Roll No.: _____

CS 228M: Logic in Computer Science Spring 2026

Quiz 1

Time: 8:30 - 9:30 *Max Marks: 30*

Instructions:

- ***Please write your roll number on all pages in the space provided at the top.***
- *Be brief, complete, and stick to what has been asked.*
- ***You must write your answer for every question only in the space allocated for answering the question. Answers written outside the allocated space risk not being graded.***
- ***You can use an extra answer book for rough calculations. You must write your roll number on the extra answer book if you are using one.***
- ***You must submit this question+answer book in its entirey along with any extra answer book for rough calculations (if you used one).***
- *Untidy presentation of answers, and random ramblings will be penalized by negative marks.*
- *Unless asked for explicitly, you may cite results/proofs covered in class without reproducing them.*
- ***If you need to make any assumptions, state them clearly.***
- ***Do not copy solutions from others. All detected cases of copying will be reported to DADAC with names and roll nos. of all involved. The stakes are high if you get reported to DADAC, so you are strongly advised not to risk this.***

DO NOT TURN THIS PAGE UNTIL YOU ARE ASKED TO.

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1. **Proof systems galore!**

   Consider the CNF formula φ obtained as conjunction of clauses C_1 through C_5 in the following table.

Clause name	Clause	Remarks
C_1	$x_1 \vee x_2 \vee x_3$	clause in φ
C_2	$\neg x_1 \vee \neg x_2 \vee \neg x_3$	clause in φ
C_3	$\neg x_1 \vee x_2$	clause in φ
C_4	$\neg x_2 \vee x_3$	clause in φ
C_5	$x_1 \vee \neg x_3$	clause in φ
C_6	$x_1 \vee \neg x_2$	C_4, C_5, pivot x_3

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   (a) Show using resolution that φ is unsatisfiable. You must fill in the table with resolvent clauses in the blank spaces, and indicate in the "Remarks" column which two clauses were resolved to obtain the resolvent, and what pivot was used (see an example in C_6). To show φ is unsatisfiable, you must eventually derive the empty clause. You need not use all rows in the table.

(b) Give a natural deduction proof of $C_4, C_5 \vdash C_6$. You must clearly list every step of your proof, and number each step of your proof clearly. You must also indicate the natural deduction proof rule applied in each step, along with the previous steps from which inferences are used in applying the current proof rule. 5

2. **A new negation normal form** 10

A propositional formula φ is said to be in Decomposable Negation Normal Form (or DNNF) if the following conditions hold:

- φ is in NNF
- The tree representation of φ has the following property:
  - For every node n in the tree, let $L(n)$ represent the set of variable names in the leaves of the sub-tree rooted at n. Then for every internal node n labeled $\wedge$ with children n_1 and n_2, we have $L(n_1) \cap L(n_2) = \emptyset$.

As an example, $(x_1 \vee x_2) \wedge (\neg x_1 \vee \neg x_3)$ is not in DNNF, since x_1 appears as label of a leaf in both sub-trees representing the sub-formulas $(x_1 \vee x_2)$ and $(\neg x_1 \vee \neg x_3)$.

On the other hand $(x_1 \vee x_2) \wedge (x_4 \vee \neg x_3)$ is in DNNF, since there is no common variable name that appears as label of a leaf in the sub-trees representing the sub-formulas $(x_1 \vee x_2)$ and $(x_4 \vee \neg x_3)$. Similarly, $(x_1 \wedge x_2) \vee (\neg x_1 \wedge \neg x_3)$ is also in DNNF.

(a) Give a DNNF formula that is semantically equivalent to $(x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee \neg x_4)$. You must represent your formula as a tree, and give justification why it is in DNNF. 5

(b) A student claims that for every propositional logic formula, there is a semantically equivalent formula in DNNF. Either provide a counterexample (i.e. a formula φ and justification why there cannot be a semantically equivalent DNNF formula), or prove the claim. You must show all steps of your answer in order to get marks. 5

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3. **Tseitin comes a-calling**

   Let φ and ψ be propositional formulas, and let $\varphi^\star$ and $\psi^\star$ be obtained by applying Tseitin encoding on φ and ψ, respectively. Thus, we know that $\varphi^\star$ is equi-satisfiable with φ, and $\psi^\star$ is equisatisfiable with ψ.

   A professor claims that $\varphi \wedge \psi$ is always equisatisfiable with $\varphi^\star \wedge \psi^\star$. Either prove the claim, or show by means of a counterexample that the professor's claim is wrong.