CS206 Spring 2007 Quiz 1

Time: 60 mins Total marks: 20

- ***Please write your Roll No. on the top right of each sheet.***
- ***You must write your answers only in the spaces provided.***
- ***The exam is open book and notes.***
- ***Results/proofs covered in class/problem sessions/assignments may simply be cited, unless specifically asked for.***
- ***Unnecessarily lengthy solutions will be penalized.***
- ***If you need to make any assumptions, state them clearly.***
- ***Do not copy solutions from others or indulge in unfair means.***

1. Prove the following sequents using natural deduction. You must use only the basic rules of natural deduction (no derived rules, including LEM, are allowed). Your proof using the basic rules must not exceed the number of steps mentioned alongside each sequent. The number of steps includes the statement of the premises. You must also annotate each step of your proof with the basic rule applied at that step.

   (a) *[5 marks]* $\phi_1 \rightarrow \neg\phi_4, \phi_3 \rightarrow (\phi_4 \rightarrow \phi_1) \vdash \phi_3 \rightarrow \neg\phi_4$ *[within 15 basic steps]*

   (b) *[5 marks]* $\phi \vee \psi, \phi \vee \neg\psi, \neg\phi \vee \psi, \neg\phi \vee \neg\psi \vdash \bot$ *[within 40 basic steps]*

2. Consider the propositional logic formula $\phi = p \vee (q \wedge (\neg p \vee (q \wedge p)))$, where p and q are propositions.

   (a) *[3 marks]* Convert the formula ϕ to an equisatisfiable formula ψ in CNF by introducing extra propositions as discussed in class.

   (b) *[3 marks]* Can we infer anything about the validity of $\neg\phi$, and hence, about the satisfiability of ϕ from the validity of ψ? Answers without brief justification will fetch no marks.

3. *[4 marks]* Given an arbitrary propositional logic formula ϕ, is it always possible to obtain a formula ψ in Disjunctive Normal Form (DNF), whose size is linear in the size of ϕ, and which is valid iff ϕ is valid? If your answer is in the negative, you must give a brief justification. Else, you must give an algorithm, in which you may reuse/cite any result/proof done in class.

Do NOT WRITE BEYOND THIS LINE.