

# Mathematics for Optimization and Modelling

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**Outline and Objectives.** The course is aimed to prepare students who wish to pursue *applied* research which uses techniques from optimization, game theory and economic modelling. The course will assume that students have an exposure to elementary mathematical notions of real analysis and linear algebra.

1. **The topology of real numbers.** Open and closed sets. Continuity. Sequences and their limit points. Compactness and the Heine-Borel theorem. Applications-existence of solutions, absolute continuity.
2. **Linear Algebra I (LA-I).** Vector spaces, linear dependence and bases. Examples. The homogenous solution and dimension. Linear maps and the rank-nullity theorem.
3. **The optimization framework and convexity.** The domain and the functional. Existence of solution. Local and global optima. Convexity. Prelude to the Karush-Kuhn-Tucker (KKT).
4. **General Domains and the KKT.** Properties of general half-spaces. Infinitesimal motions and normals. The combinatorics of tight-sets. Examples. The interior point algorithm and its implementation.
5. **Linear and Quadratic programs.** The KKT conditions. The simplex and other combinatorial methods.
6. **Flows and Paths.** A special case of LP/QP and their applications.
7. **Basic Games.** Models of strategic behaviour and their solutions. Bimatrix and combinatorial games. Optimal points versus Nash Equilibria. Infinitesimal analysis and KKT.
8. **The Bayesian Game.** The need for Bayesian analysis. The key theorems and some examples. The street-vendor problem.
9. **Games from Economics.** The bargaining game. The reputation games. The Auction. The adverse selection.
10. **Markets.** The Arrow-Debreu model and the Fisher model. The convex program for the Fisher model and KKT. Standard equilibrium and strategy. Various versions of markets and their imperfections.

11. **The placement game.** The mis-allocation, rents and all-pay auction.