CS208 HW #1

Due Date: Jan 19, 2012

Be brief, complete and stick to what has been asked. If needed, you may cite results/proofs covered in class without reproducing them.

Do not copy solutions from others
Penalty for copying: FR grade

1. *[15 marks]* Consider the NFA A with ε-transitions shown in Fig. 1.

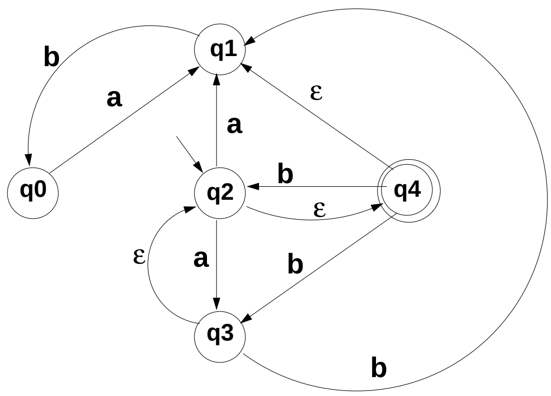

Figure 1: A non-deterministic automaton

Convert the NFA to a DFA using the subset construction method. Your solution must contain only those states in the DFA that are reachable from the initial state, and from which some final state can be reached. Any additional states in your solution will incur marks penalty.

Note we are not asking for a minimum state DFA. Your DFA must be obtained using the subset construction method, but must not contain any state that doesn't contribute to the acceptance of any string.

2. Let $A = (Q, \Sigma, Q_0, \delta, F)$ be an NFA with ε-transitions over an alphabet Σ. In each of the following subquestions, we describe how a new automaton A' is constructed starting from A, and then present a claim. You are required to either give a proof of the claim, or provide a specific counterexample that violates the claim, for each subquestion.

(a) *[5 marks]* Let $A' = (Q, \Sigma, Q_0, \delta, Q \setminus F)$.
Claim (a): $L(A') = \Sigma^* \setminus L(A)$

(b) *[5 marks]* Let $A' = (Q, \Sigma, Q \setminus Q_0, \delta, F)$.
Claim (b): $L(A) \subseteq \Sigma^* \setminus L(A)$

(c) *[5 marks]* Let $A' = (Q, \Sigma, Q_0, \delta', F)$, where $\delta' : Q \times \Sigma \to 2^Q$ is defined as: $\delta'(q, a) = Q \setminus \delta(q, a)$, for every $q \in Q$ and $a \in \Sigma \cup \{\varepsilon\}$.
Claim (c): $L(A') \cup L(A) = \Sigma^*$.

3. Let $L = \{a^n \mid i \geq 0\}$, where $a^0 = \varepsilon$ and $a^n = a^{n-1} \circ a$ for all $n \geq 1$. Thus L consists of sequences of a of all lengths, including a sequence of length 0.

(a) *[5 marks]* Let L be any finite subset of L. Show that there always exists a DFA to recognize $(L_1)^*$.

(b) *[10 marks]* Let L_2 be any infinite subset of L. Show that in this case too, there always exists a DFA to recognize $(L_2)^*$.