

CS 348: Computer Networks

- Error Detection; 7th Aug 2012

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Bit level error detection

Single-bit, multi-bit or burst errors introduced due to channel noise.

- Detected using redundant information sent along with data.
- Full Redundancy:
 - Send everything twice
 - Simple but inefficient

Error correcting codes

- **error-correcting codes:** include enough redundant information with data so receiver can recover data (useful on simplex channels where retransmission cannot be requested)
- Beyond the scope of this course!

Error detecting codes

- **error-detecting codes:** include enough redundancy to allow receiver to detect error.
- **more efficient and preferred solution**
 - parity check.
 - checksum.
 - cyclic redundancy check (CRC or polynomial code).

Parity (horizontal/vertical) ^{parity bits}

- 1 bit error detectable, not correctable
- 2 bit error not detectable

data		0101001	1
		1101001	0
		1011110	1
		0001110	1
		0110100	1
		1011111	0
parity byte		1111011	0

Rectangular Parity (LRC + VRC)

- 1 bit error correctable
- 2 bit error detectable
- Slow, needs memory

data

parity
byte

0101001	1
1101001	0
1011110	1
0001110	1
0110100	1
1011111	0
1111011	0

parity
bits

Checksum

- IP header; TCP/UDP segment checksum.
 - View message as 16-bit integers.
 - Add these integers using 16-bit ones-complement arithmetic.
 - Take the ones-complement of the result.
 - Resulting 16-bit number is the checksum.
- Can detect all 1 bit errors.
- See example in textbook.

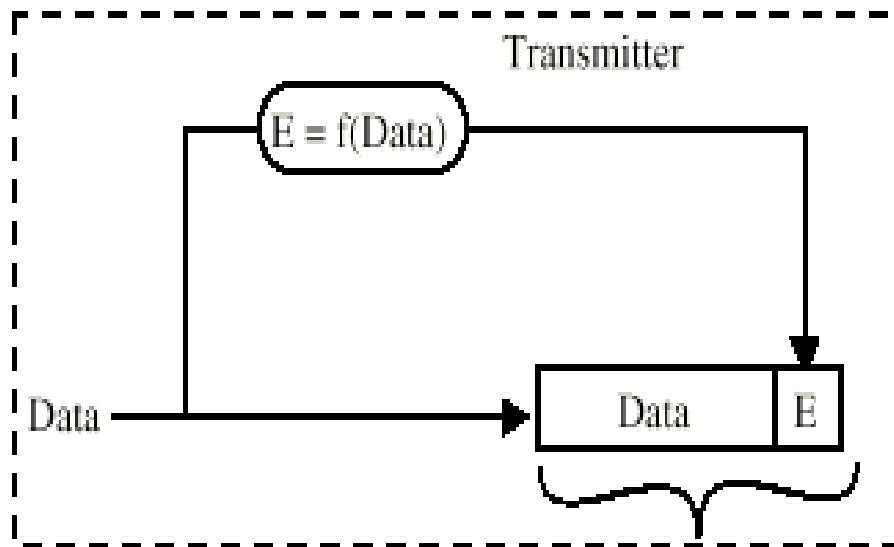
Cyclic Redundancy Check

- Based on binary division instead of addition.
- Powerful and commonly used to 'detect' errors.
- Uses modulo 2 arithmetic:
 - Add/Subtract := XOR (no carries for additions or borrows for subtraction)
 - $2^k * M$:= shift M towards left by k positions and then pad with zeros
- Digital logic for CRC is fast. no delay, no storage

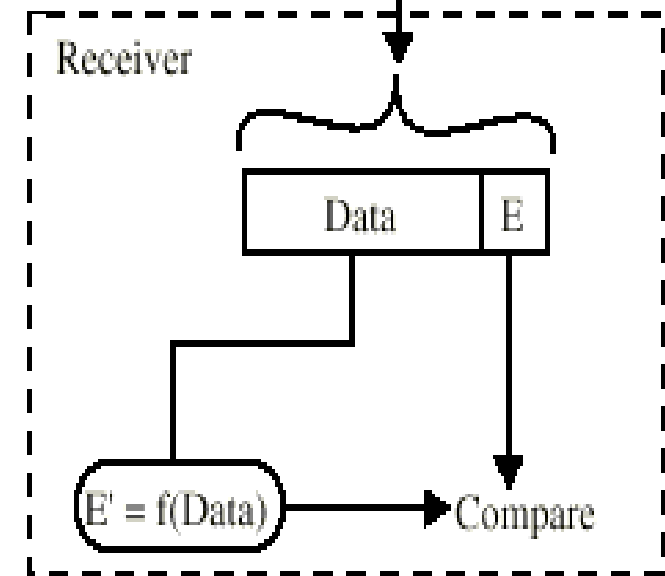
CRC algorithm

To transmit message M of size of n bits:

- Source and destination agree on a common bit pattern P of size $k+1$ ($k > 0$)
- Source: Add (in modulo 2) bit pattern (F) of size k to the message M ($k < n$), such that
 - $2^k * M + F = T$ is evenly divisible (modulo 2) by pattern P .
- Destination: check if above condition is true
 - i.e. $(2^k * M + F) / P = 0$



E, E' = error detecting codes
 f = error detecting code function



Frame check sequence (FCS)

- Given M and P , find appropriate F (frame check sequence)
 1. Multiply M with 2^k (add k zeros to end of M)
 2. Divide (in modulo 2) the product by P
 - » The remainder R is the required FCS
 1. Add the remainder R to the product $2^k * M$
 2. Transmit the resultant T

Example

- $M = 10011010$
- $P = 1101$

$$M * 2^3 = 10011010000$$

$$F = 10011010000 / 1101$$

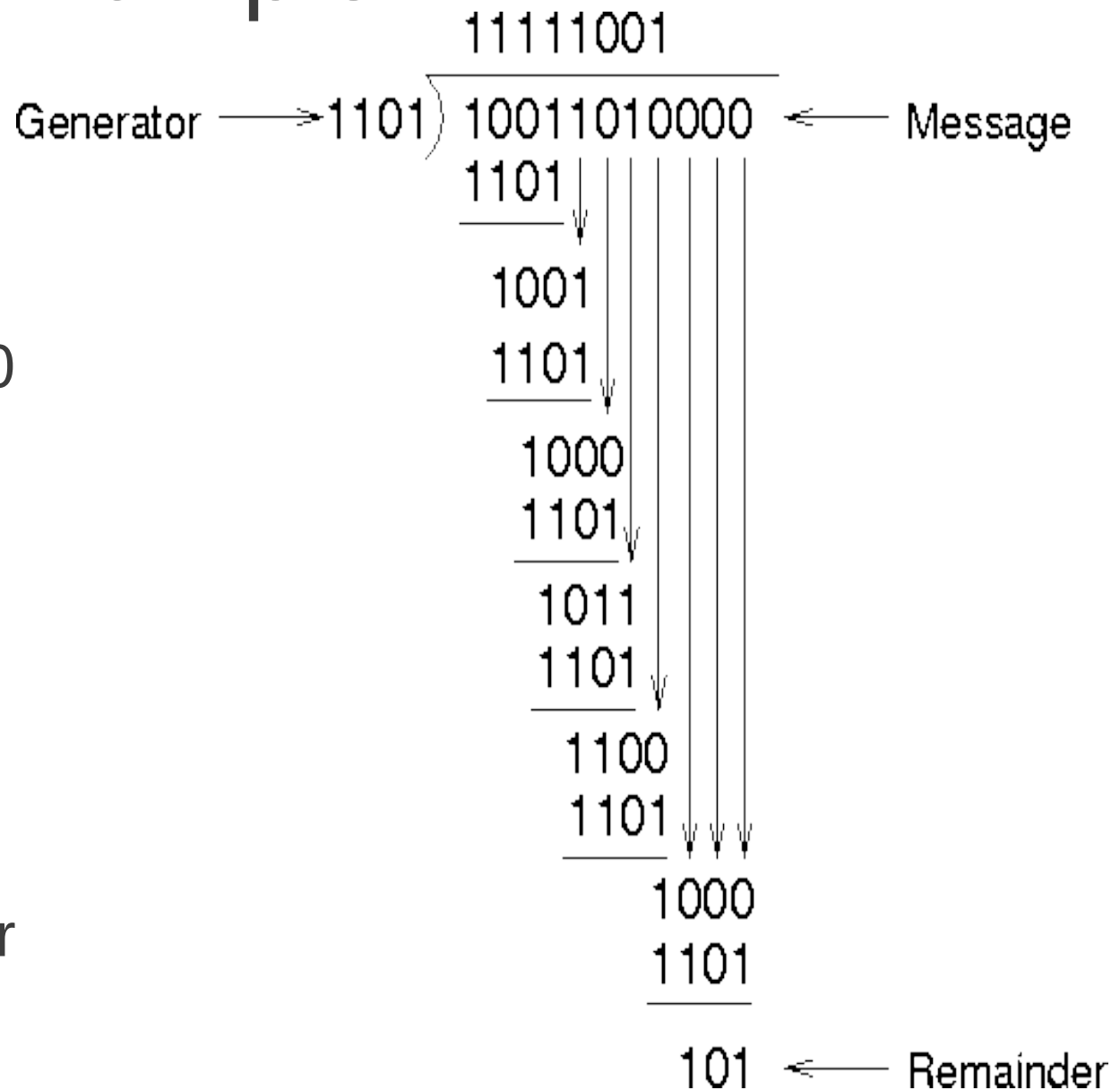
$$= 101$$

$$T = 10011010000 + 101$$

$$= 10011010101$$

At receiver

- $T/P \Rightarrow$ No remainder



Polynomial representation

- Represent n -bit message as an $n-1$ degree polynomial;
 - $M=10011010$ corresponds to
 - $M(x) = x^7 + x^4 + x^3 + x^1.$
- Let k be the degree of some divisor polynomial $C(x)$; (also called Generator Polynomial)
 - $P = 1101$ corresponds to
 - $C(x) = x^3 + x^2 + 1.$

Sender actions

- Multiply $M(x)$ by x^k ;
 - 10011010000: $x^{10} + x^7 + x^6 + x^4$
- Divide result by $C(x)$ to get remainder $R(x)$;
 - $10011010000 / 1101 = 101$
- Send $P(x)$; $10011010000 + 101 = 10011010101$

Receiver actions

- Receive $P(x) + E(x)$ and divide by $C(x)$
 - $E(x)$ represents the error with 1s in position of errors
- Remainder zero only if:
 - $E(x) = 0$ (no transmission error), or
 - $E(x)$ is exactly divisible by $C(x)$.
- Choose $C(x)$ to make second case extremely rare.

Generator polynomials

- CRC-8 $x^8 + x^2 + x^1 + 1$
- CRC-10 $x^{10} + x^9 + x^5 + x^4 + x^1 + 1$
- CRC-12 $x^{12} + x^{11} + x^3 + x^2 + 1$
- CRC-16 $x^{16} + x^{15} + x^2 + 1$
- CRC-32 $x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + x^{11} + x^{10}$
 $+ x^8 + x^7 + x^5 + x^4 + x^2 + x + 1$

CRC detects

- all single bit errors
- almost all 2-bit errors
- any odd number of errors
- all bursts up to M , where generator length is M
- longer bursts with probability 2^{-m}
- Proofs – Beyond the scope of this course.

CRC Implementation

- Hardware
 - on-the-fly with a shift register
 - easy to implement with ASIC/FPGA
- Software
 - precompute remainders for 16-bit words
 - add remainders to a running sum
 - needs only one lookup per 16-bit block

CRC calculation using a shift-register

