CS208 Quiz #1

Time: 40 mins **Date: Jan 31, 2012**

- ***The quiz is open-book, open-notes, open-material-brought-to-class.***
- ***Be brief, complete and stick to what has been asked. If needed, you may cite results/proofs covered in class without reproducing them.***
- ***Do not copy solutions from others***
- ***Penalty for copying: FR grade***

1. *[10 marks]* Let w be a finite string over $\Sigma = \{0, 1\}$. We will use the notation $\#0(w)$ and $\#1(w)$ to denote the number of 0's and 1's, respectively, in w. We will also use EVEN to denote the set of non-negative even nos.: $\{0, 2, 4, \ldots\}$, and ODD to denote the set of non-negative odd nos.: $\{1, 3, 5, \ldots\}$.

   Let $L = \{w_1 \circ w_2 \mid \#0(w_1) \in \mathsf{EVEN} \text{ and } \#1(w_2) \in \mathsf{ODD}\}$. Give an NFA with no more than 8 states for recognizing L.

2. *[10 marks]* Let $L = \{w \mid \#0(w) - \#1(w) \in \mathsf{EVEN}\}$, where we have used the notation introduced in the previous question. In other words, the difference between the count of 0's and 1's in a string $w \in L$ must be a non-negative even no. If you think L is regular, give an NFA with no more than 8 states for recognizing L. Otherwise, prove that L is not regular.

3. *[10 marks]* Give an example of languages L_1 and L_2 over the single-letter alphabet $\Sigma = \{0\}$ such that the following conditions are simultaneously satisfied:

   - Each of L_1 and L_2 is a non-regular language.
   - $L_1 \cap L_2$ is an infinite regular language.
   - $L_1 \cup L_2 = \Sigma^*$

   You must provide brief justification of why L_1 and L_2 chosen by you are non-regular, why $L_1 \cap L_2$ is an infinite regular language, and also why $L_1 \cup L_2 = \Sigma^*$. Answers without justification will fetch no marks.