At the ~Amusement Park~

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The summer vacations had just begun and already Anirudh was bored with watching TV and playing computer games. "I'll go to the amusement park and ride on the roller-coaster", he thought. He called his friends Kailash and Rajiv and they set off to have a good time.

Arriving at the park, they realized to their dismay that many people had the same bright idea. There were long lines for buying the entry tickets. Although there were six counters for selling tickets, it would be quite a while before they got theirs. "Oh well, if you don't wait at the end of the line, you'll never get to the front", said Kailash, as he got into one of the lines. "Better find out if this is the correct line", cautioned Rajiv, "The only thing worse than waiting in line is - waiting in the WRONG line!"

Meanwhile, Anirudh had been observing the lines and finally joined one that seemed to be much longer than the one Kailash was standing in. "Do you want to stand here till tomorrow?

Move to a shorter line.", Kailash jeered at him. However, it was Kailash's turn to be surprised when Anirudh 's line turned out to be a fast moving one. "Whichever line you are in, the others always move faster.", muttered Kailash. "No, No. Look at that sign at the front of Anirudh's line - 'EXACT CHANGE ONLY'. So this ticket seller takes less time to service each customer and that is why this line is moving faster.", deduced Rajiv. Soon Anirudh had obtained the tickets for the three of them and they entered the park.

"Let us go to the roller-coaster.", said Anirudh. They headed in that direction and what did they find?

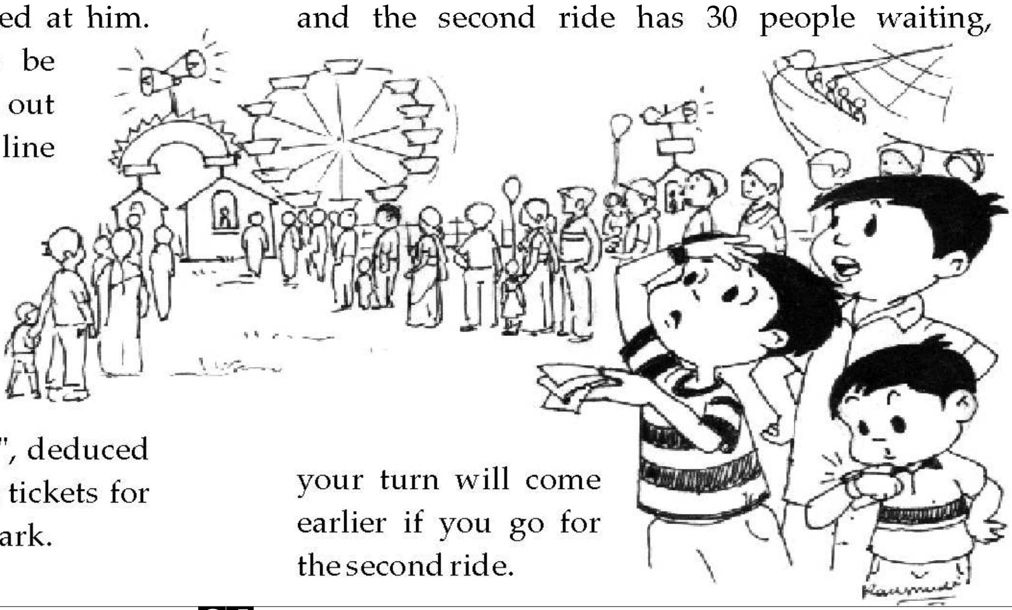
There was another long line of people waiting to get on the roller-coaster. They looked here and there and saw similar situations at the other attractions as well. "Arrgh, we will only be standing in lines all day long with a bunch of strange-looking people. Each ride will be over in a few minutes.", groaned Kailash. Anyway, since they all wanted to first ride the roller-coaster, they got into that line.

"I came here to get rides not to stand in lines. I want to get the maximum number of rides and spend minimum time waiting in these lines.", said Kailash crossly.

"That is simple. After each ride, we should join that line which has the minimum waiting time.", was Rajiv's contribution.

"The line which has the minimum number of people should have the minimum waiting time.", he concluded. "Not always.", said Anirudh, "Waiting time also depends on the duration of each ride.

For example, consider two rides both of which can accomodate 10 people at a time. If the first ride takes 10 minutes and the second ride takes only 5 minutes, the line at the second ride will move faster. Now if the first ride has 20 people waiting and the second ride has 30 people waiting, your turn will come earlier if you go for the second ride.



I was browsing the Internet yesterday and found that there is a whole subject called 'queueing theory' devoted to the study of such problems.

We can use some of the basic queueing principles to determine our waiting time.", he explained in his most professorial manner. "Have I come to an amusement park or to a lecture?" mused Kailash.

"Did you know that the word "queueing" is the only English word with five consecutive vowels.", quipped Rajiv, who had a fund of such information.

Before he could add any more, it was their turn on the roller-coaster and they went on to enjoy the ride.

Interestingly, queueing theory has a wide range of computer systems applications also. You may encounter queues of humans at a railway booking counter, a hospital, at a shopping checkout counter, etc. On the other hand, you also encounter queues when you send a request to your favourite search engine, when you access your email, when a router is forwarding your packets, etc. The main reason why you notice human queues while you do not notice the other is because the delays and response times in computer systems are negligible compared to human systems.

There are three most important characteristics of a queueing system. The first one is the *Arrival Process*, which describes the rate at which customers arrive in the system. For example, at an amusement park, there may be an average of 5 people joining the roller-coaster queue every minute.

The second one is the *Service Process*, which describes the time taken to service customers in the system.

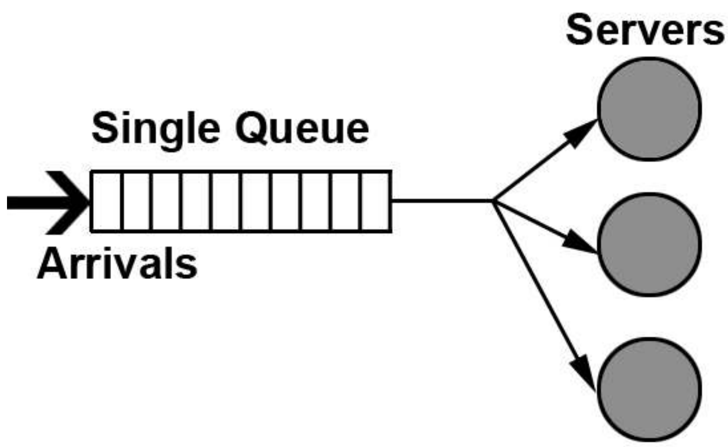
For example, at an amusement park, the roller-coaster may take an average of 10 minutes for each round of service. The third one is the *Number of Servers*, which is the number of servers available to service the customers. For example, at an amusement park, there may be 20 seats on a roller-coaster ride.

For all such queues, the same principle can be used to determine the average waiting time. This principle is called Little's Law. It states that: The average waiting time is equal to the average number of customers in the queue, divided by the the average customer arrival rate.

For example, if there are an average of 10 people waiting in a queue, and an average of 5 people join the queue per minute, then the average waiting time would be 2 minutes.

Another example to understand Little's Law is - Consider a restaurant where the customer arrival rate doubles but the customers still spend the same amount of time in the restaurant. This will double the number of customers in the restaurant.

By the same logic if the customer arrival rate remains the same but the customers service time doubles, this will also double the number of customers in the restaurant.



While this may sound simple, there a lot of intricacies. For example, what if the requests do not arrive uniformly? What if there are more arrivals during certain times of the day? What if each request takes a different amount of time to be serviced? There's a fair amount of science to all this, and there are people who make a living by studying queues and predicting the behavior of systems.

So the next time you are stuck in a queue, start counting heads and arrival timings to estimate how long it will take you to get up to the actual service. It may keep you amused till it is your turn at the server!

Queueing Theory: The mathematical study of waiting lines (or queues). There are several related processes,
arriving at the back of the queue, waiting in the queue, and being served by the server at the front of the queue.
Throughput: The rate at which a system processes the requests received.

Some interesting related websites are:
http://www2.uwindsor.ca/~hlynka/queue.html
http://www.cut-the-knot.org/content.shtml
http://www.howstuffworks.com/

Illustrations: Mrs. Kaumudi Sahasrabudhe