

1. In SVD, list all properties of Σ in $\underline{U}^\top AV = \Sigma \in \mathbb{R}^{m \times n}$, where \underline{U} and V are as derived in class.

2. In EM, show that the sequence of two steps from π^g, μ^g comprising

- estimating $q_x^g(\cdot)$ for each document x
- estimating the next π^*, μ^*

never gives a total log-likelihood $\log \Pr(X|\pi^*, \mu^*)$ any worse than $\log \Pr(X|\pi^g, \mu^g)$.

3. In EM, complete the derivation of μ^* for Poisson word generators and θ^* for multinomial word generators.

4. In the Dirichlet word generator, prove that

$$\lim_{a \rightarrow 0} B(a, x) = \frac{1}{a} \quad \text{for all } x$$

5. Using the above show that

$$\lim_{\alpha \rightarrow 0} \Pr(d|\alpha) \approx \left(\prod_{t: n(d,t) \geq 1} \frac{1}{n(d,t)} \right) (\sum_t n(d,t))! \frac{\Gamma(\sum_t \alpha_t)}{\Gamma(\sum_t \alpha_t + \sum_t n(d,t))} \exp(\sum_t \llbracket n(d,t) \geq 1 \rrbracket \log \alpha_t),$$

where $\llbracket B \rrbracket = 1$ if boolean condition B is true and 0 otherwise. $n(d,t)$ is the number of times term t appears in document d .

6. In the Lafferty-Zhai style bipartite term-document random walk, if C is stochastic and $0 < \alpha < 1$, show that $\mathbb{I} - \alpha C$ has an inverse.