Roll No.: ____________________

CS 228M: Logic in Computer Science Spring 2026

Midsem Exam

Time: 08:00 - 10:00 *Max Marks: 40*

Instructions:

- ***Please sit in the exact seat allotted to you in the published seating plan. Students found sitting in other seats risk their exams being nulified.***
- ***Please write your roll number on all pages in the space provided at the top.***
- ***Be brief, complete, and stick to what has been asked.***
- ***You must write your answer for every question only in the space allocated for answering the question. Answers written outside the allocated space risk not being graded.***
- ***You can use an extra answer book for rough calculations. You must write your roll number on the extra answer book if you are using one.***
- ***You must submit this question+answer book in its entirey along with any extra answer book for rough calculations (if you used one).***
- ***Untidy presentation of answers, and random ramblings will be penalized by negative marks.***
- ***Unless asked for explicitly, you may cite results/proofs covered in class without reproducing them.***
- ***If you need to make any assumptions, state them clearly.***
- ***Do not copy solutions from others. All detected cases of copying will be reported to DADAC with names and roll nos. of all involved. The stakes are high if you get reported to DADAC, so you are strongly advised not to risk this.***
- ***The exam is open book and notes. However, no electronic devices or gadgets are allowed.***

DO NOT TURN THIS PAGE UNTIL YOU ARE ASKED TO.

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1. **Warming up ...** [10]

Consider the propositional logic formula given below:

$$(x_1 \vee \neg(x_2 \vee (x_3 \wedge \neg(x_1 \leftrightarrow \top)))) \rightarrow (x_3 \wedge x_2)$$

Please take note of the parentheses structure carefully – this is important for answering the following sub-questions.

(a) Write a semantically equivalent formula in conjunctive normal form (CNF). You may simplify your formula using the following tautologies for any literal ℓ: $(\ell \vee \neg\ell) \leftrightarrow \top$, $(\ell \wedge \neg\ell) \leftrightarrow \bot$, $(\ell \vee \ell) \leftrightarrow \ell$ and $(\ell \wedge \ell) \leftrightarrow \ell$. [3]

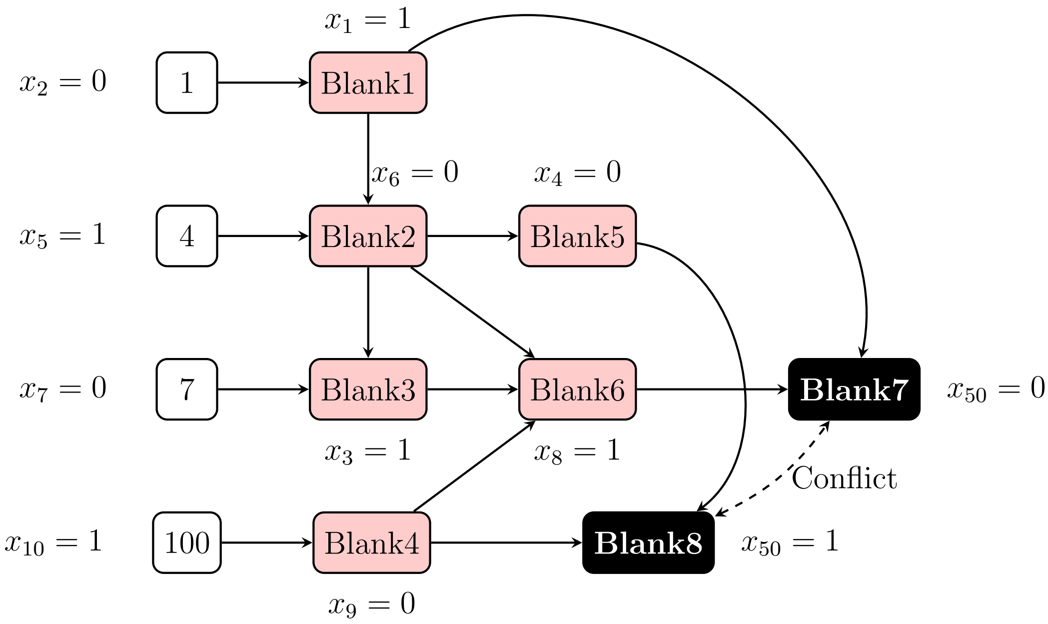
(b) Write an equisatisfiable formula obtained by applying Tseitin encoding to the above formula. You must show all steps of constructing the Tseitin encoding in order to get marks. 3

(c) Observe that the formula has three variables x_1, x_2, x_3. Show that there is an assignment of truth values to *atmost two* of these variables that renders the formula `true`, regardless of what the remaining variable(s) is/are assigned. You must indicate which variable(s) you are assigning value(s), and also what value(s) you are assigning. [2]

(d) Show that there is an assignment of values to *atmost two* of the variables x_1, x_2, x_3 that renders the formula `false`, regardless of what the remaining variable(s) is/are assigned. The variable(s) you choose to assign value(s) in this sub-question need not be the same as the one(s) you assigned value(s) in the previous sub-question. You must indicate which variable(s) you are assigning value(s), and also what value(s) you are assigning. [2]

2. **Learning from cuts** [10]

You are given a CNF formula $\varphi \equiv (\neg x_7 \vee \neg x_9) \wedge (x_3 \vee \neg x_5 \vee \neg x_7) \wedge \psi$, where ψ is itself another CNF formula. On a run of DPLL with CDCL on φ, the following implication graph is obtained on reaching a conflict. Note that in this graph, unit propagated nodes may show as shaded grey (instead of red, as shown in class) due to gray-scale printing.



[Number inside node is decision level]

(a) Fill in the correct decision levels for "Blank1" through "Blank6" in the above implication graph. [2]

Blank1:	Blank2:	Blank3 :	Blank4:
Blank5:	Blank6:	Blank7 :	Blank8:

(b) Derive a learnt clause from a cut that corresponds to the first unique implication point (1-UIP) in the above implication graph. You must clearly show your cut, identify the 1-UIP in the implication graph and write down the learnt clause. [2]

(c) Assume the DPLL+CDCL algorithm backtracks to the second most recent decision level in the learnt clause. Show that the sequence of assignments that results due to this non-chronological backtrack, and subsequent unit-propagations again leads to a conflict. You must provide your answer in the form of an implication graph that shows the new conflict that is obtained. You must label your implication graph completely (decision levels of nodes, all edges corresponding to clauses that result in unit propagation, assignments corresponding to nodes and conflicting nodes) in order to get marks. You need not color or shade the nodes. 6

3. **Are you seated in the right seat?** [10]

In this question, we wish to use propositional satisfiability solving to find if there is a way to seat students for CS228M midsem exam in a room, while respecting constraints imposed by the department. Given below is the set of constraints.

C1: There are n students enrolled in CS228M, who are eligible to take the midsem exam.

C2: The room has $4n$ seats. However, students of another course are also seated in the room for their midsem exam; hence not all seats are available for CS228M students.

C3: Seats in the room are arranged in a $n \times 4$ matrix (n rows, numbered 1 through n; 4 columns, numbered 1 through 4).

C4: No two students of CS228M must be seated in adjacent columns of the same row.

C5: No two students of CS228M who are seated in adjacent rows can sit in columns that differ by 1. Thus, if CS228M student X is seated in row 1, column 2, then no CS228M student Y can be seated in row 2, column 1, or in row 2, column 3 (rows 1 and 2 are adjacent, columns 1 and 3 differ from column 2 by 1). However, student Y can be seated in row 2, column 2 or in row 2, column 4. This is done to prevent students from peeking into the answerbooks of others seated diagonally across from them.

C6: The availability of seats for CS228M students is indicated by a $n \times 4$ matrix A of true/false values. $A[i, j] = \mathsf{true}$ if and only if the seat in the i^{th} row and j^{th} column may be used to seat a CS228M student, if all other constraints are satisfied.

Assume we have $4n$ propositional variables $a_{i,j}$, where $1 \leq i \leq n$ and $1 \leq j \leq 4$. Variable $a_{i,j}$ is assumed to be true if and only if a CS228M student can be seated in row i and column j, while respecting all constraints above.

Assume we have a formula ψ on the $a_{i,j}$ variables that evaluates to true iff exactly n of the $a_{i,j}$ variables are set to true. This formula therefore ensures that no more than n seats of the class are used for seating CS228M students.

You are required to construct propositional formulas $\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5$ (described below) such that every satisfying assignment of $\psi \wedge \bigwedge_{i=1}^{5} \varphi_i$ gives a way of seating all CS228M students for their midsem exam, while satisfying constraints C1 through C6.

(a) **Boundary case 1:** Construct a propositional formula φ_1 such that φ_1 is satisfiable iff the seating of CS228M students in row 1 satisfies all constraints. [3]
[We will assume that you can write a similar formula φ_2 for boundary case 2, which is satisfiable iff the seating of CS228M students in row n satisfies all constraints.]

(b) **Boundary case 3:** Construct a propositional formula φ_3 such that φ_3 is satisfiable iff the seating of CS228M students in column 1 satisfies all constraints. 3
[We will assume that you can write a similar formula φ_4 for boundary case 4, which is satisfiable iff the seating of CS228M students in column 4 satisfies all constraints.]

(c) **Inside case 5:** Construct a propositional formula φ_5 such that φ_5 is satisfiable iff the seating of CS228M students in row i, column j, where $1 < i < n$ and $1 < j < 4$ satisfies all constraints. 4

4. **Logical interpolation** 10

(a) Consider the propositional formulas $\varphi \equiv (x_1 \vee y_1) \wedge (\neg x_1 \vee y_2)$, and $\psi \equiv (y_1 \vee y_2 \vee z_1)$. Show that there exists a formula $\widehat{\varphi}$ that depends only on y_1 and y_2 such that $(\varphi \rightarrow \widehat{\varphi}) \wedge (\widehat{\varphi} \rightarrow \psi)$ is a tautology. You must give the formula $\widehat{\varphi}$ (1 mark) and also show that the above formula is a tautology (2 marks) using any method that is convenient to you. 1

(b) Let $X = \{x_1, \dots x_n\}$, $Y = \{y_1, \dots y_m\}$ and $Z = \{z_1, \dots z_r\}$ be disjoint sets of propositional variables. Define the *support* of a propositional formula as the set of variables that appear in the formula. Let φ be a formula with support $X \cup Y$ and ψ be a formula with support $Y \cup Z$. Suppose further that you are told that $\varphi \wedge \neg\psi$ is unsatisfiable. Prove that there exists a propositional logic formula $\widehat{\varphi}$ with support Y such that $(\varphi \rightarrow \widehat{\varphi}) \wedge (\widehat{\varphi} \rightarrow \psi)$ is a tautology. You must clearly show all steps of your reasoning in order to get marks. [7]