

TD 610
Tutorial 3
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1. Consider a function $f(x, y)$ and a starting point p . Suppose that the objective is to find a zero of f . How would you choose the next iterate? Consider the case when $f = x$ and $g = x^3$. When will your algorithm be faster?
2. Let $x_1, \dots, x_k$ be points in $\mathbb{R}^m$. We say that y is a convex combination of the x_i 's if there are non-negative reals λ_i such that $\sum_i \lambda_i x_i = y$ and $\sum_i \lambda_i = 1$. Try your hand at understanding the collection of all convex combinations of $[0, 0]$, $[0, 1]$, $[1, 0] \in \mathbb{R}^2$.
3. As above, let $y_1, \dots, y_r$ each be convex combinations of the x 's and let z be a convex combination of the y_j 's. Show that z is a convex combination of the x_i 's.
4. Let $y = \sqrt{x}$ be the production function of a rice-farmer, where x is the input (in days) and y is the output in Rs '000. Suppose the farmer makes Rs. 100 a day in basket-weaving. Find his best operating point for the farm. Next, suppose that the landlord imposes a tax of 25% on the output. What would be the new operating point and what would be the landlord's earning? Suppose that the landlord had imposed a tax of that amount, what would be the extra value created?
5. Show that the function $\sin x + \cos x$ is *concave* between 0 and $\pi/2$.
6. Let $C \subseteq \mathbb{R}^m$ be compact and convex and let $y \notin C$. Show that there is a linear function ℓ such that $\ell(y) > \ell(x)$ for all points $x \in C$. Thus a *hyperplane* separates y from C . Try this out for $m = 2$.
7. For a commodity C (say onions), whose price p varies with time t . Suppose that there is a decreasing function $d(p)$ which gives us the *static* demand for the commodity, when the price is static and equals p . However, many times, the demand also depends on $p'(t)$, the derivative of the price. Thus, suppose that the demand is given by $d(p) - m \cdot p'(t)$. Given a cyclical price function $p(t)$, find the demand as a function of time.