CS208 Quiz (Spring 2012)

Time: 60 mins **Date: April 18, 2012**

- ***The exam is open-book, open-notes, open-material-brought-to-exam-hall.***
- ***Do not copy solutions from others***
- ***Penalty for copying: FR grade (and you know we mean it!)***

MARKING SCHEME (B):

- ***This is multiple-choice type quiz. Each question has at least one, and possibly many right answers. You are required to indicate ALL correct answers for each question.***
- ***If you have indicated a wrong answer for a question, your marks for the entire question becomes zero.***
- ***If you have not indicated any wrong answer for a question, you will be awarded 2 marks for every correct answer chosen by you.***

1. Consider the CFG G given below, where A and X are non-terminals, A is the start symbol, and $\{0, 1\}$ is the set of terminals.

   $$A \rightarrow AXAX \mid 1$$
   $$X \rightarrow XX \mid 0 \mid \varepsilon$$

   Let $L(G)$ denote the language generated by the above grammar. Which of the following are true of $L(G)$? Put a "×" on the index ((a), (b), ...) of each correct answer.

   (a) $L(G)$ can be generated by another CFG that has a single non-terminal

   (b) $L(G)$ is not regular

   (c) $\{0, 1\}^* \setminus L(G)$ is not context-free

   (d) If we apply PL for CFLs to $L(G)$, the shortest string to which PL can be applied is of length 3.

   (e) There exists a deterministic PDA with 1 state and 2 stack symbols that accepts $L(G)$ by final state.

   (f) There exists a deterministic PDA with 1 state and 2 stack symbols that accepts $L(G)$ by empty stack.

2. When we say that a TM M accepts a language L by halting, we mean that if M is started with a word w on its tape, if halts if and only if $w \in L$. In class, we have seen how every binary string can be thought of as encoding a deterministic Turing machine (DTM) that accepts by halting. Recall also that with the encoding studied in class, every DTM can be encoded as a binary string with no two consecutive 0's. Similarly, every binary string can be thought of as an input string on a given (possibly different) alphabet. We will use $[\![w]\!]_{TM}$ to denote the Turing machine encoded by $w \in (\mathbf{0}+\mathbf{1})^*$. Similarly, we will use $[\![w]\!]_\Sigma$ to denote the input string encoded by $w \in (\mathbf{0}+\mathbf{1})^*$.

   Let L_1 and L_2 be as defined below:

   - $L_1 = \{w \mid w \in (\mathbf{0}+\mathbf{1})^+$ and there is at least one input string on which $[\![w]\!]_{TM}$ doesn't halt$\}$.
   - $L_2 = \{w_1 00 w_2 \mid w_1, w_2 \in (\mathbf{0}+\mathbf{1})^*, w_1$ has no two consecutive zeros, and $[\![w_1]\!]_{TM}$ does not halt on $[\![w_2]\!]_\Sigma\}$.

   Which of the following statements are true? Put a "×" on the index ((a), (b), ...) of each correct answer.

   (a) If L_1 is recursively enumerable, then L_2 is recursive.

   (b) L_1 is co-recursively enumerable

   (c) The language $L_2 = \{ww^R \mid w \in L_1\}$ is recursive. Here, w^R means the reverse of the word w.

3. For each of the following languages, indicate if they are recursive. If not, please also indicate if they are recursively enumerable (RE). Put a "×" against the correct answer for each question.

   (a) $\{w \mid w \in (\mathbf{0}+\mathbf{1})^*$ and $[\![w]\!]_{TM}$ halts within 100 steps when started with the all blank tape$\}$.
   Ans: [ ] Recursive, [ ] Not recursive, but RE [ ] Not RE

   (b) $\{w \mid w \in (\mathbf{0}+\mathbf{1})^*$, the language accepted by $[\![w]\!]_{TM}$ has < 100 words$\}$
   Ans: [ ] Recursive, [ ] Not recursive, but RE [ ] Not RE

   (c) $\{w \mid w \in (\mathbf{0}+\mathbf{1})^*, [\![w]\!]_{TM}$ accepts by halting the same language as accepted by the universal Turing Machine by halting$\}$
   Ans: [ ] Recursive, [ ] Not recursive, but RE [ ] Not RE

   (d) $\{w \mid w \in (\mathbf{0}+\mathbf{1})^*, [\![w]\!]_{TM}$ accepts $[\![w^R]\!]_\Sigma$ by halting$\}$. Here, w^R means the reverse of the word w.
   Ans: [ ] Recursive, [ ] Not recursive, but RE [ ] Not RE