Roll No.: ______________________

CS 208X : Automata Theory and Logic (Parallel Stream) Spring 2024

End-semester Exam

Time: 14:00 - 17:00 *Max Marks: 50*

Instructions:

- ***Please write your roll number on all pages in the space provided at the top.***
- *Be brief, complete, and stick to what has been asked.*
- ***You must write your answer for every question only in the space allocated for answering the question. Answers written outside the allocated space risk not being graded.***
- ***Extra sheets are provided at the end of your answer book for rough calculations. If you need rough sheets beyond these, please ask an invigilator. You must write your roll number on all rough calculation sheets.***
- ***You must submit this question+answer book in its entirey along with any extra answer book for rough calculations (if you used one).***
- *Untidy presentation of answers, and random ramblings will be penalized by negative marks.*
- *Unless asked for explicitly, you may cite results/proofs covered in class without reproducing them.*
- ***If you need to make any assumptions, state them clearly.***
- ***Do not copy solutions from others. All detected cases of copying will be reported to DADAC with names and roll nos. of all involved. The stakes are high if you get reported to DADAC, so you are strongly advised not to risk this.***

DO NOT TURN THIS PAGE UNTIL YOU ARE ASKED TO.

Marks:

1a	1b	1c	2a	2b	2c	3a	3b	4	5a	5b

1. **A dose of logic** [10]

Consider the propositional logic formula φ obtained by conjoining (i.e. "and"-ing) the following subformulas.

Subformula name	Subformula
φ_1	$\big(x_1 \rightarrow (x_2 \vee x_3)\big)$
φ_2	$\big(x_2 \rightarrow (\neg x_3 \vee \neg x_4)\big)$
φ_3	$\big(x_3 \rightarrow (\neg x_2 \vee x_4)\big)$
φ_4	$\big(x_4 \rightarrow \neg(x_1 \vee x_2)\big)$
φ_5	$\big(\neg x_4 \rightarrow \neg(\neg x_3 \vee x_2)\big)$

(a) The formula φ is clearly not in negation normal form (NNF). Construct an NNF formula that is semantically equivalent to φ. You can proceed by constructing an NNF subformula equivalent to each subformula φ_i, and then conjoining the resulting NNF subformulas. You must show all steps of your working; solutions without steps will fetch 0 marks. [3]

(b) Convert the NNF formula obtained above into a semantically equivalent formula in conjunctive normal form (CNF). Please keep in mind that Tseitin encoding gives you an equi-satisfiable, but not necessarily a semantically equivalent formula. Hence, don't try using Tseitin encoding. Instead, use distributive laws of propositional logic. 3

(c) Find a satisfying assignment of φ by applying the DPLL algorithm to the CNF formula obtained in the previous sub-question. When applying DPLL, you must first try to simplify the formula using unit clauses, then using pure literal elimination and only after that use branching. When branching, you must branch on variables in the following order: $x_1 < x_2 < x_3 < x_4$, i.e. you must branch on x_1 if it is not assigned so far before branching on any other variable, and so on. Finally, when branching, use the 1 (or True) value of the branched variable first before using the 0 (or False) value of the same variable after backtracking. [4]
You must show all steps of your application of DPLL algorithm, showing the simplified formula at each step. You must also indicate which of {unit clause (UC), pure literal elimination (PL), branching (BR) or backtracking (BT)} you are applying in each step. Answers without clear explanations will fetch no marks.

2. **Regular (or not?) languages** [10]

   For each of the languages given below, indicate whether it is regular or not. If you think the language is regular, you have to give an NFA/DFA/regular expression for the language and argue why your NFA/DFA/regular expression recognizes all and only the strings in the given language. If you think the language is not regular, you must prove using Pumpling Lemma for regular languages that the language is not regular.

   All languages below are over the alphabet $\{0, 1\}$. Moreover, for a word $w \in \{0, 1\}^*$, we use $n_0(w)$ and $n_1(w)$ to denote the count of 0s and 1s respectively in w.

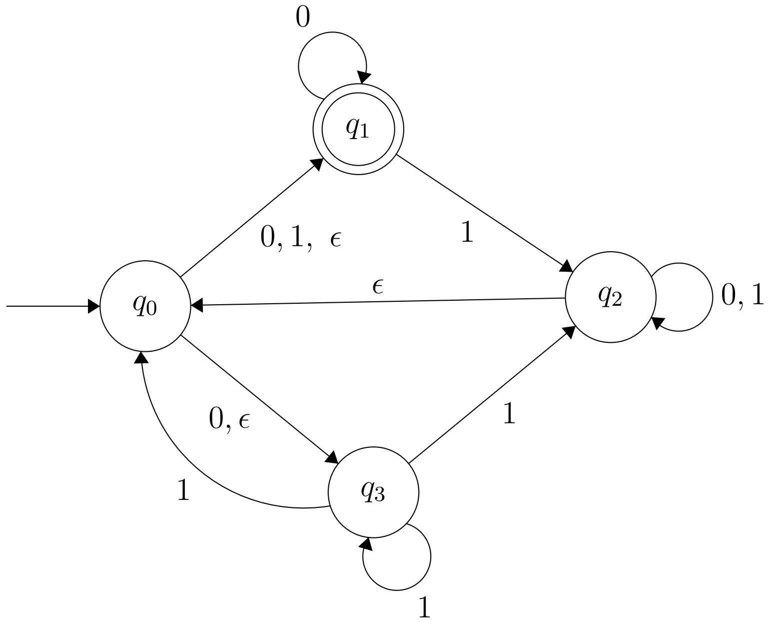
   (a) $L_1 = \{w \mid (n_0(w) + n_1(w)) \mod 3 = 0\}$ [3]

(b) $L_2 = \{w \mid \exists v \text{ s.t. } v \in \{0,1\}^* \text{ and } w.v \in L(\mathbf{0^*1^* + 1^*0^*})\}$. Here, $L(\mathbf{0^*1^* + 1^*0^*})$ denotes the language represented by the regular expression $\mathbf{0^*1^* + 1^*0^*}$. $\boxed{3}$

(c) $L_3 = \{0^n 1^m 0^n \mid m, n \geq 1\}$ $\boxed{4}$

3. **Determinizing the world** [10]

Consider the non-deterministic finite automaton A shown below. This automaton has several ε transitions.



(a) Construct a non-deterministic finite automaton that accepts the same language as A but has no ε transitions. You must clearly show the steps of your construction. Simply giving the final automaton without showing steps will fetch no marks. [5]

(b) Construct a deterministic finite automaton that accepts the same language as A by applying the subset construction. You must clearly show the subset of NFA states represented by each state in your DFA. Simply giving the final DFA without showing the subset of NFA states corresponding to each DFA state will fetch no marks. $\boxed{5}$

4. **SAT in practice** [10]

You are given a box that can hold items upto a total weight of 10 kilograms. The box has to be filled up with (possibly a subset of) 8 packets given to you. The packets are labeled $P_1, \ldots P_8$, and the weight of packet P_k is exactly k kilograms. We must fill packets in the box such that the total weight of packets placed in the box doesn't exceed 10 kilograms. Additionally, there are some (in)compatibility restrictions among packets. Specifically, packets P_1, P_4 and P_6 cannot be placed together in the box at the same time, and similarly packets P_2, P_3 and P_5 cannot be placed together in the box at the same time.

Our goal is to fill packets in the box in such a manner that no additional packet can be added to the packets already placed in the box without violating either the total weight restriction of the box, or without violating the compatibility restrictions mentioned above. Such a filling of packets in the box is called a *maximal packing*.

Let $p_i,\ i \in \{1, \ldots 8\}$ be propositional variables with the following interpretation: p_i is set to True iff packet i is placed in the box. You are required to write a propositional formula φ using the variables p_i for $i \in \{1, \ldots 8\}$ such that (a) every satisfying assignment of φ gives a maximal packing of the box, and (b) every maximal packing of the box corresponds to a satisfying assignment of φ.

In order to get marks for this question, you must give the formula φ correctly and prove that the above two conditions (a and b) are satisfied. Answers without justification will fetch no marks. If you are using additional variables, you must clearly indicate the interpretation of each additional variable beyond $p_1, \ldots p_8$.

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5. **Determinism can be costly** [10]

   Let $\Sigma_n = \{0, 1, 2, 3, \ldots n\} \cup \{\#\}$, where $n \geq 1$. Define $L_n = \{w \in \Sigma_n^* \mid$ every non # symbol that appears after the first # in w also appears before the first # in $w\}$. For example, $012\#0\#21 \in L_2$, but $01\#021 \notin L_2$.

   (a) Construct a deterministic finite automaton (DFA) for the language L_1. [4]

(b) Show that any deterministic finite automaton for the language L_n must have at least 2^{n+1} states. You must provide clear justification for your arguments; otherwise your answer will fetch no marks. $\boxed{6}$