# CS208 HW #3 Supplement

## Due Date: Mar 12, 2012

- ***Be brief, complete and stick to what has been asked. If needed, you may cite results/proofs covered in class without reproducing them.***
- ***Do not copy solutions from others***
- ***Penalty for copying: FR grade***

1. *[7.5 + 7.5 marks]* Let $A = (Q, \Sigma, \delta, q_0, F)$ be a DFA, where $Q = \{q_0, q_1, \ldots q_r\}$. The *unique run* of $A$ on a word $w = a_1 \ldots a_k \in \Sigma^*$ is the unique sequence of states $(q_{i_0} q_{i_1} q_{i_2} \ldots q_{i_k})$ such that $q_{i_0} = q_0$ and $q_{i_{j+1}} = \delta(q_{i_j}, a_{j+1})$ for $0 \leq j \leq k-1$. We will denote the unique run of $A$ on $w$ as $\rho_{A,w}$. Note that $\rho_{A,\varepsilon} = (q_0)$. Normally, we say that the word $w = a_1 \ldots a_k$ is accepted by $A$ if the last state in the unique run $\rho_{A,w}$ belongs to the set $F$.

   In this question, we will define an alternative notion of acceptance. Let $w \in \Sigma^*$ and let $\rho_{A,w}$ be the unique run of $A$ on $w$. For each $i \in \{0, \ldots r\}$ (recall that $Q = \{q_0, q_1, \ldots q_r\}$), let $n_i(\rho_{A,w})$ denote the number of occurrences of $q_i$ in $\rho_{A,w}$. It is easy to see that $0 \leq n_i(\rho_{A,w}) \leq |w| + 1$ and $n_0(\rho_{A,w}) \geq 1$ for every $w \in \Sigma^*$.

   Let $f : \mathbf{N}^{r+1} \to \mathbf{N}$ be a $r+1$-ary function on natural numbers (consider $\mathbf{N} = \{0, 1, 2, \ldots\}$). Given $w$, $A$ and $f$, we will say that $w$ is accepted by $A$ by virtue of $f$ if $f\left(n_0(\rho_{A,w}), n_1(\rho_{A,w}), \ldots n_r(\rho_{A,w})\right) = 0$. The language accepted by $A$ by virtue of $f$ is $L(A, f) = \{w \mid w \in \Sigma^* \text{ and } w \text{ is accepted by } A \text{ by virtue of } f\}$.

   (a) Give an example of a DFA $A$ on $\Sigma = \{0, 1\}$ with no more than 4 states, and a function $f : \mathbf{N}^r \to \mathbf{N}$ (where $r$ is the actual no. of states of $A$) such that $L(A, f)$ is not regular. You must clearly explain why $L(A, f)$ is not regular for your choice of $A$ and $f$. Note that the set of final states in $A$ is of no consequence here, so you need not specify this set.

(b) Now consider the function $g : \mathbf{N}^4 \to \mathbf{N}$ given by $g(n_0, n_1, n_2, n_3) = (n_0 + n_1 + n_2) \pmod 4$. Give an example of a DFA $A$ on $\Sigma = \{0, 1\}$ with exactly 4 states, such that $L(A, g)$ is regular and infinite. You must clearly explain why $L(A, g)$ is regular for your choice of $A$. Once again, the set of final states in $A$ is of no consequence here, so you need not specify this set.

2. *[10 marks]* Consider the context-free grammar $G$ given below, where $S$ is the start symbol, $\{S, X, Y\}$ are non-terminals, and $\{0, 1\}$ are terminals.

$$
\begin{array}{lcl}
S & \to & X \cdot X \mid 0 \cdot X \cdot Y \cdot 1 \\
X & \to & 0 \cdot X \mid \varepsilon \mid 1 \cdot Y \\
Y & \to & 1 \cdot Y \mid 1 \cdot X \mid 0
\end{array}
$$

Is the language $L(G)$ regular? If so, give a DFA/NFA for $L(G)$, explaining what each state of your DFA/NFA represents (you must have a line or two of explanation for every state). Otherwise, show that $L(G)$ violates the pumping lemma for regular languages.