TD 610

Tutorial 4

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1. Consider the 3-D domain given by $x^2 + y^2 + z^2 \geq 4$, $0 \leq x \leq 1$, $0 \leq y \leq 1$ and $0 \leq z \leq 3$. Show that the domain D above has 8 vertices and 12 edges. Illustrate points x in the domain so that $|I(x)| = 0, 1, 2, 3$. What changes if $z \leq 3$ is changed to $z \leq 2$? *Hint: Degeneracy.*

2. Given 3 vectors v_1, v_2 and v_3 in $\mathbb{R}^3$, how would you determine if a given vector v is in the cone formed by v_1, v_2, v_3. Try this out for $[1, 1, 0], [0, 1, 1], [1, 0, 1]$ with $v = [1, 1, 1]$ and $v = [4, 1, 1]$. Given $v = [a, b, c]$, what are the conditions on a, b, c for v to belong to the cone formed by v_1, v_2, v_3? Note that these are linear conditions. Also, generalize when v_i are general.

3. Do the above when only two vectors are given. This arises when $I(x)$ has only two tight equations.

4. Generalize the above in $\mathbb{R}^n$.

5. Let an ellipsoid in 3-space be given by

$$x^2 + 2y^2 + 3z^2 = 6$$

   Find the point on the ellipsoid maximizing the function $x+2y+3z$ and also the general $ax+by+cz$. Suppose now that we have the additional constraint that $x \leq 0.5$. Devise a scheme to compute the optimal point for the function $x + 2y + 3z$.

6. Compute the closest point to $(1, 2, 3)$ on the above ellipse with and without the additional constraint of $x \leq 0.5$.

7. Wine has about thrice as much alochol as Beer. There is a daily quota (or weekly or monthly depending on, well, other factors) on the total alcohol that can be consumed. My happines function is $2\sqrt{w} + \sqrt{b}$. A bottle of beer costs Rs. 100 and Wine costs about 4 times as much as Beer. I am allowed to spend about Rs. 200 per day *on the average* on alochol and I dont really like to drink the same thing every day. Also, I like to party once a week. Please adivse. The Wine maker approached me yesterday and said that a new technology will reduce the price of Wine by a third but that it needs investments. I said I will support her by investing with the Wine company. Of course,

it is always good to invest in new technology, especially for alcohol, but I am nagged by the doubt of *Was I wrong*?

8. Consider the equations below:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \leq \begin{bmatrix} 4 \\ 4 \\ 3 \\ 6 \\ 5 \\ 5 \\ 8 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

   Note that the last 3 constraints are that $x \geq 0, y \geq 0, z \geq 0$. Verify that $(0, 0, 0)$ is a vertex. Our objective is to maximize $x + 2y + 3z$. Start with $(0, 0, 0)$ and arrive at the optimum vertex. At each step, compute the cone $Dir(x)$ and choose a direction of motion. Move along that and arrive at the next vertex and so on.

9. Suppose that the problem above had the last 3 constraints removed, i.e., $x \geq 0, y \geq 0, z \geq 0$. How would the KKT algorithm run when started with $(0, 0, 0)$?

10. Let A be an $m \times n$ matrix with $m < n$. Show that there is a non-zero vector x which is $n \times 1$ such that Ax is the zero vector. *Hint: Use induction.*

11. Compute the row echelon form of the matrix A:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 1 & 0 & 3 & 5 \end{bmatrix}$$

    Compute a basis of the spave x such that $Ax = 0$.