CS206 Spring 2007 Quiz 1

Time: 60 mins | Total marks: 20

- ***Please write your Roll No. on the top right of each sheet.***
- ***You must write your answers only in the spaces provided.***
- ***The exam is open book and notes.***
- ***Results/proofs covered in class/problem sessions/assignments may simply be cited, unless specifically asked for.***
- ***Unnecessarily lengthy solutions will be penalized.***
- ***If you need to make any assumptions, state them clearly.***
- ***Do not copy solutions from others or indulge in unfair means.***

1. Prove the following sequents using natural deduction. You must use only the basic rules of natural deduction and at most one application of LEM (no derived rules, including more than one application of LEM, are allowed). Your proof using the basic rules must not exceed the number of steps mentioned alongside each sequent. The number of steps includes the statement of the premises. You must also annotate each step of your proof with the basic rule applied at that step.

   (a) *[5 marks]* $\neg\phi_3 \ \rightarrow\ \phi_2 \ \vdash\ \phi_1 \ \rightarrow\ \phi_2 \vee \phi_3$ *[within 15 basic steps]*

   (b) *[5 marks]* $(\neg\phi_1 \wedge \phi_2) \ \rightarrow\ (\phi_3 \wedge \phi_4),\ \phi_2 \wedge \phi_3 \ \rightarrow\ \phi_1 \wedge \phi_4,\ \phi_1 \ \rightarrow\ \phi_5 \ \vdash\ \phi_2 \ \rightarrow\ \phi_5$ *[within 20 basic steps]*

2. *[5 + 5 marks]* Let $\phi(x, y, z)$ be a propositional logic formula, and let $\psi(x, y, z) = \phi(x, y, \phi(x, y, z))$.

(a) Show using natural deduction that $z \rightarrow \phi(x, y, z) \vdash z \rightarrow \psi(x, y, z)$.

(b) Give an equisatisfiable formula for $\psi(x, y, z)$ in which ϕ is not fed as an argument to itself. You must give complete justification why you think your formula is equisatisfiable with $\psi(x, y, z)$.