CS208 HW #2

Due Date: Feb 2, 2012

- ***Be brief, complete and stick to what has been asked. If needed, you may cite results/proofs covered in class without reproducing them.***
- ***Do not copy solutions from others***
- ***Penalty for copying: FR grade***

1. *[5+5+5 marks]* For each of the following languages, either prove that the language is regular (by giving an NFA/DFA recognizing the language) or prove that the language is not regular.

   (a) $\{x \mid x \in (\mathbf{0}+\mathbf{1})^*, x \neq x^R\}$, where x^R denotes the reverse of x. Thus, $01101^R = 10110$.

   (b) $\{0^{2n}1^{n+2k} \mid n \geq 0, k \in \mathbf{Z}, n+2k+1 \geq 0\}$, where $\mathbf{Z}$ denotes the set of (positive or negative) integers.

   (c) $\{w \mid w$ is the shortest binary encoding of $5k, k \geq 1\}$. Thus, the language under consideration is $\{0, 101, 1010, 1111, 10100, \ldots\}$ and does not contain strings like $00, 00101$ etc.

2. *[5+5+5 marks]* Suppose r_1 and r_2 are regular expressions over the same alphabet Σ. We say $r_1 = r_2$ to denote equality of the languages represented by r_1 and r_2. In other words, every string in the language represented by r_1 is also included in the language represented by r_2 and vice versa. For each of the following pairs of regular expressions over $\Sigma = \{0, 1\}$, either prove that they represent the same language, or give a string that is present in the language of one but not in the language of the other. In the latter case, you must also describe why your solution string is in the language of one regular expression, but not in that of the other.

   (a) $r_1 = \mathbf{1}^*(\mathbf{1}+\mathbf{0})^*\mathbf{0}^*$ and $r_2 = (\mathbf{0}^*\mathbf{1}^*)^*$

(b) $r_1 = ((\mathbf{0}+\mathbf{1})^*\mathbf{0})^*\mathbf{0}$ and $r_2 = (\mathbf{0}+\mathbf{1})^*\mathbf{0}^*\mathbf{0}$

(c) $r_1 = (\mathbf{0}+\mathbf{1})^*\mathbf{01}(\mathbf{0}+\mathbf{1})^*$ and $r_2 = \mathbf{1}^*(\mathbf{0}+\mathbf{1})^*\mathbf{0}(\mathbf{0}+\mathbf{1})^*\mathbf{1}$

3. *[7.5 + 7.5 marks]*

   (a) Give as small an upper bound as you can of the number of *distinct* languages over $(\mathbf{0}+\mathbf{1})^*$ that can be recognized by DFAs using at most n states. Your answer must be a function of n. You must also clearly explain your reasoning.

   (b) Give an example of a language that satisfies the Pumping Lemma for regular languages, but is not regular. You must clearly show why the language satisfies the Pumping Lemma. You must also argue why the language is not regular.