CS206 Homework #1

Max marks: 50 **Due Feb 13, 2008**

- *Be brief, complete and stick to what has been asked.*
- ***Do not copy solutions from others.***

1. *[10 marks]* In a banquet hall, there are $n(> 1)$ persons including you. Each person (including you) is either (i) a *liar*, and always gives wrong answers to questions, or (ii) *truthful*, and always gives right answers to questions. Each person has an identity card that clearly indicates whether he/she is a liar/truthful. However, these identity cards are not publicly visible, and one must ask a person to show his/her identity card to find out whether he/she is a liar/truthful. You are told that the total number of liars in the hall is even. Of course, you know whether you are a liar or truthful, and this immediately tells you whether the remaining number of liars/truthfuls are even/odd.

   The banquet hall has two exit doors, numbered 1 and 2. One of these doors leads to a beautiful bed of flowers, while the other leads to a torture chamber (which of course doesn't have any flower beds). You wish to know which door leads to the bed of flowers without opening any of the doors. Every person other than you in the hall knows which door leads where, so it would be prudent to ask one or more of them about this. The catch however is that you do not know a priori who among the others is a liar and who is truthful.

   Suppose you are allowed to ask *at most* one question to every other person in the hall. Thus, you may or may not choose to ask a person. However, if you do choose to ask a person, you can only ask the following question.

   Q1: Does door 1 lead to the flower bed?

   In addition, you are allowed to see *at most* one person's identity card. However, if you see a person's identity card, you can no longer ask question Q1 to him/her.

   Use propositional logic to come up with a strategy such that you can correctly find out which door leads to the flower bed. You must provide complete justification of your strategy, and also provide a proof that your strategy always leads to the correct answer about which door leads to the flower bed.

   Hint: Treat Q1 as a proposition. In addition, consider propositions L_i, that indicate whether the i^{th} person is a liar or not.

2. *[7.5 + 7.5 marks]* Let p, q be propositions, and ϕ_1 and ϕ_2 be propositional logic formulae on p, q.

   (a) Consider the following definitions for ϕ_1 and ϕ_2

   - $\phi_1 = (p \rightarrow \neg\phi_2)$
   - $\phi_2 = (q \rightarrow \neg\phi_1)$

   Show that there are exactly two pairs of propositional logic formulae (ϕ_1, ϕ_2) which satisfy the above definitions. Also, justify your answer.

(b) If the definition of ϕ_1 above was changed to $\phi_1 = (p \rightarrow \phi_2)$, and the definition of ϕ_2 was left unchanged, is it possible to find propositional logic formulae on propositions p and q that satisfy the modified definitions? If your answer is in the negative, you must explain why. Otherwise, you must give the propositional logic formulae ϕ_1 and ϕ_2 in terms of p and q, and provide justification for your answer.

3. *[5 + 5 marks]* Prove using natural deduction the following sequents. You may not use LEM either directly or indirectly (i.e,. by giving a derivation of LEM and then using LEM):

   (a) $x_1 \rightarrow x_2 \vee x_3 \vee x_4,\ x_2 \rightarrow \neg x_1 \vee \neg x_4,\ x_3 \vee x_4 \rightarrow x_2 \ \vdash x_1 \rightarrow \neg x_4$.

   (b) $x_1 \rightarrow x_2 \vee x_3,\ x_2 \rightarrow \neg x_1 \vee \neg x_4,\ x_3 \rightarrow \neg x_1 \vee \neg x_4,\ x_4 \rightarrow x_1 \wedge x_5,\ x_5 \rightarrow x_1 \wedge x_4,\ x_1 \ \rightarrow x_4 \vee x_5 \ \vdash \neg x_1$.

4. *[7.5 + 7.5 marks]* Consider the propositional logic formula ϕ_a obtained by conjuncting all the premises of problem 3(a) above. Similarly, let ϕ_b be the propositional logic formula obtained by conjuncting all the premises of problem 3(b) above.

   (a) Apply the DPLL Algorithm to ϕ_b and find a satisfying assignment.

   (b) Construct an ROBDD of ϕ_a using the proposition ordering $x_2 < x_3 < x_4 < x_1$ and determine the total number of paths from the root of the ROBDD to its False leaf. You must clearly show the steps of constructing your ROBDD.