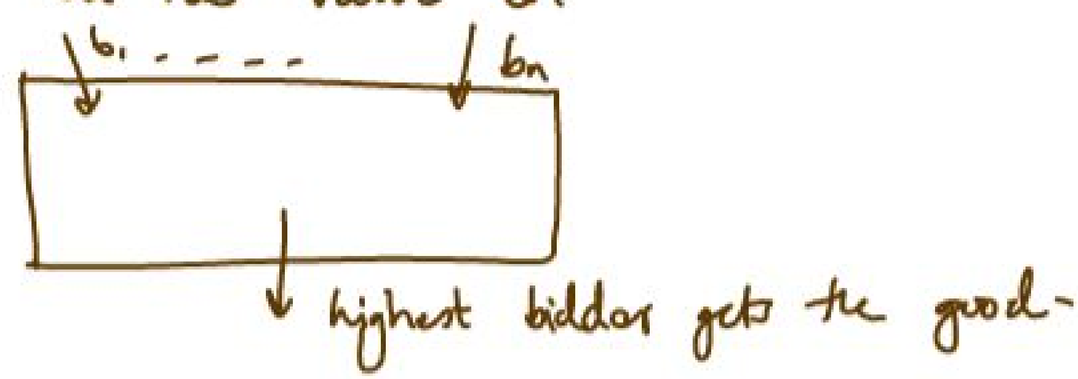
In fact it has no nash-equilibrium.

07/04/2014

Last time : We saw prisoner's dilemma.

& Saw an auction setting where

a_i has value v_i:



We saw that this has no NE.

Today : 2^{nd} price auction.

highest bidder wins the good but pays the bid of the 2^{nd} highest.

Claim : $(v_1, v_2, \ldots, v_n)$ is the NE in the 2^{nd} price auction.

proof : $b_1 = v_1, \ldots, b_n = v_n$.

Suppose $\exists a_j$ $(j \neq 1)$ who deviates

$(v_1, v_2, \ldots, v_j^*, v_{j+1}, \ldots, v_n)$

For the outcome to change $v_j^* > v_1$

$\therefore$ Outcome for a_j : $\underbrace{v_j}_{\text{true value}} - \underbrace{v_1}_{\text{pays}} < 0$

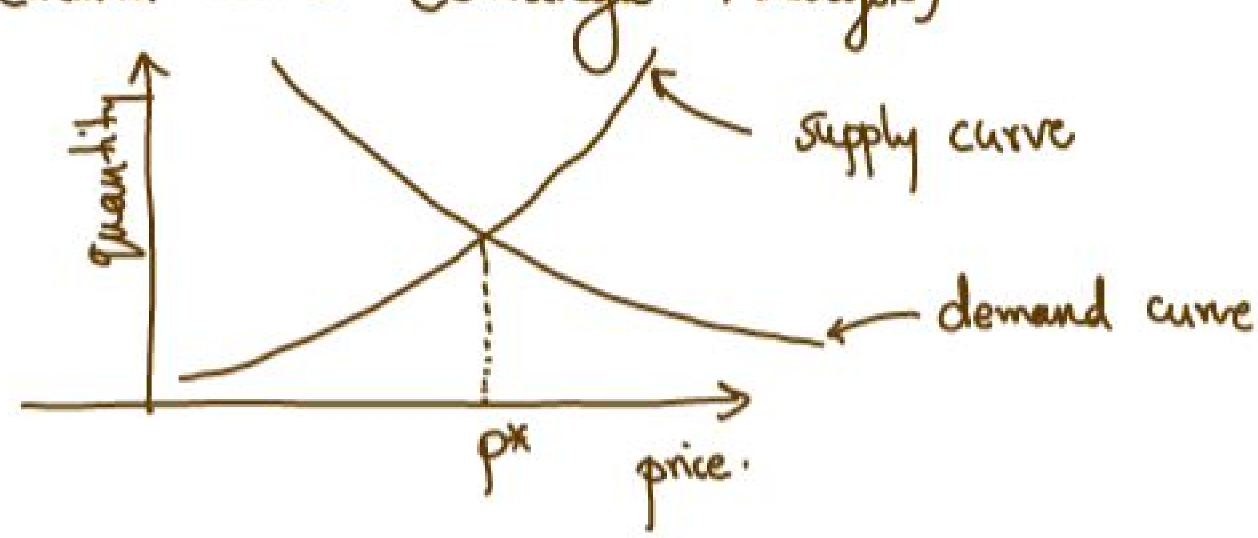
$\because$ this becomes <0, no incentive to move.

Now let us consider a_1 : $(v_1^*, v_2, \ldots, v_n)$

as long as a_1 wins, her outcome will be $(v_2 - v_1)$, i.e. does not change.

$\therefore$ she too has no incentive to move.

Supply Demand Curve (Strategic Analysis)



p^* : supply = demand and the market clears.

Tatonnement process

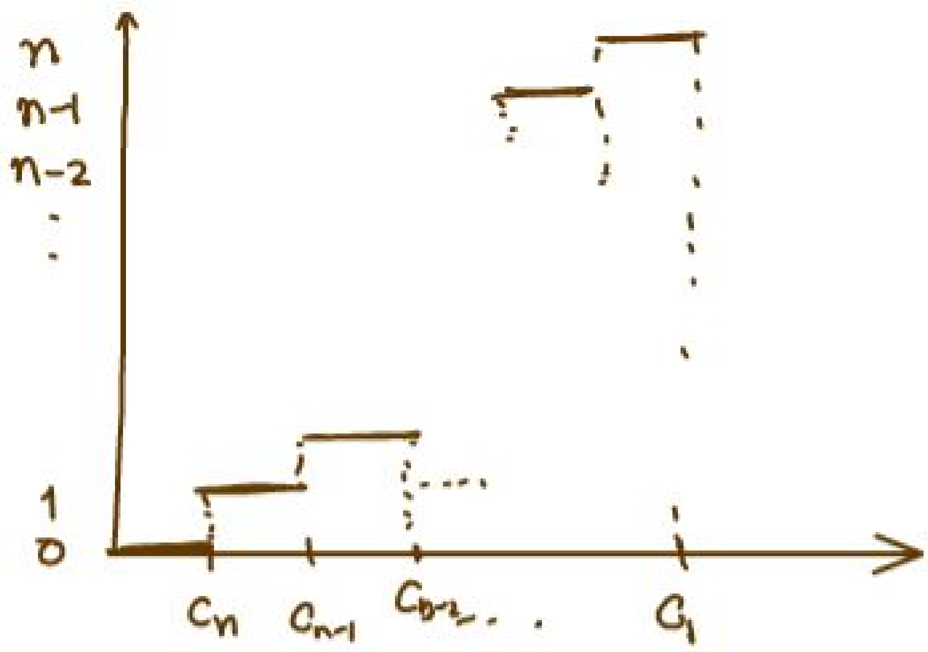
Suppose $b_1, b_2, \ldots, b_n$ buyers

$p_1 > p_2 > \cdots > p_n$ prices

$a_1, a_2, \ldots, a_n$ producers

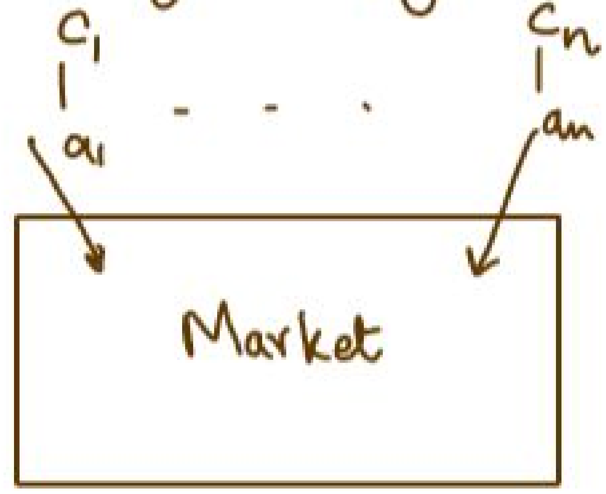
$c_1 > c_2 > \cdots > c_n$ costs of production

Demand Curve : (Not continuous)



Supply Curve.

How should we design a payoff



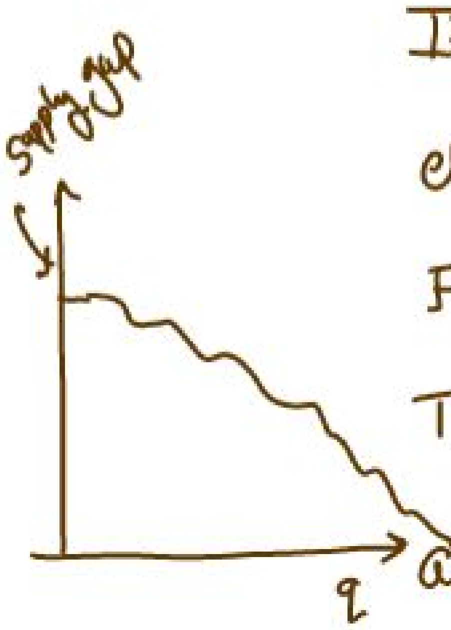
Suppose a ~~s~~ supplier bids q.

Let $D(q)$ = # of buyers who can affor q

$= |\{ p_i \mid p_i \geq q \}|$

$S(q)$ = #suppliers who bit lower than q

$= |\{ q_i \mid q_i \leq q \}|$



If $D(q) < S(q)$ then outcome $= 0$

else $(D(q) \geq S(q))$ then $q - c$

For a price q, let us compute $D(q) - S(q)$

This supplygap (q) is $\downarrow$ function, which drops by 1 at each value p_i, c_i

Now the values of q s.t. $S(q) = D(q)$ be p^*

<u>Claim</u>: Given that $(c_1, c_2, \ldots, c_n)$

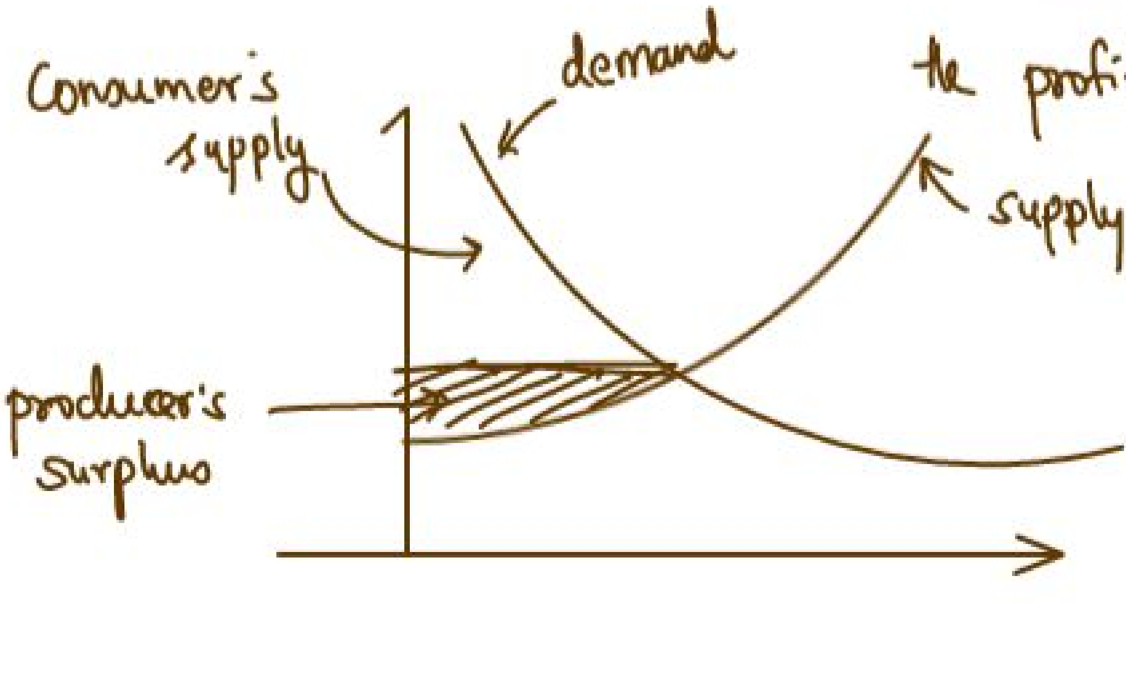
$NE = (c_1, c_2, \ldots, c_k, p^*, p^*, \ldots, p^*)$, where k such that $c_k > p^* > c_{k+1}$.

<u>proof</u>: $D(p^*) = S(p^*) = n-k$.

$1 \le i \le k$: i^{th} player sells at lower cost they may sell the lemon but at a loss. $\therefore$ They will not lower their bid to p^*.

$k+1 \le i \le n$: no incentive to increasing the bid beyond p^* $\because$ increasing it will lead to "no sell".

no incentive to drop prices as the profit will be reduced.



— Market clearing prices are established under

1. Risk taking sellers
2. Completely price dependent buyers
3. The primary source of producers, consumer surplus.
4. Gap of anarchy is huge if the consumers are varied.

10/04/2014

n sellers $\quad c_1 > c_2 \dots > c_n$

n buyers $\quad p_1 > p_2 \dots > p_n$

Market players/agents : sellers

Strategy $s_i : [0, \infty]$

Mechanism : Sellers are risk averse (i.e. they value guaranteed outcome).

NE : p^* price where supply(p^*) = demand(p^*).

Note that in the above mechanism

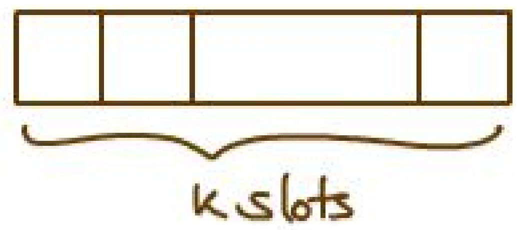
* Market clears (there may be unmet demand and unmet capacity to produce). S', B'

* matching with the least cost producer & the most able consumers

* S', B' will not be able to conduct any additional transactions. All "redundancy" is allotted as consumers & producers surplus.

* Big gap between "optimal" vs NE.

"Mandi"



$p_1 > p_2 \dots > p_n$
$c_1 > c_2 \dots > c_n$

k : limit on transactions (#sellers) & $n > k$

(i) Parrabhani Muncipal Corporation (PMC): auction the stalls where sellers bid.

(ii) k chosen sellers fight the market game.

$a_1 \quad a_2 \quad \dots \quad a_n$

$v_1 \quad v_2 \quad \dots \quad v_n$ || k winners

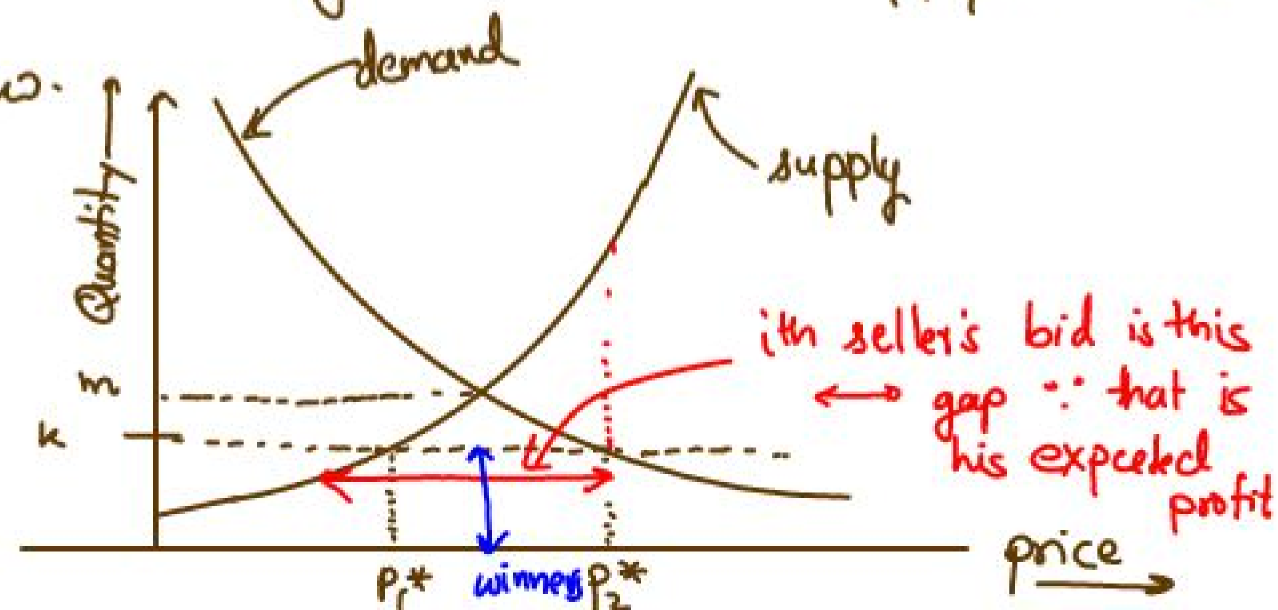
$b_1 \quad b_2 \quad \dots \quad b_n$

$$u_i = \begin{cases} v_i - b_i & \text{if you get a slot} \\ 0 & \text{o/w.} \end{cases}$$

$\boxed{\text{PMC}}$ top k bids administer price b_{k+1}.

<u>Observe</u>: NE will be for $b_i = v_i \ \forall i$

Let us understand the game from the supply-demand point of view.



Note that such a situation may arise when there is no "mandi" just because buyers have a tendency to buy from the first k sellers.

<u>Seller's analysis</u> : p_i^* is possible $\because$ $D(p_i^*) \geq k$.

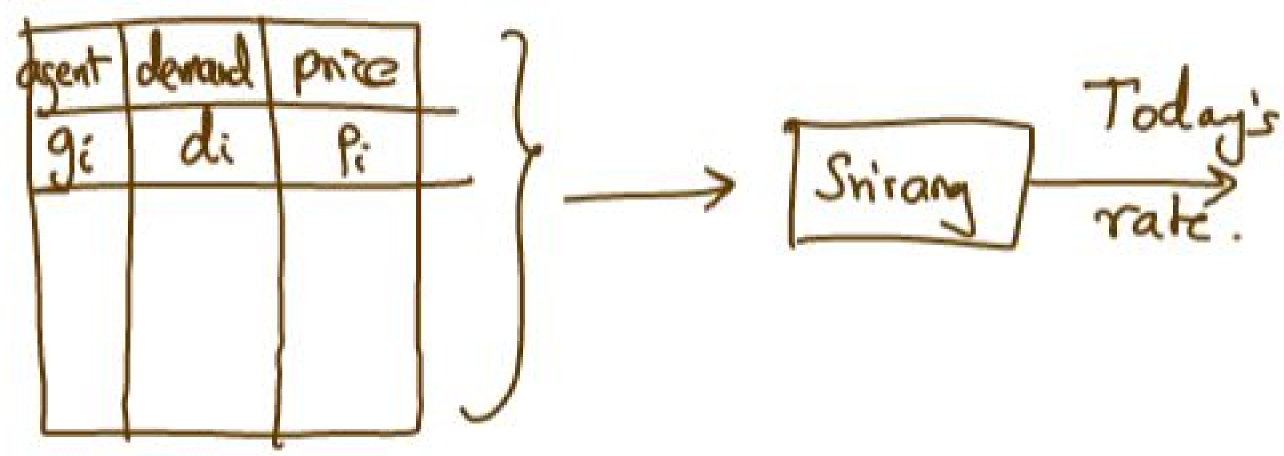
Now the value of each seller is : $v_i = p_i^* - c_i$

Net Revenue

The chosen sellers will be the lowest k cost price sellers i.e. and the bid administered will be $p_i^* - p_s^*$ where p_s^* is such that $S(p_s^*) = k$.

<u>Srirang's game</u> :

Cow A 50 litres of milk



This is a monopoly

The market may not clear if you wish to maximize profit.

Now suppose two sellers.

Sri Sid

g_i = gopis are the agents

$s_i = \{sid, sri\}$

This game has asymmetric nash equilibria.