#### **Structured learning**

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#### Structured models

Standard classification



Structured prediction



Feature function vector  $f(x,y) = f_1(x,y), f_2(x,y), \dots, f_K(x,y),$  Structured y

- 1. Vector:  $y_1, y_2, ..., y_n$
- 2. Segmentation
- 3. Tree
- 4. Alignment
- 5. ..

#### Structured model

- Score of a prediction y for input x:
  - s(x,y) = w. f(x,y)
- Prediction problem: find highest scoring output
  - $\mathbf{y}_* = \operatorname{argmax}_{\mathbf{y}} \mathbf{s}(\mathbf{x}, \mathbf{y})$
  - Space of possible y exponentially large
  - Exploit decomposability of feature functions
    - $f(x,y) = \sum_{c} f(x,y_{c},c)$
- Training problem: find w given many correct input-output pairs (x<sub>1</sub> y<sub>1</sub>), (x<sub>2</sub> y<sub>2</sub>), ..., (x<sub>N</sub> y<sub>N</sub>)

### Outline

- Applications
- Inference algorithms
- Training objectives and algorithms

## Information Extraction (IE)

#### Find structure in unstructured text

According to Robert Callahan, president of Eastern's flight attendants union, the past practice of Eastern's parent, Houston-based Texas Air Corp., has involved ultimatums to unions to accept the carrier's terms



Others

- Disease outbreaks from news articles
- Addresses/Qualifications from resumes for HR DBs
- Room attributes from hotel websites
- Proteins and interactions from bio-medical abstracts

#### Clues that drive extraction

- Orthographic patterns: names have two capitalized words.
- Keywords: "In" is within 1—3 tokens before location
- Order of entities: Titles appear before Journal names
- Position: Product titles follow a N(4in,1) distance from top
- Dictionary match: Authors have high similarity with person\_name column of DB
- Collective: All occurrences of a word prefer the same label

#### Learning models for IE

- Rule-based models (1980s)
  - Too brittle, not for noisy environment.
- Classifiers for boundaries (1980s)
  - Could give inconsistent labels
- Hidden Markov Models (1990s)
  - Generative model, restrictive features
- Maxent Taggers (1990s) & MeMMs (late 1990)
  - Label bias problem.
- Conditional Random Fields (2000s)
- Segmentation models. (2004)

#### Sequence labeling

My review of Fermat's last theorem by S. Singh

#### Sequence labeling

My review of Fermat's last theorem by S. Singh

t	1	2	3	4	5	6	7	8	9			
x	My	review	of	Fermat's	last	theorem	by	S.	Singh			
y	Other	Other	Other	Title	Title	Title	other	Author	Author			



#### HMM for IE

- The two types of parameters
  - $Pr(x_i | y_i)$ 
    - Multinomial distribution of words in each state
  - Pr(y<sub>i</sub> | y<sub>i-1</sub>)



#### Structured learning for IE

My review of Fermat's last theorem by S. Singh

x v	Other	Other Other Other		Title	Title	Title	other	Author	Author		
•	My	review	of	Fermat's	last	theorem	by	S.	Singh		
t	1	2	3	4	5	6	7	8	9		

$$y_1 - y_2 - y_3 - y_4 - y_5 - y_6 - y_7 - y_8 - y_9$$
  
 $f(y_i, y_{i-1}, i, \mathbf{x})$ 

Features decompose over adjacent labels.

$$\mathbf{f}(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{|\mathbf{x}|} \mathbf{f}(y_i, y_{i-1}, i, \mathbf{x})$$

MAP can be found in O(nm<sup>2</sup>) time



Feature vector for each position



 $f_2(y_i, \mathbf{x}, i, y_{i-1}) = 1$  if  $y_i$  is Person &  $x_i$  is Douglas  $f_3(y_i, \mathbf{x}, i, y_{i-1}) = 1$  if  $y_i$  is Person &  $y_{i-1}$  is Other

#### Features in typical extraction tasks

- Words
- Orthographic word properties
  - Capitalized? Digit? Ends-with-dot?
- Part of speech
  - Noun?
- Match in a dictionary
  - Appears in a dictionary of people names?
  - Appears in a list of stop-words?
- Fire these for each label and
  - The token,
  - W tokens to the left or right, or
  - Concatenation of tokens.

#### **Publications**

- Cora dataset
  - Paper headers: Extract title, author affiliation, address, email, abstract
    - 94% F1 with CRFs
    - 76% F1 with HMMs
  - Paper citations: Extract title,author,date, editor,booktitle,pages,institution
    - 91% F1 with CRFs
    - 78% F1 with HMMs

#### Peng & McCallum 2004

#### IE as Segmentation

y	Other	Other	Other		Title		other	Author				
x	Му	review	of	Fermat's	last	theorem	by	S.	Singh			

- Output y is a sequence of segments s<sub>1</sub>,...,s<sub>p</sub>
- Feature f(x,y) decomposes over segment and label of previous segment

$$\mathbf{f}(\mathbf{x}, \mathbf{y}) = \sum_{j=1}^{p} \mathbf{f}(\mathbf{x}, s_j, y_{j-1})$$

- MAP: easy extension of Viterbi O(m<sup>2</sup> n<sup>2</sup>)
  - m = number of labels, n = length of a sequence

#### **Some Results**



#### **Collective labeling**

- Y does have character.
- Mr. X lives in Y.
- X buys Y Times daily.



 $\mathbf{f}(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{D} \sum_{j=1}^{|D_i|} \mathbf{f}(\mathbf{x}, y_{ij}, y_{ij-1}, i) + \sum_{x_{ij} = x_{i'j'}} f_e(y_{ij}, y_{i'j'})$ 

#### Starting graphs (.. of an extraction task from addresses)

9	0	0	0	0	0	•	•	0	0	0	0	•	0	0	0	0	0	0	0	•	0	0	0	Ø	•	0	•	0	•	0	0	0	ø
•	0	6	0	6	6	\$	\$	•	6	0	\$	6	0	6	0	6	0	0	\$	\$	6	0	0	6	\$	0	\$	\$	•	6	0	\$	\$
	•	•	•	•	•	•	\$	\$	•	•	\$	•	•	•	•	•	•	•	\$	\$	•	0	\$	•	\$	•	\$	\$	\$	•	•	\$	\$
Þ	•	•	0	0	0	•	•	•	0	0	•	•	0	0	0	0	0	0	\$	•	0	0	0	0	•	0	\$	\$	•	0	0	\$	•
Þ	•	\$	•	\$	\$	\$	\$	•	0	•	\$	•	0	\$	•	0	•	\$	\$	\$	\$	0	\$	0	\$	•	\$	•	\$	\$	•	\$	\$
₽	•	•	0	•	•	•	\$	•	0	0	\$	•	0	0	0		0	0	0	•	•	0	0		•	0	\$	\$	•	0	0	0	\$
Þ	•	0	0	\$	\$	\$	\$	•	0	0	\$	•	0	\$	0		0	0	\$	\$	0		•		•		\$	•	•	\$	•	\$	\$
Þ	•		0	0	0	0	0	0	0	0	0	•	0	0	0			0	0	0			0		6		0	•	•	0	0	0	0
Þ	•		0	0		•	•	•		0	•	•	0	0	0			0		•			•				•	0	•	0	0	•	•
Þ	•		0			0	ø	•			ø	0	0	0	0			0					•				0		•		0	0	ø
Þ	0					0	•	•						0				0					•				•		•		0	0	•

#### Graph after collective edges



#### A closer look at the graph...



#### Our approach

#### BP on clusters of cliques and chains with single node separators



- Basic MP step: Compute max-marginals for a separator node → MAP for each label of the node.
- MAP algorithms for chains  $\rightarrow$  easy and efficient.
- MAP algorithms for cliques 
   Design new combinatorial algorithms

### **Clique inference**

Given a clique c with n nodes, m labels

- $\phi_{\mathbf{u}}(\mathbf{y}_{\mathbf{u}})$  Node Potentials for each node  $\mathbf{u} \in \mathbf{c}$
- cp(y) Clique Potential over all nodes in c
- Find MAP labeling y<sup>\*</sup> as
  - $\mathbf{y}^* = \operatorname{argmax}_{\mathbf{y}} (\Sigma_{\mathbf{u}} \phi_{\mathbf{u}}(\mathbf{y}_{\mathbf{u}}) + \mathbf{cp}(\mathbf{y}))$
- Two properties of clique potentials
  - Associative
  - Symmetric: depends only on label counts
    - $CP(y_1,...,y_n) = f(n(y)) = f(n_1,n_2,...,n_m)$

#### **Cardinality-based Clique Potentials**

MAX	$\max_{y} f(n_{y})$	Optimal
MAJ	$\sum_{y} W \alpha_{y} n_{y}$ ( $\alpha = \operatorname{argmax} y n_{y}$ )	Optimal based on Lagrange relaxation
CLINA	$\sum_{y} n_{y}^{2}$ (POTTS)	13/15 Approx
SUM	$\sum_{y} n_{y} \log n_{y}$ (Entropy)	<sup>1</sup> / <sub>2</sub> Approx O(nlog n) time.

#### The $\alpha$ -pass Algorithm



Labeled with best label  $\neq \alpha$ 

1. For every  $\alpha$ , sort nodes by  $\phi_u(\alpha)$ -max<sub>y≠ $\alpha$ </sub> $\phi_u(y)$ 

1. For all  $1 \le k \le n$ 

1. Label first k nodes with  $\boldsymbol{\alpha}$ 

 $O(mn\log n)$ 

2. Label the rest with their best non- $\alpha$  labels.

2. Pick the best solution across all ( $\alpha$ ,k) combinations.

#### Parse tree of a sentence

- Input x: "John hit the ball"
- Output y: parse tree



- Features decompose over nodes of the tree
- MAP: Inside/outside algorithm O(n<sup>3</sup>)
- Better than Probabilistic CFGs (Taskar EMNLP 2004)

### Sentence alignment

- Input x: sentence pair
- Output y : alignment

A <mark>fair</mark> deal and prosperity go hand in hand.

एक <mark>अच्छा</mark> सौदा और <mark>समृद्धि साथ</mark> - साथ चलते हैं ।

•  $y_{i,j} = 1$  iff word i in 1st sentence is aligned to word j in  $2^{nd}$ 

Features vector decompose over each aligned edge

- $f(x, y) = \sum_{yi,j=1} g(x, i,j)$
- **g**(**x**, i,j): various properties comparing i-th and j-th word
  - Difference in the position of the two words
  - Is part of speech of the two words the same?
- MAP: Maximum weight matching

### Ranking of search results in IR

- Input x: Query q, List of documents d<sub>1</sub>, d<sub>2</sub>,..., d<sub>n</sub>
- Output **y**:
  - Ranking of documents so that relevant documents appear before irrelevant ones
  - y<sub>i</sub> = position of document d<sub>i</sub>
- Feature vector f(x, y) defined as follows
  - $\mathbf{g}(d_i, q) = \text{vector of properties relating } d_i$  to q
    - Jaccard similarity between query words and document
    - Popularity of document d<sub>i</sub>
  - $f(x, y) = \sum_{y_i < y_j} (g(d_i, q) g(d_j, q))$
- MAP: rank documents on w.g(d<sub>i</sub>, q)

#### Markov models (CRFs)

- Application: Image segmentation and many others
- **y** is a vector y<sub>1</sub>, y<sub>2</sub>, .., y<sub>n</sub> of discrete labels
- Features decompose over cliques of a triangulated graph
- MAP inference algorithms for graphical models, extensively researched
  - Junction trees for exact, many approximate algorithms
    - Special case: Viterbi
- Framework of structured models subsumes graphical models

#### Structured model

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#### Max-margin loss surrogates

True error 
$$E_i(\operatorname{argmax}_{\mathbf{y}} \mathbf{w}.\mathbf{f}(\mathbf{x}_i, \mathbf{y}))$$
  
Let  $\mathbf{w}.\delta \mathbf{f}(\mathbf{x}_i, \mathbf{y}) = \mathbf{w}.\mathbf{f}(\mathbf{x}_i, \mathbf{y}_i) - \mathbf{w}.\mathbf{f}(\mathbf{x}_i, \mathbf{y})$   
1. Margin Loss  
 $\max_{\mathbf{y}} [E_i(\mathbf{y}) - \mathbf{w}.\delta \mathbf{f}(\mathbf{x}_i, \mathbf{y})]_+$   
2. Slack Loss  
 $\max_{\mathbf{y}} E_i(\mathbf{y})[1 - \mathbf{w}.\delta \mathbf{f}(\mathbf{x}_i, \mathbf{y})]_+$   
 $\operatorname{E}(\mathbf{y})=4$   
 $E(\mathbf{y})=4$   
 $E(\mathbf{y})=4$   

#### **Final optimization**

Margin

$$\min_{\mathbf{w},\xi} \frac{1}{2} ||\mathbf{w}||^2 + \mathbf{C} \sum_{i=1}^{\mathbf{N}} \xi_i$$
  
s.t.  $\mathbf{w}.\delta \mathbf{f_i}(\mathbf{y}) \ge \mathbf{E_i}(\mathbf{y}) - \xi_i \quad \forall \mathbf{y} \neq \mathbf{y_i}, \mathbf{i}: \mathbf{1} \dots \mathbf{N}$   
 $\xi_i \ge 0 \quad i: 1 \dots N$ 

Slack

$$\begin{split} \min_{\mathbf{w},\xi} \frac{1}{2} ||\mathbf{w}||^2 + \mathbf{C} \sum_{i=1}^{\mathbf{N}} \xi_i \\ \text{s.t. } \mathbf{w}.\delta \mathbf{f_i}(\mathbf{y}) \ge \mathbf{1} - \frac{\xi_i}{\mathbf{E_i}(\mathbf{y})} \quad \forall \mathbf{y} \neq \mathbf{y_i}, \mathbf{i}: \mathbf{1} \dots \mathbf{N} \\ \xi_i \ge 0 \quad i: 1 \dots N \end{split}$$

Exponential number of constraints -> Use cutting plane

## Margin Vs Slack

- Margin
  - Easy inference of most violated constraint for decomposable **f** and E

$$\mathbf{y}^M = \operatorname{argmax}_{\mathbf{y}}(\mathbf{w}.\mathbf{f}(\mathbf{x_i}, \mathbf{y}) + \mathbf{E_i}(\mathbf{y}))$$

- Too much importance to y far from margin
- Slack
  - Difficult inference of violated constraint

$$\mathbf{y}^{S} = \operatorname{argmax}_{\mathbf{y}}(\mathbf{w}.\mathbf{f}_{\mathbf{i}}(\mathbf{y}) - \frac{\xi_{\mathbf{i}}}{\mathbf{E}_{\mathbf{i}}(\mathbf{y})})$$

- Zero loss of everything outside margin
  - Higher accuracy.

#### Accuracy of Margin vs Slack



Slack scaling up to 25% better than Margin scaling.

#### **Approximating Slack inference**

- Slack inference: max<sub>y</sub> s(y)-ξ/E(y)
  - Decomposability of E(y) cannot be exploited.

- $-\xi/E(\mathbf{y})$  is concave in  $E(\mathbf{y})$
- Variational method to rewrite as linear function

$$-\frac{\xi}{E(\mathbf{y})} = \min_{\lambda \ge 0} \lambda E(\mathbf{y}) - 2\sqrt{\xi\lambda}$$

#### Approximating slack inference

- $s(y)-\xi/E(y)$  is concave in E(y)
- Its variational form.



#### **Approximating Slack inference**

#### Now approximate the inference problem as:

$$\max_{\mathbf{y}} \left( s(\mathbf{y}) - \frac{\xi}{E(\mathbf{y})} \right) = \max_{\mathbf{y}} \min_{\lambda \ge 0} s(\mathbf{y}) + \lambda E(\mathbf{y}) - 2\sqrt{\xi\lambda}$$
$$\cdot \min_{\lambda \ge 0} \max_{\mathbf{y}} s(\mathbf{y}) + \lambda E(\mathbf{y}) - 2\sqrt{\xi\lambda}$$
Same tractable MAP as in Margin Scaling

#### Approximating slack inference

#### Now approximate the inference problem as:



#### Slack Vs ApproxSlack



ApproxSlack gives the accuracy gains of Slack scaling while requiring same the MAP inference same as Margin scaling.

## Limitation of ApproxSlack

 Cannot ensure that a violating y will be found even if it exists

• y3

◆ y2

◆ y1

-0.5

s(y)

- No  $\lambda$  can ensure that.
- Proof:
  - $s(y_1) = -1/2$   $E(y_1) = 1$
  - $s(y_2) = -13/18$   $E(y_2) = 2$
  - $s(y_3) = -5/6$   $E(y_3) = 3$
  - s(correct) = 0
  - ξ = 19/36
  - $y_2$  has highest  $s(y)-\xi/E(y)$  and is violating.
  - No  $\lambda$  can score  $y_2$  higher than both  $y_1$  and  $y_2$

4

3

2

Щ Х

Correct

0

## Max-margin formulations

Margin scaling

$$egin{aligned} \min_{\mathbf{w},\xi} rac{1}{2} ||\mathbf{w}||^2 + C \sum_{i=1}^N \xi_i \ ext{s.t. } \mathbf{w}.\mathbf{f}(\mathbf{x}_{ ext{i}},\mathbf{y}_{ ext{i}}) &\geq ext{E}_{ ext{i}}(\mathbf{y}) + \mathbf{w}.\mathbf{f}(\mathbf{x}_{ ext{i}},\mathbf{y}) - \overbrace{\xi_{ ext{i}}}^N orall \mathbf{y} 
eq \mathbf{y}_{ ext{i}}, orall \mathbf{i} \ &\xi_i \geq 0 \quad orall i \end{aligned}$$

Slack scaling

$$\begin{split} \min_{\mathbf{w},\xi} \frac{1}{2} ||\mathbf{w}||^2 + C \sum_{i=1}^{N} \xi_i \\ \text{s.t. } \mathbf{w}.\mathbf{f}(\mathbf{x}_i, \mathbf{y}_i) \ge 1 + \mathbf{w}.\mathbf{f}(\mathbf{x}_i, \mathbf{y}) - \underbrace{\xi_i}_{E_i(\mathbf{y})} \quad \forall \mathbf{y} \neq \mathbf{y}_i, \forall i \\ \xi_i \ge 0 \quad \forall i \end{split}$$

# The pitfalls of a single shared slack variables

Inadequate coverage for decomposable losses



Margin/Slack loss = 1. Since  $y_2$  non-separable from  $y_0$ ,  $\xi$ =1, Terminate. Premature since different features may be involved.

#### A new loss function: PosLearn

Ensure margin at each loss position

$$\begin{split} \min_{\mathbf{w},\xi} & \frac{1}{2} ||\mathbf{w}||^2 + C \sum_{i=1}^N \sum_c \xi_{i,c} \\ \text{s.t} & \mathbf{w}.\mathbf{f}(\mathbf{x}_i,\mathbf{y}_i) \geq 1 + \mathbf{w}.\mathbf{f}(\mathbf{x}_i,\mathbf{y}) - \frac{\xi_{i,c}}{E_{i,c}(\mathbf{y}_c)} \quad \forall \mathbf{y} : \mathbf{y}_c \neq \mathbf{y}_{i,c} \\ & \xi_{i,c} \geq 0 \quad i : 1 \dots N, \forall c \end{split}$$

Compare with slack scaling.

$$egin{aligned} \min_{\mathbf{w}, \xi} rac{1}{2} ||\mathbf{w}||^2 + C \sum_{i=1}^N \xi_i \ ext{s.t. } \mathbf{w}. \mathbf{f}(\mathbf{x}_{ ext{i}}, \mathbf{y}_{ ext{i}}) \geq 1 + \mathbf{w}. \mathbf{f}(\mathbf{x}_{ ext{i}}, \mathbf{y}) - rac{\xi_{ ext{i}}}{ ext{E}_{ ext{i}}(\mathbf{y})} \ \ orall \mathbf{y} 
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Margin/Slack loss = 1.

Since  $y_2$  non-separable from  $y_0$ ,  $\xi=1$ , Terminate. Premature since different features may be involved.

PosLearn loss = 2 Will continue to optimize for  $y_1$  even after slack  $\xi_3$ becomes 1

#### **Comparing loss functions**



PosLearn: same or better than Slack and ApproxSlack

### Inference for PosLearn QP

#### Cutting plane inference

For each position c, find best y that is wrong at c

$$\max_{\mathbf{y}:\mathbf{y}_c\neq\mathbf{y}_{i,c}} \left( s_i(\mathbf{y}) - \frac{\xi_{i,c}}{E_{i,c}(\mathbf{y}_c)} \right) = \max_{\mathbf{y}_c\neq\mathbf{y}_{i,c}} \left( \frac{\max s_i(\mathbf{y})}{\mathbf{y}\sim\mathbf{y}_c} - \frac{\xi_{i,c}}{E_{i,c}(\mathbf{y}_c)} \right)$$

Small enumerable set

MAP with restriction, easy!

- Solve simultaneously for all positions c
  - Markov models: Max-Marginals
  - Segmentation models: forward-backward passes
  - Parse trees

## Running time



Margin scaling might take time with less data since good constraints may not be found early PosLearn adds more constraints but needs fewer iterations.

## Summary of training

- 1. Margin scaling popular due to computational reasons, but slack scaling more accurate
  - A variational approximation for slack inference
- 2. Single slack variable inadequate for structured models where errors are additive
  - A new loss function that ensures margin at each possible error position of y