Programming Languages (CS329)
Homework 1 Solutions

1. HALF = (λ $\overline{n}$ (RIGHT ($\overline{n}$ f PAIR($\overline{0}$ $\overline{0}$))))

   ```
   f      = (λ p (IF (EQUAL ? (LEFT p) (RIGHT p))
                     (PAIR (INC(LEFT p)) (RIGHT p))
                     (PAIR (LEFT p) (INC(RIGHT p)))
                 )
             )
   ```

   Alternate solution:

   HALF = (λ $\overline{n}$ (RIGHT ($\overline{n}$ f PAIR($\overline{0}$ $\overline{FALSE}$))))

   ```
   f      = (λ p (IF (RIGHT p)
                     (PAIR (LEFT p) TRUE)
                     (PAIR (INC(LEFT p)) FALSE)
                 )
             )
   ```

2. LOG2 = (λ $\overline{n}$ (RIGHT ($\overline{n}$ f PAIR($\overline{n}$ $\overline{0}$))))

   ```
   f      = (λ p (IF (EQUAL ? (LEFT p) 1)
                     p
                     (PAIR (HALF(LEFT p)) (INC(RIGHT p)))
                 )
             )
   ```

   Alternate solution:

   ```
   LOG2 = (λ n (IF (ZERO? n)
                   1
                   RIGHT(n f PAIR(0 0))
               )
          )

   f      = (λ p (IF (EQUAL ? (EXP(INC(RIGHT p))) (INC(LEFT p)))
                     (PAIR(INC(LEFT p) INC (RIGHT p)))
                     (PAIR (INC(LEFT p)) (RIGHT p))
                 )
             )
   ```

3. (a)

(b) $\overline{7}$ = (λ f(λ a(f (f (f (f (f (f (f a)))))))))
= (λ f(λ a(($\overline{2}$ f) (f (f (f (f (f a))))))))
= (λ f(λ a(($\overline{2}$ f) (($\overline{2}$ f) (f (f (f a)))))))
= (λ f(λ a(($\overline{2}$ ($\overline{2}$ f)) (f (f (f a))))))
= (λ f(λ a(($\overline{2}$ ($\overline{2}$ f)) (($\overline{2}$ f) (f a)))))
= (λ f(λ a(($\overline{2}$ $\overline{2}$)f)) (($\overline{2}$ f) (f a)))
= $\underline{7}$

(c) Parse tree of $\overline{n}$ has $n+2$ nodes. If leaves are also counted as nodes then answer is $2n+5$.
Number of nodes in parse tree of $\underline{n}$: 1. $F(n)+2$ (2 for Abstraction) where
$F(n) = 1 + \lfloor \log_2(n) \rfloor + F(n - 2^{\lfloor \log_2(n) \rfloor})$
$F(0) = 0$
2. If leaves are also counted as nodes then $f(n)+5$ where
$f(n) = 1 + (2\lfloor \log_2(n) \rfloor + 1) + F(n - 2^{\lfloor \log_2(n) \rfloor})$
$f(0) = 0$