Some Computational Problems

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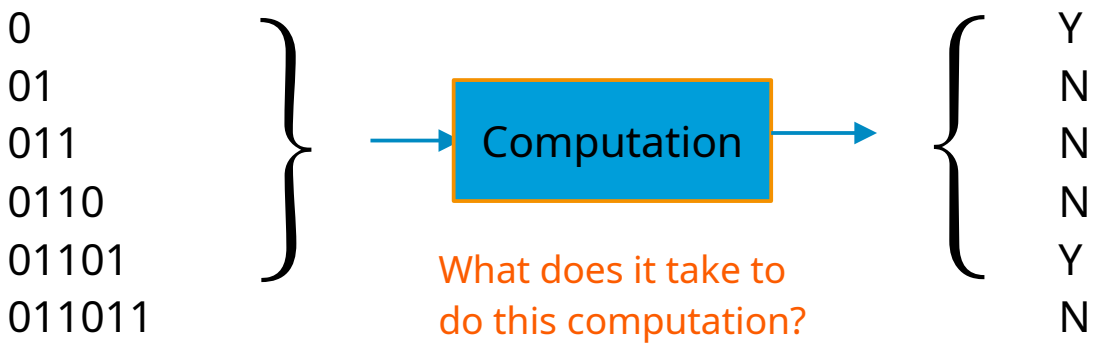
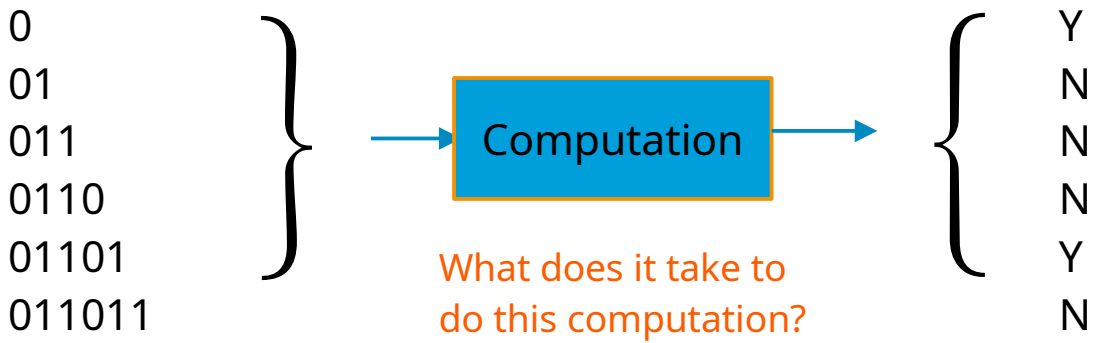
Prob 1: Counting #1‘s modulo k

- Given a stream of 0‘s and 1‘s, output whether the count of 1‘s seen so far is a multiple of k
- Example: k = 3



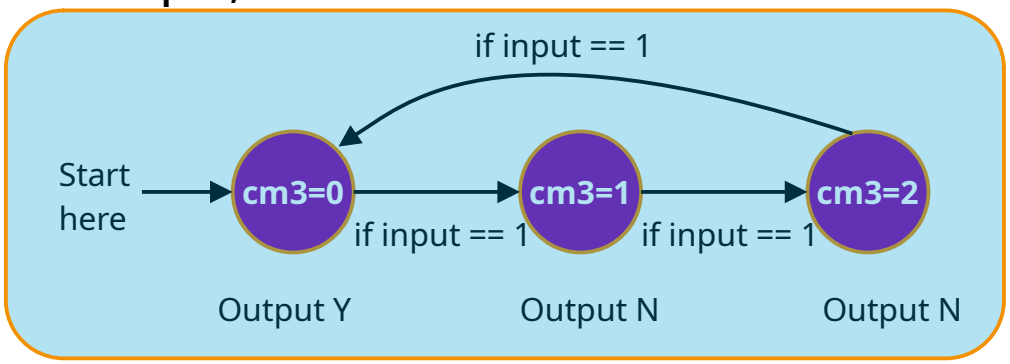
Prob 1: Counting #1's modulo k



Intuitive „code" view

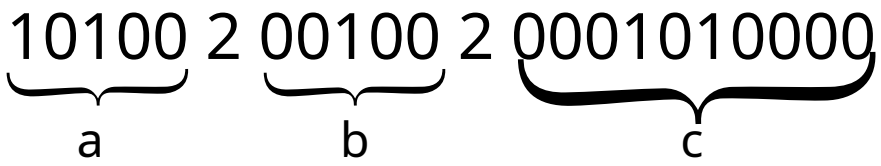
```
cm3 = 0;  // cm3 : count of 1's mod 3

Repeat until no symbols to read
   Read next input symbol;

 if (input == 1)
     cm3 = (cm3 + 1) mod 3;

 if (cm3 == 0) output Y
 else output N
```

„State transition" (automata) view
Compact, mimics intuitive code view

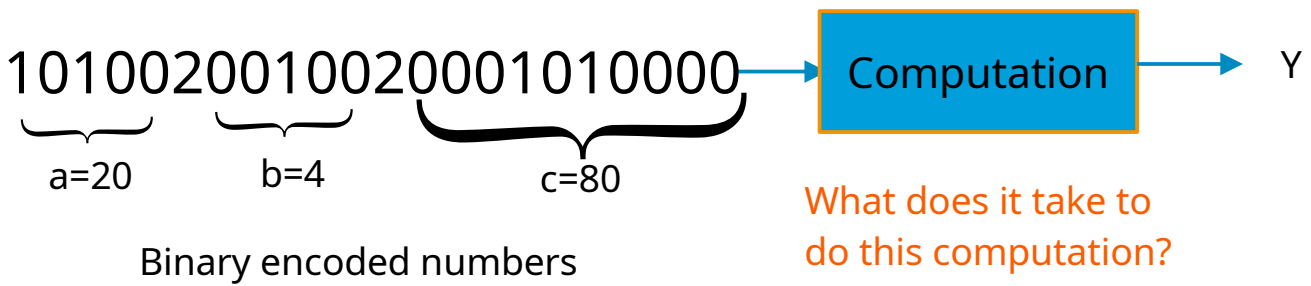


Prob 2: Checking for product triple

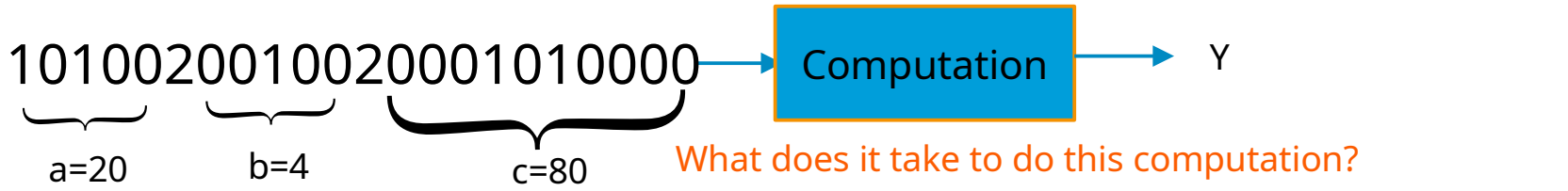
- Given a stream of 0‘s, 1‘s and 2‘s in following format



- Output **Y** if a x b = c and **N** otherwise
- Assume a and b are always k bits long and c is 2k bits long
- Example, k = 5



Prob 2: Checking for product triple

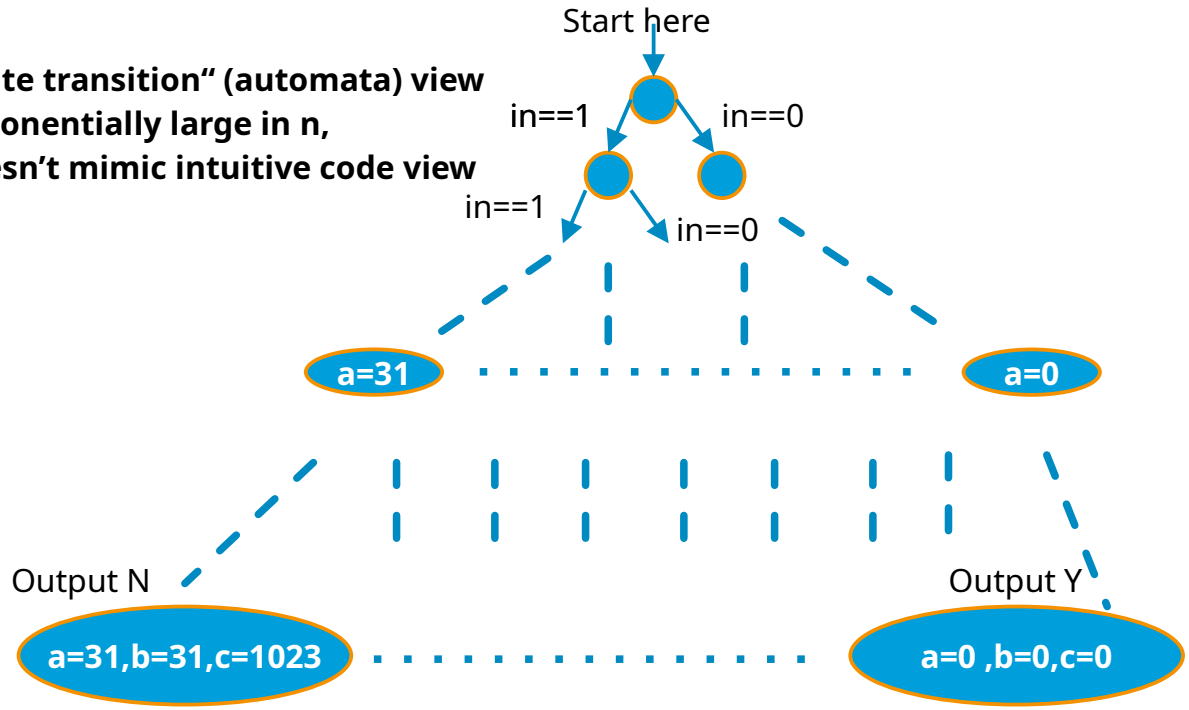


What does it take to do this computation?

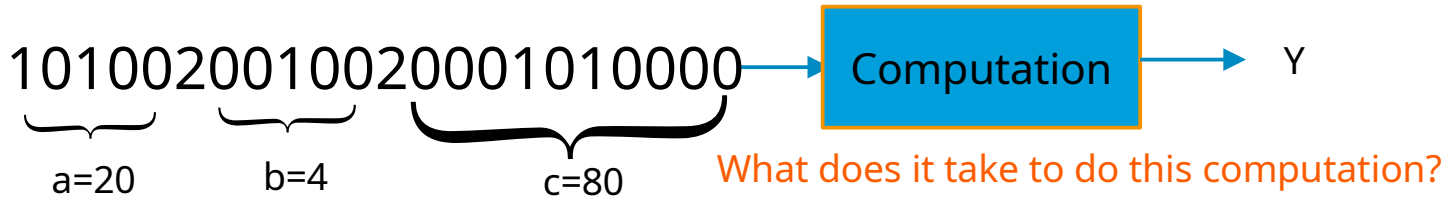
Intuitive „code“ view

```
Read first k symbols;
   Get value of a;
Read next k symbols after 2;
   Get value of b;
Read next k symbols after 3;
   Get value of c;

If (a x b == c) output Y
else output N
```

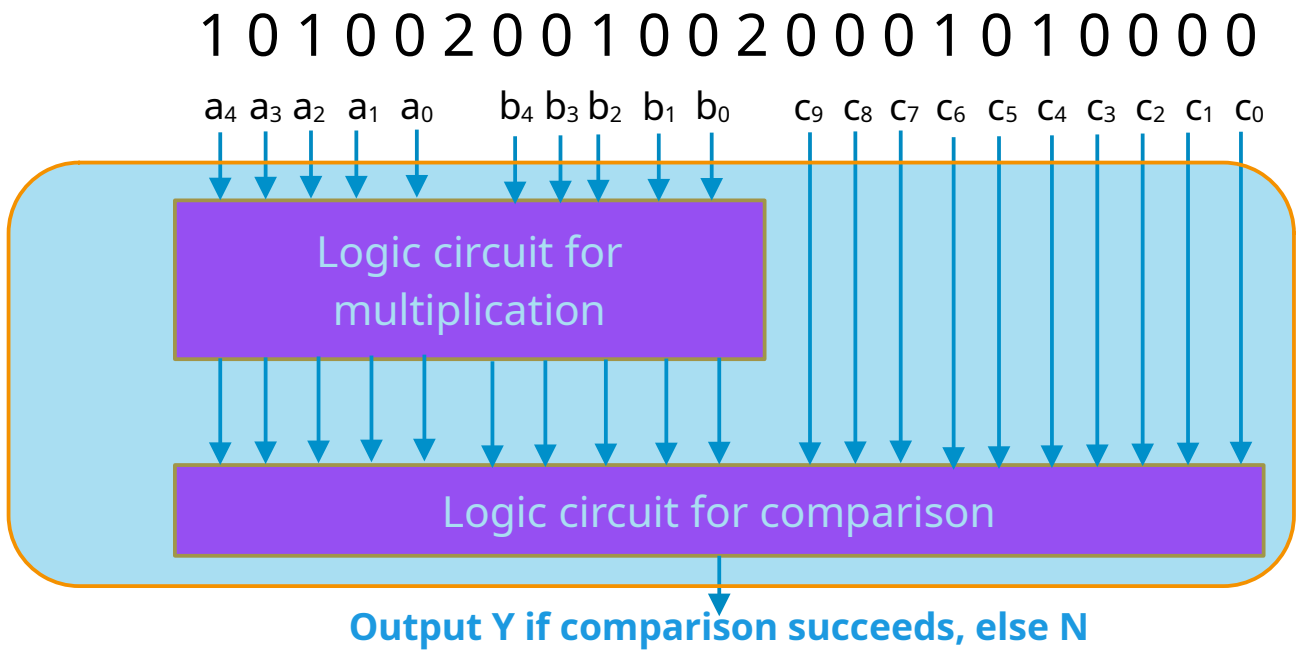
„State transition“ (automata) view
Exponentially large in n,
doesn’t mimic intuitive code view



Prob 2: Checking for product triple



Entire circuit representable by logic formula of size quadratic in n, mimics intuitive code view



Logic and Automata

- State transition centric view: Automata theory
- Logic formula centric view: Logical reasoning
- Both important, and also deeply connected
  - At times, it helps to view computation through lens of automata
    - See Prob 1 in earlier slides
  - At other times, through lens of logic
    - See Prob 2 in earlier slides