CHAPTER 5

Other Relational Languages

Solutions to Practice Exercises

5.1 a. \( \{ t \mid \exists q \in r \ (q[A] = t[A]) \} \)
   b. \( \{ t \mid t \in r \land t[B] = 17 \} \)
   c. \( \{ t \mid \exists p \in r \exists q \in s \ (t[A] = p[A] \land t[B] = p[B] \land t[C] = p[C] \land t[D] = q[D] \) 
      \land t[E] = q[E] \land t[F] = q[F] \} \)
   d. \( \{ t \mid \exists p \in r \exists q \in s \ (t[A] = p[A] \land t[F] = q[F] \land p[C] = q[D]) \} \)

5.2 a. \( \{ < t, p, q > \mid \exists p, q \ (< t, p, q > \in r_1) \} \)
   b. \( \{ < a, b, c > \mid < a, b, c > \in r_1 \land b = 17 \} \)
   c. \( \{ < a, b, c > \mid < a, b, c > \in r_1 \lor < a, b, c > \in r_2 \} \)
   d. \( \{ < a, b, c > \mid < a, b, c > \in r_1 \land < a, b, c > \in r_2 \} \)
   e. \( \{ < a, b, c > \mid < a, b, c > \in r_1 \land < a, b, c > \notin r_2 \} \)
   f. \( \{ < a, b, c > \mid \exists p, q \ (< a, b, p > \in r_1 \land < q, b, c > \in r_2) \} \)

5.3 a. \( \{ a \mid \exists b \ (a < b, b > \in r \land b = 7) \} \)
   
   i. \[ r \quad A \quad B \]
      \[ P \quad 17 \]

   ii. query (X) :- r (X, 17)
b. \( \{ <a, b, c> \mid <a, b> \in r \land <a, c> \in s \} \)

i.\[
\begin{array}{|c|c|c|}
\hline
r & A & B \\
\hline
 & -a & -b \\
\hline
\end{array}
\]
\[
\begin{array}{|c|c|c|}
\hline
s & A & C \\
\hline
 & -a & -c \\
\hline
\end{array}
\]
\[
\begin{array}{|c|c|c|}
\hline
result & A & B & C \\
\hline
 & -a & -b & -c \\
\hline
\end{array}
\]

ii. query\((X, Y, Z) : r(X, Y), s(X, Z)\)

c. \( \{ <a> \mid \exists c \left( <a, c> \in s \land \exists b_1, b_2 \left( <a, b_1> \in r \land <c, b_2> \in r \land b_1 > b_2 \right) \right) \} \)

i.\[
\begin{array}{|c|c|c|}
\hline
r & A & B \\
\hline
 & -a & -b > -s \\
\hline
 & -c & -s \\
\hline
\end{array}
\]
\[
\begin{array}{|c|c|c|}
\hline
s & A & C \\
\hline
 & -a & -c \\
\hline
\end{array}
\]

ii. query\((X) : s(X, Y), r(X, Z), r(Y, W), Z > W\)

5.4 a. Query:

query\((X) : p(X)\)
p\((X) : \text{manages}(X, "Jones")\)
p\((X) : \text{manages}(X, Y), p(Y)\)

b. Query:

query\((X, C) : p(X), \text{employee}(X, S, C)\)
p\((X) : \text{manages}(X, "Jones")\)
p\((X) : \text{manages}(X, Y), p(Y)\)

c. Query:

query\((X, Y) : p(X, W), p(Y, W)\)
p\((X, Y) : \text{manages}(X, Y)\)
p\((X, Y) : \text{manages}(X, Z), p(Z, Y)\)

d. Query:
5.5 A Datalog rule has two parts, the head and the body. The body is a comma separated list of literals. A positive literal has the form \( p(t_1, t_2, \ldots, t_n) \) where \( p \) is the name of a relation with \( n \) attributes, and \( t_1, t_2, \ldots, t_n \) are either constants or variables. A negative literal has the form \( \neg p(t_1, t_2, \ldots, t_n) \) where \( p \) has \( n \) attributes.

We consider only safe rules; see Section 5.4.4 for the definition of safety of Datalog rules. Further, we assume that every variable that occurs in an arithmetic literal also occurs in a positive non-arithmetic literal.

Consider first a rule without any negative literals. To express the rule as an extended relational-algebra view, we write it as a join of all the relations referred to in the (positive) non-arithmetic literals in the body, followed by a selection. The selection condition is a conjunction obtained as follows. If \( p_1(X, Y) \), \( p_2(Y, Z) \) occur in the body, where \( p_1 \) is of the schema \((A, B)\) and \( p_2 \) is of the schema \((C, D)\), then \( p_1.B = p_2.C \) should belong to the conjunction. The arithmetic literals can then be added to the condition.

As an example, the Datalog query

\[
query(X, Y) :- \text{works}(X, C, S_1), \text{works}(Y, C, S_2), S_1 > S_2, \text{manages}(X, Y)
\]

becomes the following relational-algebra expression:

\[
E_1 = \sigma_{(w1\text{.company\_name} = w2\text{.company\_name} \land w1\text{.salary} > w2\text{.salary} \land \\
\text{manages.person\_name} = w1\text{.person\_name} \land \text{manages.manager\_name} = w2\text{.person\_name})} \rho_{w1} \bowtie \rho_{w2} \text{works} \times \text{manages}
\]

Now suppose the given rule has negative literals. First suppose that there are no constants in the negative literals; recall that all variables in a negative literal must also occur in a positive literal. Let \( \neg q(X, Y) \) be the first negative literal, and let it be of the schema \((E, F)\). Let \( E_i \) be the relational algebra expression obtained after all positive and arithmetic literals have been handled. To handle this negative literal, we generate the expression

\[
E_j = E_i \bowtie (\Pi_{A_1, A_2}(E_i) - q)
\]

where \( A_1 \) and \( A_2 \) are the attribute names of two columns in \( E_i \) which correspond to \( X \) and \( Y \) respectively.

Now let us consider constants occurring in a negative literal. Consider a negative literal of the form \( \neg q(a, b, Y) \) where \( a \) and \( b \) are constants. Then, in the above expression defining \( E_j \) we replace \( q \) by \( \sigma_{A_1=a \land A_2=b}(q) \).

Proceeding in a similar fashion, the remaining negative literals are processed, finally resulting in an expression \( E_w \).
Finally the desired attributes are projected out of the expression. The attributes in $E_w$ corresponding to the variables in the head of the rule become the projection attributes.

Thus our example rule finally becomes the view:

```
create view query as
    \Pi_{w_1.p\_name, w_2.p\_name}(E_2)
```

If there are multiple rules for the same predicate, the relational-algebra expression defining the view is the union of the expressions corresponding to the individual rules.

The above conversion can be extended to handle rules that satisfy some weaker forms of the safety conditions, and where some restricted cases where the variables in arithmetic predicates do not appear in a positive non-arithmetic literal.