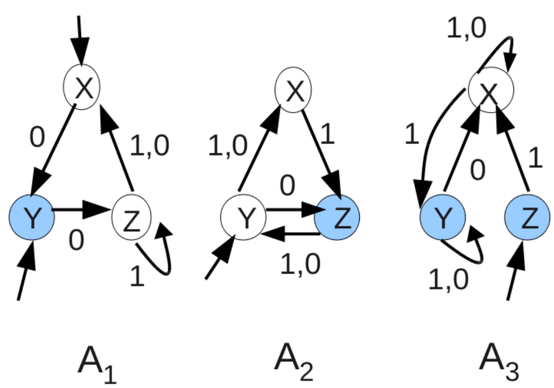
CS208 In-class Quiz 2 (Spring 2013)

Time: 55 mins **Date: Jan 21, 2013**

- ***The exam is open book.***
- ***Be brief, complete and stick to what has been asked. If needed, you may cite results/proofs covered in class without reproducing them.***
- ***Do not copy solutions from others***
- ***Penalty for copying: FR grade (and you know we mean it!)***

1. *[10 marks]* Let L be defined as $\{w \mid w \in \{0,1\}^*$ and w **cannot** be written as $u.v.u$, for any strings $u, v \in \{0,1\}^*$, where $|u| \leq 2\}$. Give a DFA or NFA (with or without ε transitions) that accepts the language L.

2. *[15 marks]* Let A_0, A_1 and A_2 be the NFAs shown in Fig. 2.



For $i \in \{0,1,2\}$, we will represent each NFA A_i as $(Q, \Sigma, Q_{0,i}, \delta_i, F_i)$, where $Q = \{X, Y, Z\}$, $\Sigma = \{0,1\}$, $\delta_i : Q \times \Sigma \rightarrow 2^Q$ is the state transition function of A_i, $Q_{0,i}$ is the set of starting states of A_i and $F_i \subseteq Q$ is the set of final states of A_i (shown shaded). Given a string $w_1 \ldots w_r \in \Sigma^*$, we will say that $w_1 \ldots w_r$ is accepted by the *interleaving of* (A_1, A_2, A_3) iff there exists a sequence of states $(q_0, q_1, q_2, \ldots q_r)$, where each $q_i \in Q$ and satisfies the following three conditions: (i) $q_o \in Q_{0,0}$, (ii)$q_i \in \delta_{(i-1) \bmod 3}(q_{i-1}, w_i)$ for $1 \leq i \leq r$, and (iii) $p_r \in F_{r \bmod 3}$. We will also say ε is accepted by the interleaving of (A_1, A_2, A_3) iff $Q_{0,0} \cap F_0 \neq \emptyset$.

Construct an NFA (with or without ε-transitions) accepting every string accepted by the interleaving of (A_1, A_2, A_3). You must clearly indicate what each state of the NFA represents, its initial and final states, its transitions and their labels. You may choose to either represent the NFA as a diagram or represent it using the five-tuple notation studied in class.

3. *[10 marks]* We have mentioned in class that the language $L = \{0^n 1^n \mid n \geq 0\}$ cannot be accepted by any NFA (although we haven't covered its proof yet). Using the above fact, show that there exists an infinite family of languages $\mathcal{L} = \{L_i \mid i \geq 0\}$ such that all the following conditions hold:
   - Each L_i can be accepted by an NFA
   - The set $\bigcap_{i \geq 0} L_i = \{w \mid w \in L_i$ for all $i \geq 0\}$ cannot be accepted by any NFA